A

TREATISE

ON

ASTRONOMY,

IN WHICH

THE ELEMENTS OF THE SCIENCE

ARE DEDUCED

IN A NATURAL ORDER,

FROM THE APPEARANCES OF THE HEAVENS

TO

AN OBSERVER ON THE EARTH;

DEMONSTRATED

ON MATHEMATICAL PRINCIPLES;

AND EXPLAINED

BY AN APPLICATION

TO

THE VARIOUS PHENOMENA.

BY OLINTHUS GREGORY,

TEACHER OF MATHEMATICS, CAMBRIDGE.

Idem, propinqua nocte, stellatas vigil
Cum vesper accendit faces,
Ut gaudet immortale mirari jubar
Terraque majores globos,
Et per cadentes intueri lacrymas
Rimosa lucis atria! CASIMIR.

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1802.

[T. Davison, White-Friars.]

Sirfojee Rajah. 1830
2036

To

CHARLES HUTTON, *Esq. LL.D. F.R.S.*

Professor of Mathematics

In the Royal Military Academy, Woolwich:

In Testimony

Of Admiration and Esteem

For the eminent Talents and Genius

He has so frequently and usefully displayed

In his public Capacity;

And for those generous and amiable Qualities

Which shed a Lustre over the Path

Of private Life;

The following Sheets are inscribed,

By his

Affectionate Friend, and obliged Servant,

THE AUTHOUR.

PREFACE.

WHEN an authour ushers a performance into the world on a subject that has been frequently treated upon, and often with considerable ability and success, it is natural for him to frame an apology for his undertaking, either by writing a panegyric upon the subject he attempts to discuss, or by pointing out those particulars in which his own work differs from others, and shewing how far there is a probability that his deviations from the usual modes of arrangement, or discussion, will be productive of advantage. The subject of the treatise now presented to the public, is ASTRONOMY, a science which needs no laboured eulogium; for its intrinsic excellence, and its attendant advantages, are such, that it has been studied, encouraged, and promoted, by the most learned and eminent men of all ages and all countries. Astronomy not only contributes to the improvement and perfection of Chronology, Geography, and Navigation, to the extension of Trade and Commerce, and the consequent comfort of life; but it is also of admirable use, in strengthening the mind by proper exercises, in arming the reason against the vanity of scepticism and the delusions of sophistry, and in adding fresh power to the native force and penetration of the understanding. Besides these benefits, which all are willing to ascribe to Astronomy, there are others which are not so universally acknowledged, yet ought not to be neglected or forgotten:—this most delightful, this noblest of the sciences, subjects, as it were, the economy of the universe to our contemplation: it enables us to obtain a nearer acquaintance with the most numerous, the most stupendous and magnificent scenes in the whole creation; to trace out their mutual intercourses, their certain and determined motions, their regular returns, their stated periods; to discover the inviolable laws which regulate the heavens, and the admirable harmony which pervades the universe: and while we are thus engaged,

while we are permitted "to scale the etherial towers, and "freely range through the celestial fields," is it possible that we should not be impressed with a sense of the unlimited power, unbounded wisdom, and infinite goodness, of the adorable Creator and Governour? The great excellence of Astronomy then, even as a promoter of morality and devotion, must be admitted, if it appear that it furnishes us with stronger arguments to prove the existence of a supreme, intelligent First Cause—shews more effectually his power and wisdom—or gives us more clear and just notions of his other attributes and perfections, than any other knowledge which is attained by the mere light of nature: and that it accomplishes this, nay, more than this, cannot fairly be denied, since many, who are no mean judges in this respect, are constantly ready to assert it, on the fullest conviction. "If we," says the pious and learned Dr. *Watts*, "look upward with *David* to the worlds above us; if *we consider the Heavens as the work of the finger of God, and the moon and the stars which he hath ordained;* What amazing glories discover themselves to our sight? What wonders of wisdom are seen in the exact regularity of their revolutions? Nor was there ever any thing that has contributed to enlarge my apprehensions of the immense power of God, the magnificence of his creation, and his own transcendent grandeur, so much as that little portion of Astronomy which I have been able to attain. And I would not only recommend it to *young students* for the same purposes, but I would persuade *all mankind* (if it were possible), to gain some degree of acquaintance with the vastness, the distances, and the motions of the planetary worlds, on the same account. It gives an unknown enlargement to the understanding, and affords a divine entertainment to the soul and its better powers. With what pleasure and rich profit would men survey those astonishing spaces in which the planets revolve, the hugeness of their bulk, and the almost incredible swiftness of their motions? When we muse on these things, we may lose ourselves in holy wonder, and cry out with the Psalmist, "*Lord, what is man, that thou art mindful of him; and the son of man, that thou shouldst visit him?*"

Since, then, the science of Astronomy is so powerfully recommended to our attention and regard, by its tendency to improve the understanding, to delight the imagination, and to ameliorate the heart; it is no wonder that publications, in which its principles have been laid down, its elements demonstrated, or its phenomena explained, have become very

numerous. The Authour of the present undertaking has no wish to depreciate the character of any of these publications, or to lower their claims upon general esteem, in order to induce the public to transfer their exclusive encouragement to his own performance; on the contrary, he cannot refrain from giving his suffrage to the real value and obvious utility of the labours of several of his predecessors in this department of science. The works of Mr. *Bonnycastle*, Mrs. *Bryan*, Mr. *Ferguson*, and Dr. *Long*, are admirably calculated to convey in a popular manner, but in a manner which often unites familiarity with elegance, a clear and correct statement of the various facts which Astronomy brings to light, and ingenious explanations of the principal phenomena of the Heavens. Among the scientific treatises, those of Dr. *Gregory*, M. *de la Caille*, and Mr. Professor *Vince*, merit distinguished approbation: Dr. *Gregory's Elements of Astronomy, Physical and Geometrical*, though published at a time when the physical principles of Newton were comparatively but little known, and where known, opposed, richly deserved the esteem in which they were held by Newton himself, as a most excellent explanation and defence of his philosophy: in almost every page they display strong traits of originality, and of ripened judgment, and will remain a lasting monument of the admirable genius and profound knowledge of their authour.—*The Elements of Astronomy* by M. *de la Caille*, have also been long admired, on account of the elegance, the accuracy, and the scientific skill with which they are exhibited: and *The complete System of Astronomy*, by Mr. *Vince*, is generally and justly acknowledged to be the most extensive and useful body of information on this science, which has yet been published in Britain. Other performances likewise, particularly those of Mr. *Emerson* and Dr. *Keill*, have much merit *. But, with the exception of the *System* of Professor Vince, none of these suit the present matured state of the science: books of astronomy must, from the nature of the subject on which they treat, have a very contracted period of utility, in comparison with that which is allotted to works on pure and abstract science: the latter may be advanced to a very high degree of perfection by the exertions of an individual, and may

* The Authour speaks here of Astronomical works published in England; for the excellent ones published in France, and not translated, are by no means so generally known in this country as might be wished; indeed, most of them are so expensive, that there is but little prospect of their obtaining a place in the library of every mathematician.

obtain a celebrity which, after the lapse of ages, shall have suffered no diminution; but the former depend upon circumstances, over which a single man, though of gigantic intelligence, has no control. Astronomy is "a science which has increased not only by the accumulation of its own revenues, but by the tributary contributions of other sciences and arts;" * and is, therefore, receiving constant accessions from almost every quarter: fresh inventions in mechanics, more perfect instruments in optics, ingenious improvements in analysis, more simple formulæ in calculation, more profound acquaintance with physical principles; all contribute to the real extension of Astronomy, by facilitating and rendering more accurate the necessary computations, and by giving precision and success to every diligent and judicious observation. It is, hence, manifest that treatises upon this science, however accurate in theory, however ingenious in their arrangement, however copious and clear in the developement of principles, or the elucidation of appearances, cannot alone continue long in use; but must give way to others which are enriched with the result of the most successful series of observations, and the newest and most brilliant discoveries. The only scientific work on Astronomy, published in this kingdom, in which the new improvements and discoveries are incorporated in a manner adequate to their real importance and value, is that by Professor Vince, before mentioned: but it is not to be expected that all who are desirous of increasing their astronomical knowledge, by the result of the laudable exertions of *Bode*, *Herschel*, *La Lande*, *Maskelyne*, *Zach*, and others, will have it in their power to consult so voluminous a work as the Professor's; though even that would be far more practicable than to have recourse to the numerous volumes of *Memoirs*, *Acts*, *Transactions*, &c. through which this valuable information is scattered. From these considerations, the Authour of the following sheets has been induced to endeavour to supply, what is thought by many besides himself to be a deficiency, and to lay before the public a treatise on Astronomy, which shall in some measure correspond with the advanced state of the science, and at the same time occupy an intermediate station, between those which give merely a concise view of the elements, and those which by their voluminousness and consequent expensiveness, are prevented from obtaining a general circulation among students of mathematics.

* *Monthly Review*, vol. xxvii. N. S.

In the prosecution of the Authour's design, the chief difficulty consisted in adopting a proper mode of arrangement, and in allotting to each of the various celestial bodies, and its phenomena, a just and proper share in the work. With respect to arrangement, it appears to the Authour to have been customary for the great facts of Astronomy to be stated too early, that is, long before the mind is prepared to give them admission. The truths in Astronomy, as laid before us in the System of Copernicus, although they cannot but afford delight by their simplicity, uniformity, and harmony, yet are presented in a garb so totally different from that which nature *appears* to wear, that many persons are unwilling to yield them any farther assent, than that which is grounded upon the opinion of others, who, they suppose, from the unremitting attention they have paid to the subject, may probably be right. According to this mode of arrangement, the student must submit to two operations of intellect, both, in some measure, painful; he must not wait for the association between the ideas suggested by the propositions set before him, and the sensations or internal feelings which accompany his ideas of truth, though this association lies at the foundation of rational assent; but he must give assent at once, to that which is contrary to the evidence of his senses; and then, he must trace out the agreement of what he reluctantly assented to with the phenomena of nature. Instead of a process like this, is it not much more natural, and much more satisfactory to the mind, to develope gradually the real system, as far as possible, from our observations upon its component parts? And with respect to the assignment of the separate portions into which the consideration of the heavenly bodies may be divided, the Authour cannot help thinking, that many have been seduced, by contemplating the beauty and regularity which are manifest in the order and motions of the primary and secondary planets, to give to the fixed stars an inferiour place in the science, to conceal them as it were in a corner, as though they were the most insignicant bodies in the universe, and of no importance in the study of Astronomy; whereas, according to the just remark of Dr. *Keill*, "It is upon the observation of the fixed stars, "as upon immoveable pillars, that the whole science of As-"tronomy is erected, and by them it is sustained."

With these views the Authour could not avoid attempting the success of a different kind of arrangement, such as appeared to him more consistent with the fabric of the universe, more congenial to the satisfactory operations of the

reasoning powers, and, he hopes, more calculated to disseminate a knowledge of the science, and to stamp conviction upon the minds of ingenuous enquirers. The nature of the plan he has adopted may be understood from the following concise account of it:—The work commences with a determination of the figure and dimensions of the earth; which is followed by an explanation of terms relating to some imaginary points, lines, and circles on the earth, and their corresponding ones in the heavens: the apparent diurnal motions of the heavenly bodies are then briefly described, the diurnal and annual apparent motions of the sun are more particularly pointed out, and the method of ascertaining the situation of the ecliptic (or circle, in which the sun's annual motion appears to be performed) is explained. This is followed by an elucidation of the seasons, a determination of the length of the year, and an account of the precession of the equinoxes: these are succeeded by a description of the methods by which the relative situations of the fixed stars have been ascertained, the nature and necessity of the artificial distribution of them into constellations is shewn, and an enumeration of the constellations, and the most noted stars in each, is given. The Authour then explains the nature of parallax, refraction, and the equation of time, since the corrections depending upon them are of so much consequence; and exemplifies the use of that part of astronomy which determines the apparent motion of the sun, and the relative situation of the fixed stars, by a collection of problems, in which are given the methods of determining the rising, culminating, setting, &c. of both the sun and the stars: this part of the work includes as much of the science as could be known previous to the discovery of the planets, or the determination of the orbit and motions of the moon. After this the astronomy of the planets is commenced: the most striking of their apparent motions are described, and the description is followed by a concise sketch of the most celebrated systems which have been invented to account for the various phenomena; and reasons are assigned for assuming the system of Copernicus, as improved by Kepler and Newton. The theory of apparent motions is then laid down, and applied to the phenomena of the planetary motions: the law is shewn by which the planets are retained in their orbits, and its conformity with the law of gravity is rendered obvious. To this succeeds the determination of the orbits of the planets, and the various elements of their theory, from observation: in order to effect this in the most natural way, the situation and

magnitude of the earth's orbit are first established, as a proper basis for those geometrical operations which lead to the determination of the orbits of the other planets. The apparent and real diameters, the times of rotation of the sun and planets, and the inclinations of their axes, are then ascertained; and the illustration of their phases, stations, and other appearances, is completed. These are followed by the astronomy of the moon and satellites, and the ring of saturn —by the doctrine of solar and lunar eclipses, and occultations of fixed stars by the moon—by an explanation of the nature of the transits of mercury and venus over the sun's disc, and the method of deducing the sun's parallax from observations on these transits. After these are given three chapters, on the astronomy of comets, the aberration of light, and the determination of terrestrial latitudes and longitudes. In the last chapter, the contemplation of the fixed stars is resumed; an enquiry is made into their distances, magnitudes, nature, number, and motion; and this naturally suggests some reflections on the immensity of the universe, and some arguments to prove the existence and attributes of the great First Cause: with which the treatise concludes. Throughout the whole, it is supposed that the reader has obtained a previous knowledge of the principles of algebra, plane and spherical geometry and trigonometry, conic sections, mechanics, optics, and the projections of the sphere: the doctrine of fluxions is only made use of in one or two instances, and those respecting matters of mere curiosity.

The Authour is aware, that plans of a very different nature from that which he has adopted might have been resorted to with a considerable share of success; but he was induced to proceed in the manner he has just described, by a hope that he should thereby be enabled to develope the principles, discover the facts, and establish the prevailing system of astronomy, without requiring the assent of the student to what was not previously shewn to be consistent with truth, or the allowed principles of subsidiary branches of knowledge. In order to prevent the admission of impossible or contradictory suppositions, or to avoid that errour which, it is to be lamented, is sometimes found, even in the writings of mathematicians, namely, that of reasoning in a circle, it is universally allowed to be a good rule, not to suppose any thing to be done, or the existence of any thing to be established, until the mode of accomplishing the former, or the great probability of the existence of the latter, is satisfactorily shewn. To this rule it is, perhaps, impossible entirely to adhere, since

even *Euclid*, that most accurate and rigorous geometrician, was sometimes obliged to deviate from it; yet, as it is certainly far better to keep it in view than totally to disregard it, the Authour has set it constantly before him, and has, he believes, departed from it but in very few instances. He acknowledges that he has, in all his directions for determining the theory of the sun and planets, supposed the latitude and longitude of the place of observation to be known: but this, he conceives, will be readily granted him, since something must be assumed, and these particulars are ascertained, with tolerable accuracy, in most places where his treatise is likely to be circulated.

Besides the determination of the orbits, distances, magnitudes, and motions, of the heavenly bodies, and other particulars which belong to the elements of astronomy, the Authour was desirous of incorporating in his performance various problems that are intimately connected with the subject, and that may be of real advantage; and he has, in a few places, inserted problems, more on account of their curiosity, or the entertainment they afford, than their actual utility. He has also taken the liberty to insert one or two erroneous methods of performing problems, and then to shew their fallacy; but this is only done where it appeared absolutely necessary, in order to caution the student against the adoption of rules which are strongly recommended, either by their apparent simplicity and accuracy, or the celebrity of those authours who have made use of them.

As far as the Authour's judgment enabled him to perform it properly, he has assigned to every part of astronomy its due proportion in his treatise, and he shall be happy if it be found that he has discussed the various subjects which came under his notice, with accuracy and perspicuity; but, as he is conscious that some persons will wish for more profound or more extensive information, on many points, than could possibly be given within the bounds he has prescribed himself, he has frequently referred to publications, where the thirst of the ardent enquirer after knowledge may be more completely satisfied.

It would ill become the Authour to presume that his work is without faults: he has, it is true, used every exertion to avoid errours; still he is fearful they will be met with too frequently. He has only one apology to offer, but that will, he hopes, ward off the severity of censure: the treatise was composed during the short intervals of leisure which could be snatched from the employment of a large school; an em-

ployment which requires the Authour's persevering attention for more than eight hours in the day, and leaves him scarcely leisure sufficient to remove, by exercise, the injury done to health by such close confinement. Nothing but an earnest desire to promote the cause of science could have tempted him to relinquish the advantages of exercise, and appropriate his leisure to the composition of this treatise; and if, through the want of either time or ability, he has failed of accomplishing the purpose he had in view, he trusts it will be forgiven, when he assures the public he should never have attempted to supply the want of a work of this kind, had it appeared probable that any other person would undertake it speedily.

The Authour cannot conclude this Preface before he has acknowledged his obligations to preceding writers on Astronomy, from whose labours he has, in many respects, derived considerable benefit: indeed, he is free to confess, that his treatise has few claims to originality, except in point of arrangement; and, if the plan he has pursued be, on the whole, productive of advantage to the student or the science, his grand object will be attained. Nevertheless, as he is solicitous to improve his work as far as possible, he earnestly intreats that those who discover any errours therein, either in method, in principle, or in computation, will have the goodness to make them known to the Authour, that they may be corrected in a future edition, should the performance be so far approved as to render another edition necessary.

Cambridge, May 20, 1801.

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A

TREATISE

ON

ASTRONOMY.

CHAPTER I.

On the Figure and Dimensions of the Earth.

ART. 1. TO determine the distances, magnitudes, and orbits, of the heavenly bodies; to describe their various apparent motions, and explain them by shewing their dependence upon the real motions from which the appearances result; to exhibit the laws by which these bodies are regulated; and to trace the relations and dependences of these laws so far as we are enabled to develope them, is the business of that branch of natural philosophy which is called ASTRONOMY: a science which, from the certainty and evidence of its demonstrations, from the great and manifest advantages with which it furnishes us, and from the vast amplitude and sublimity of that universe which it proposes for our contemplation, is admirably calculated

erish a love for the native elegance of truth, to stimulate to diligent enquiry and useful invention, and to ravish the mind by displaying the harmony of nature, and pourtraying, as it were, in living characters, the wisdom and power of the ALMIGHTY FRAMER OF ALL.

2. The present undertaking will be confined in its object, chiefly, to that part of astronomy which directs us how to ascertain, from properly conducted observations, the magnitudes, relative and absolute situations, and motions, of the heavenly bodies: *Physical Astronomy*, or that part where the nature of these bodies is examined, the causes of their motions investigated, and the laws by which they appear to be regulated, minutely explored, will be but occasionally touched upon; and references will be made to those authors who have given the most accurate, profound, and satisfactory information, on that part of the science.

3. The earth being the place from whence we behold the heavenly bodies, and the appearance of them being manifestly different at different parts of it; the inhabitants of Russia, for instance, observing the sun, moon, and stars, to rise and set at different times and situations from what they would appear to the inhabitants of New Holland; and the cause of this variation in appearances being obviously the figure of the earth; an astronomer, therefore, who rightly directs his enquiries, will first endeavour to determine *the shape and magnitude of the earth.*

4. In early times, as might naturally be expected, many fanciful and absurd notions respecting the figure of the earth prevailed; some of these were adopted because they agreed with the slight and inadequate observations of the rude and unlearned; and others represented this matter in the way which best accorded with their preconceived opinions in philosophy or religion; they " divined Nature, instead of interro-

"gating her by experiments; they constructed the "universe, when they should have observed it;" and, as the ancient sage remarks, "sought truth in their "own little worlds, not in the great world." Thus, according to *Cosmas Indopleustes*, the earth was an immense plane, extending much farther in length than in breadth, and environed by an unpassable ocean. He placed a huge mountain towards the north, around which the sun and stars performed their diurnal revolutions; and from the conical shape which he ascribed to it, with the oblique motion of the sun, he accounted for the inequality of the days and the variation of the seasons. The vault of heaven, as he conceived, leaned upon the earth extended beyond the ocean, being likewise supported by two vast columns: beneath the arch, angels conducted the stars in their various motions: above it were the celestial waters, and over all he placed the supreme heavens...... But, quitting these fictions of the imagination, let us endeavour to discover the real shape of the earth by means of such arguments as naturally result from general observation, and universally admitted facts.

5. It is an allowed principle in optics, that amongst objects at equal distances (supposing them to be equally illuminated, and that there are no intervening obstacles) those which are of the greatest bulk may be the most clearly discerned, and *vicè versa;* and that some objects of considerable magnitude may be very conspicuous, whilst smaller objects of similar materials, at an equal distance, on the same plane, cannot be seen. Now, it is frequently obferved by mariners when they approach the shores of countries, *wherever situated*, that the points of high rocks, steeples of churches, and other thin but lofty objects, come in sight much sooner than houses and other buildings and objects, of greater magnitude but less height. In like manner, by persons on the shore, the masts and rigging of an approaching ship are discerned some

time before the hull and lower parts of the vessel, though much larger, come into view. Seamen, it is well known, frequently discover distant lands from the tops of a ship's masts, long before they are visible to those who stand upon deck. These circumstances, and many others of a similar nature, immediately suggest the idea that the earth is not a vast plane interspersed with hills, dales, and seas, but of some curvilinear figure, the convexity of which rises up and prevents the prospect of distant objects. And since circumstances alike both in kind and degree, have thus been observed on all known parts of the earth, it is natural to infer that it is in the shape of a solid of regular and equal curvature, or that it is a sphere. The experience of those who are employed in the business of levelling, tends greatly to confirm the justice of this inference; for, if the earth be a sphere, two or more places are on a true level when they are equally distant from its centre; but the line of sight, or apparent level, will be a tangent to its surface, and consequently different parts of that line will be at unequal distances from the centre. Thus, in fig. 1, Pl. I. A D G, or a curve parallel thereto, will be the line of *true level;* and, supposing the operation of levelling to commence at the point D, the tangent D H I will be the line of *apparent level.* Now, it is constantly found by persons engaged in this part of surveying, that in levelling the bottoms of canals to convey water to distant places, the allowance which they are obliged to make for the differences F H and G I between the apparent and true level, at the distances D F and D G, agrees exceedingly nearly with what would be required were the earth an exact sphere. It is also found that the distance A D, to which a person can see from the top of an eminence A P, corresponds very accurately with the distance as calculated on the supposition of the earth being a sphere of a known diameter. To these observations we must likewise add, that

several navigators have sailed quite round the earth; not in an exact circle it is true, for this the winding of the shores would not admit of; but going in and out as the shores happened to lie, they have held on the same course, and arrived at their native country on a different side from that on which they first began their voyage: thus *Ferdinand Magellan*, setting out on the west side of Spain, continued shaping his course westward, till he returned home on the south-eastern side of Spain; and thus also have *Drake*, *Dampier*, *Cooke*, and others, circumnavigated the earth: and when, in addition to all these facts, it is recollected, that all the rules of navigation are conformable to the opinion of the earth being nearly globular, and that these rules never lead the mariner into material error, even in the longest and most complicated voyages; these well-known circumstances, without adducing others, though others equally or more forcible might be adduced, must sufficiently establish the belief in the mind of every impartial and competent judge, that the earth deviates but very little in its form from that of a sphere.

6. Were the earth exactly of this form, if it were cut by a plane passing through the centre, the section would be a complete circle, and then its diameter might be ascertained by means of accurate instruments for taking angles, in the following manner: Suppose the exact height A P (fig. 1, Pl. I.) of a mountain standing near the sea (the Pike of Teneriffe, for instance) was known to be four English miles, and that the angle A P D made by a plumb-line and a line P I conceived to touch the surface of the water in the farthest visible point D, was found to be $87° 25' 55''$; then, since A E drawn perpendicular to A P is equal to E D, as is known from the principles of geometry, it would be, as radius : tangent of A P E : : A P (4) : A E = E D = 89·184; and as radius : secant of A P E : : A P : P E = 89·274: whence P D = 178·458. Lastly, in the triangle C P D,

as radius : tangent of A P D : : P D : D C = 3978·9, the double of which is 7957·8, the earth's diameter.

7. But, since a meridian (Art. 18.), or any other circle on a sphere, may be conceived to be divided into 360 equal parts, called degrees, and these into minutes and seconds, as explained by the writers on trigonometry, the circumference of the earth, and thence its diameter, may be determined by measuring the length of a degree on the meridian or any other great circle (Art. 12.). To perform this important problem, there have been various methods invented by different philosophers of early and later times; one of these methods, which unites considerable accuracy with great facility, will be readily understood from fig. 2, Pl. I. where P B and S T represent two mountains or very high buildings, the distance P S between which must be very nicely determined by longimetry: then, by measuring the angles R B T and R T B with an accurate instrument, their sum taken from 180° leaves the angle B R T, which is measured also by the arc P S; whence P S is known in parts of the whole circle. Thus, if the angle B T R be 89° 45′ 32″, the angle T B R 89° 54′ 28″, and the distance P S $23\frac{1}{15}$ English miles; then the angle R or arc P S being equal to $180° - \overline{89° 45' 32'' + 89° 54' 28''} = 20'$, it will be, as 20′ : 60′ or 1° : : $23\frac{1}{15}$: $69\frac{1}{5}$ English miles, length of a degree. Hence, the circumference of the earth is (according to this example) 24912 miles, and its diameter nearly 7930 miles..... A material advantage attending this method is, that there is no occasion to measure the altitudes of the mountains, an object which can seldom be attained without considerable difficulty.

8. The method which is given above is, it must be confessed, as well as all the other methods which aim at the measurement of a degree without having recourse to the heavenly bodies, liable to some inac-

curacy: for, by reason of the changes in the state of the atmosphere, distant terrestrial objects never appear in their true places; they always seem more or less elevated or distant, according to the nature of the season, and the time of the day. On this account—and because it could not escape observation, that as persons changed their situation on the earth by moving towards the north or the south, the stars and other heavenly bodies either increased or decreased their apparent altitudes proportionally — the measurement of a degree was attempted, even by the earliest philosophers, by means of known fixed stars; (Art. 27.). Every person who is acquainted with plane trigonometry will admit, that the distance of two places, north and south of each other, may be accurately measured by a series of triangles: for if we measure the distance of any two objects, and take the angles which each of them make with a third, the triangle formed by the three objects will become known; so that the other two sides may be as truly determined by calculation, as if they had been actually measured. And by making either of these sides the base of a new triangle, the distances of other objects may be found in the same manner; and thus by a series of ti iangles, properly connected at their bases, we might measure any part of the circumference of the earth. And if these distances were reduced to the north and south, or meridian line (Art. 18.), and the altitude of some star was measured at the extremities of the distance, the difference of the altitudes would be equal to the length of the grand line in degrees, minutes, &c. whence the length of a degree would be known. This method was, we believe, first practised by *Eratosthenes* in Egypt; and has been frequently used since with greater and greater accuracy, in proportion as the instruments for taking angles became, by gradual improvements, more exact and minute.

9. By this method, or some others not widely different, and which it is needless here to explain, the length of a degree has been measured in different parts of the earth; the results of the most noted of these admeasurements it may be proper to give.

Snell found the length of a degree by two different methods: by one method he made it 57064 Paris toises, or 342384 feet; and by the other 57057 toises, or 342342 feet.

M. *Picard*, in 1669, found by mensuration from Amiens to Malvoisin, the quantity of a degree to be 57060 toises, or 342360 feet; being nearly an arithmetical mean between the numbers of Snell.

Our countryman *Norwood*, about the year 1635, by measuring between London and York, determined a degree at 367196 English feet, or 57300 Paris toises, or 69 miles 288 yards.

M. *Muschenbroek*, in 1700, with a view of correcting the errors of Snell, found by particular observations that the degree between Alcmaer and Bergen-op-zoom contained 57033 toises.

Messrs. *Maupertuis*, *Clairaut*, *Monnier*, and others from France, were sent on a northern expedition, and began their operations in July 1736; they found the length of a degree in Sweden to be 57439 toises, when reduced to the level of the sea. About the same time Messrs. *Godin*, *Bouguer*, and *Condamine*, from France, with some philosophers from Spain, were sent to South America, and measured a degree in the province of Quito in Peru; the medium of their results gives about 56750 toises for a degree.

M. *de la Caille*, being at the Cape of Good Hope in 1752, found the length of a degree on the meridian there to be 57037 toises. In 1755 father *Boscovich* found the length of a degree between Rome and Rimini in Italy to be 56972 toises.

In 1764, *F. Beccaria* measured a degree near Tu-

rin; from his measurement he deduced the length of a degree there 57024 toises. At Vienna the length of a degree was found 57091 toises.

And in 1766 Messrs. *Mason* and *Dixon* measured a degree in Maryland and Pennsylvania, North America, which they determined to be 363763 English feet, or 56904½ Paris toises.

10. The differences between the measure of a degree on the meridian, at different parts of the earth, being in some cases considerable, must lead us to suppose that the earth is not strictly spherical; and indeed it was the idea that the earth deviated from a sphere, that occasioned a degree to be measured in so many different places: for M. *Richer*, in a voyage made to Cayenne, found that the pendulum of his clock did not vibrate so frequently there, as it did when at Paris; but that it was necessary to shorten it by about the eleventh part of an inch to make it vibrate in exact seconds. From this circumstance, though apparently trifling, resulted more precise knowledge respecting the figure of the earth: for, since it is evident from the nature of pendulums, that where they perform their vibrations slower, the velocity of the descent of bodies by their gravity is slower in the same proportion; and since a difference in the force of gravity was supposed to indicate an alteration in the distance from the centre of the earth, it was concluded that those parts where the pendulum vibrated most slowly were farthest removed from the centre; and, on the contrary, where the vibrations were quickest, the distance from the centre was concluded to be least. Therefore, from observations on the motions of pendulums at different parts of the earth, properly combined with the mensuration of a degree at the same places, the figure and dimensions of the earth are at length ascertained, with probably as much accuracy as the nature of the case will admit of. The figure is now supposed to

correspond very nearly with that of a spheroid generated by the motion of a semi-ellipsis about its minor axis; and the two axes, as calculated by the principles of conic sections from the different measurements of a degree, are found to be 7940 and 7977 English miles respectively; being nearly in the ratio of 214 to 215.

The principles and nature of the calculation may be explained by this analysis: Let a P A p (fig. 1, Pl. II.) be an ellipsis representing a section of the earth through its shorter diameter P p, the greater diameter being A a; let E and F be two places, where the measure of a degree has been taken; these measures are proportional to the radii of curvature in the ellipsis at those places; and if C Q, C R, are conjugates to the diameters, the vertices of which are E and F, C Q will be to C R in the subtriplicate ratio of the radius of curvature in E to the radius of curvature in F (as is proved by the writers on conics), and therefore in a given ratio one to the other. Then Q V and R Z S being drawn parallel to P p, and Q X Y W parallel to a A, the angles Q C P, R C P, are the latitudes (Art. 19.) at E and F; so that these angles and the ratio of C Q to C R being given, the rectilinear figure CVQXRY is given in species; and the ratio of $VC^2 - ZC^2 (= QX \times XW)$ to $RZ^2 - QV^2 (= RX \times XS)$ is given, which is the ratio of VC^2 to CP^2: consequently the ratio of C A to C P is given. Hence, if the sine and co-sine of the greater latitude be each augmented in the subtriplicate ratio of the measure of the degree in the greater latitude to that in the less, then the difference of the squares of the augmented sine, and the sine of the less latitude, will be to the difference of the squares of the co-sine of the less latitude and the augmented co-sine, in the duplicate ratio of the longer to the shorter diameter. For C q being taken in C Q equal to C R, and q v drawn parallel to Q V; C v and q v, C Z and Z R, will be the sines and co-sines of the respective

latitudes to the same radius; and C V, V Q, will be the augmentations of C v and $v q$ in the ratio named. . . . Then, if an angle be taken, the tangent of which shall be to the tangent of either latitude, as the less axis to the greater, the measure of a degree in that latitude will be to the measure of a degree at the equator (Art. 16.) in the triplicate ratio of the co-sine of this angle to the co-sine of the latitude. Thus, E being one of the places measured, the tangent of the angle Y C l will be to the tangent of Y C Q, the latitude of the point E, as Y l to Y Q, that is, as the less axis to the greater; and C Q : C l : : sine C l Y : sine C Q Y : : co-sine l C Y : co-sine Q C Y; C Q^3 being to C l^3 (C P^3) as the radius of curvature in E to the radius of curvature in A; that is, as the measure of a degree in E, to the measure of a degree in A: and thus, not only the ratio of the diameters, but their real lengths may be deduced from any two measurements given.

The same ratio is also deduced from the experiments on pendulums, provided it be allowed that the earth be a true spheroid, which can only take place in the case of a uniform gravity in all parts of the earth. However, we may now look upon the dimensions and shape of the earth as *known*; since they are determined with an accuracy amply sufficient for all practical purposes in either Navigation or Astronomy *.

* After the greater part of this treatise was written, the author perused the learned *Bishop Horsley's Elementary Treatises on the Fundamental Principles of Mathematics*: at the end of theſe Treatises, there is a ſmall tract on *The Figure of the Earth as deducible from Observation*, where the general problem (on the ſuppoſition that the earth is a ſpheroid), together with some subordinate problems having a dependence upon the ſame principle, have received a complete discussion. Other ingenious discussions on the figure of the earth may be found in *De La Lande's Astronomy*, tom. iii. p. 83—126, and in the *Abridgement of the Phil. Trans.* vol. vi. p. 363, &c.

CHAPTER II.

Explanation of Terms relating to ſome imaginary Points, Lines, and Circles, on the Earth, and in the Heavens.

ART. 11. SINCE the figure of the earth varies so little from that of a sphere, it is usual in the greater part of the enquiries and calculations of Astronomers to proceed as though it were a sphere in reality; and, since to an observer on the earth the heavens appear as a very large concave sphere, every part of which is equidistant from him, it has been found expedient to imagine various lines and circles to be described upon the earth, and the planes of several of them to be extended every way until they mark lines and circles, similar and concentric to themselves, upon the imaginary concave sphere of the heavens. Some of these circles, &c. it is our business now to explain.

12. The section which is made on a sphere by a plane passing through the centre is called a *great circle* of the sphere; if the plane do not pass through the centre, the section is called a *less circle*: hence, a *great circle* is that which divides the sphere into two equal parts; and a *less circle* that which divides the sphere into two unequal parts.

13. Those two points on the surface of a sphere which are equidistant from every part of the circumference of one of its great circles, are called the *poles* of that great circle; and if the sphere be cut by a section passing through these *poles*, it will form another

great circle, forming what is called a *secondary* to the former great circle.

14. The *axis* of the earth is an imaginary line, passing through the centre, about which it is supposed (Art. 390.) to revolve.

15. The extremities of this axis are called the *poles* of the earth, or the *poles of the equator* (Art. 16.).

16. That great circle, the poles of which (Art. 13.) are the poles of the earth, is called the *equator*: or the equator is the circumference of an imaginary great circle, perpendicular to the axis, and at equal distances from the poles.

17. If the axis of the earth be supposed produced both ways as far as the apparent concave surface of the heavens, it is then called the *axis of the heavens*; its extremities are called the *poles of the heavens*; and the circumference formed by extending the plane of the equator to the same concave surface is called the *equator in the heavens*, or the *equinoctial*.

18. A secondary to the equator drawn through any place on the earth, and consequently through the poles (Art. 13.), is the *meridian* of that place: and, as it passes through the poles, which are called *north* and *south* respectively, every meridian is a *north* and *south* line.

19. The *latitude* of any place upon the surface of the earth is its distance from the equator, measured on an arc of the meridian passing through it. A less circle passing through any place parallel to the equator, is called a *parallel of latitude*: hence all places lying on the same parallel of latitude have the same latitude. If the place lie between the equator and the north pole it has *north latitude*; if it be toward the south pole, it has *south latitude*.

20. All places that lie under the same meridian have the same *longitude*; and all places which lie under different meridians, are said to have different longitude. *The difference of longitude* between any two

places, is the distance of their meridians measured in degrees, &c. upon the equator. It is usual for geographers to pitch upon the meridian of some remarkable place for the *first meridian*, and to reckon the longitudes of other places by measuring the distances of their meridians from the *first meridian* on the equator. If the meridian of the place lie towards the right-hand of the first meridian, the observer standing with his face towards the north, the place is said to be in *east longitude*; but if the meridian is towards the left hand, the place is in *west longitude*. For a considerable time the first meridian was made to pass through *Teneriffe*, one of the *Canary Isles*; being the most western part of the earth then discovered, from which all other places were estimated in east longitude. But it is customary now for the first meridian to be reckoned by every geographer, as passing through the respective metropolis of his native country; thus, in England the first meridian is made to pass through London, or rather the Royal Observatory at Greenwich.

21. The horizon is either *sensible* or *rational*: the *sensible horizon* is a circle, the plane of which is supposed to touch the spherical surface of the earth, in the place of the spectator whose horizon it is, and is extended to the heavens; the *rational horizon* is a circle, whose plane passes through the centre of the earth, parallel to the plane of the sensible horizon, and is also continued to the heavens. The earth is so small in comparison of the immense largeness of the apparent heavens, that the planes of the sensible and rational horizons are, in many astronomical enquiries, supposed to coincide, without producing any material error.

22. Great circles which are drawn as secondaries to the rational horizon, are called *vertical circles*, and serve to measure the altitude or depression of any celestial object.

23. The two points where all the vertical circles which can be drawn to any rational horizon meet, are called, the one above the spectator, the *Zenith*, and that which is below him, the *Nadir*. Corresponding to the Nadir on the imaginary celestial sphere, is the *Antipode* on the earth.

24. These definitions and remarks may be illustrated by referring to fig. 3, Pl. I. where $p\ e\ p\ q\ p$ represents a vertical section of the globe of the earth, and P E P Q P a section of the concave sphere of the heavens. The axis of the earth is denoted by the line $p\ p$, whose extremes are the poles of the earth; and the axis of the heavens is P P, whose extremes are its poles. The circle $e\ q$ is the equator, and the circle E Q, corresponding to it in the heavens, is the equinoctial. If S be the place of a spectator, p S $p\ q$ is the meridian of that place; or P Z P Q, the same circle continued to the heavens, is the meridian of that place, and p S $p\ q$ on the earth is a *circle of longitude*. In like manner, P O P, P U P, are meridians; and $p\ o\ p$, $p\ u\ p$, circles of longitude. The latitude of the place S, is measured by the arc e S, its distance from the equator: S m and n A are parallels of latitude; all the places which lie on either of them have the same latitude. The sensible horizon is the circle, the diameter of which is $h\ r$; it touches the earth in the place S of the spectator: the circle whose diameter is H R, passing through the centre of the earth parallel to the former, is the rational horizon, Z is the zenith, or point over the head of the spectator; N the nadir, or point under his feet: Z H N, Z V N, &c. are vertical circles: and A is the place on the earth diametrically opposite to the spectator, or his antipode.

25. The altitude of one pole, or the depression of the other, at the rational horizon of any place on the earth's surface, is equal to the latitude of that place. For the latitude of the place S is measured by the arc e S, or the arc E Z containing an equal number of de-

grees: but P R or H P is the arc of altitude or depression of the poles; and because H Z, H N, Z R, and E P, are arcs of 90° each, if from either of these be taken Z P or P N, we have the remainders P R, E Z, and H P, equal to each other.

26. The elevation of the equator at any place is equal to the complement of the latitude of that place. For, by the last article, H P is equal to E Z the latitude; and since P E is an arc of 90°, E H is the complement of H P; that is, the elevation of the equator is equal to the complement of the latitude. The young astronomer will do well to remember this article and the last, as they will be often of advantage in subsequent parts of this work.

CHAPTER III.

On the apparent Motion of the heavenly Bodies, particularly of the Sun; the Ecliptic; Cause of Seasons; Length of the Year; and Precession of the Equinoxes.

Art. 27. A PERSON who observes the heavens with a tolerable degree of attention, will soon perceive that all the heavenly bodies, the sun, moon, and stars, appear to move round the earth, in circles parallel to the equinoctial, once in the compass of about twenty-four hours: and though, as will be shewn hereafter (Art. 205. &c.), these *apparent* motions are almost entirely to be accounted for by the *real* motions of the earth; yet it will enable us to proceed with more ease and regularity, if we, for the present, consider the various heavenly bodies to be really moving round the earth, in the manner they appear to do. It will not be long before the observer discovers that by far the greater number of them never change their relative situations, each (so long as the observer keeps in the same place) rising and setting at the same interval of time, and at the same points of the horizon; these are, therefore, called *fixed stars;* but that a few other stars, called *planets*, are constantly changing their situations, each continually rising and setting at different points of the horizon, and at varying intervals of time;—this is also the case with the *sun* and *moon*.

28. If the observer were at one of the poles of the earth; he would have the celestial pole corresponding thereto in his zenith, the other in his nadir;

the equinoctial would coincide with his horizon, and all the heavenly bodies, moving in circles parallel to the equinoctical, would appear to him neither to rise nor set. This will be elucidated by referring to fig. 4, Pl. I. and is called a *parallel sphere*, because the diurnal motion of all the bodies is parallel to the horizon.

29. Were the observer situated on the equator; both the celestial poles would be in his horizon, and the equinoctial would then be perpendicular to the horizon; consequently the circles *a a*, *b b*, *x x*, &c. (fig. 5, Pl. I.) in which the heavenly bodies move, are perpendicular to the horizon. The bodies rise at right angles to the horizon, move gradually to the meridian, and spend each as much time in descending to the horizon as they did in passing from it to the meridian: they also set at right angles to the horizon. Hence this situation is called a *right sphere*.

30. If the observer be placed any-where between the equator and the poles; then the celestial equator cuts his horizon obliquely, and all the heavenly bodies rise, set, and perform their diurnal motions obliquely: this is therefore called an *oblique sphere*: see fig. 6, Pl. I. Here the paths of some of the celestial bodies are entirely above the horizon, as *c c*; others are entirely below it, as *z z*; and others are partly above and partly below it, as *b b*, *a a*, *x x*, *y y*. With regard to those which *cut* the horizon, it is observed that they take just the same interval to rise from the horizon to the meridian, as to pass from the meridian to the horizon again.

31. The largest parallel which appeared entire above the horizon of any place in north latitude, was by the ancient astronomers called the *arctic circle* of that place; between this circle and the north pole are comprehended all the stars which never set in that place, and are called *circumpolar stars*; the largest parallel which was entirely hid below the horizon of

that place was called the *antarctic circle;* between this and the south pole are comprehended all the stars which never rise at that place. But by the moderns the former of these circles is called the circle of *perpetual apparition*, and the latter the circle of *perpetual occultation*. The names of *arctic* and *antarctic* are now applied to two circles, which are described at about 23° 28′ from the north and south poles respectively.

32. Since the sun, stars, &c. are observed to be on the meridian at *half* the interval during which they appear above the horizon, or very nearly so; and since it was alſo remarked, that each of them was at its greatest altitude when on the meridian, it was natural to conclude that one of the best methods of determining the situation of any of them, was to mark the *time* when it came to the meridian, and to measure its *altitude* at that time. This method we shall proceed to exemplify; but it is previously necessary to point out how to find a meridian line, which may be done as below.

33. On the supposition that the sun is at equal distances from the meridian, and at equal altitudes, when it is equal times before and after the meridian— a supposition which, though not strictly true (as will soon appear), is yet sufficiently accurate for our present purpose—a meridian line may be found thus: On an horizontal plane describe three or four concentric circles, as E, G, H, fig. 8, Pl. I. and in the common centre fix perpendicularly a wire C B, having a well-defined point. When the sun shines in the morning, observe where the shadow of the top of the wire, as C D, touches one of the circles; and in the afternoon mind where the extremity of the shadow C F just touches the same circle: then through the centre C draw the line C E, bisecting the arc D F, and C E will be a meridian, as required. If the same be done with as many of the

circles as the shining of the sun will admit of, and the mean of all the bisecting lines C E be chosen as a meridian, there will be no doubt of its accuracy, particularly if the observations be made about Midsummer, which is the best time.

34. After a meridian line is thus found, another parallel to it may be readily drawn at any convenient distance: the method is this: Hang a thread and plummet exactly over the south end of the known meridian line, and let another thread and plummet be hung over the south end of the plane upon which a meridian is to be drawn; then let a person observe when the shadow of the thread falls on the given meridian, and immediately give a signal to another person, who must at that moment mark two points on the shadow of the second thread, through which two points the new meridian must be described. The same thing might be accomplished by other methods; but this is probably the most accurate.

35. The most simple method of finding the *sun's altitude*, when on the meridian, is that which was practised by the ancients: they erected an upright pillar at the south end of a meridian line, and when the shadow of it exactly coincided with that line, they accurately measured the shadow's length, and then, knowing the height of the pillar, they found, by an easy operation in plane trigonometry, the altitude of the sun's upper limb: whence, after allowing for the apparent semidiameter, the altitude of the sun's centre was known.

36. But the methods now adopted are much more accurate. In a known latitude, a large astronomical quadrant, of 6, 8, or 10 feet radius, is fixed truly upon the meridian; the limb of this quadrant is divided into minutes, and smaller subdivisions, by means of a vernier; and it is furnished with a telescope (having cross hairs, &c.) turning properly upon the centre. By this instrument the altitude of the sun's

centre is very carefully measured, and the proper deductions are made for refraction and parallax (Art. 94. and 80.).

37. With an instrument such as just mentioned we may ascertain the apparent motion of the sun in the following manner: our observations must be begun when the sun is known to be in the equinoctial, that is (Art. 26.) when the altitude of his centre while on the meridian is equal to the complement of the latitude; the time when this is the case, is about the 20th of March. On this day we must note some fixed star which comes to the meridian exactly at the same time as the sun does; for the stars may be seen in the day-time with an astronomical telescope: on the following day, both the altitude of the sun, and the situation of the stars when the sun is on the meridian, must be observed; the sun's meridian altitude will be about 23′ 40″ greater than on the former day, and the star will be found on the meridian about $3^m\ 39^s$ in time before the sun. Make similar observations for a few days, and it will be found at the end of a week, that the sun's meridian altitude will be increased 2° 46′, and the star will be on the meridian $25^m\ 26^s$ in time before the sun, or it will be 6° 21½′ westward of the meridian when the sun is upon it. During this period of seven days, therefore, the sun has been moving apparently towards the east, and has increased his altitude by regular gradations: and because the motions of all the heavenly bodies appear to be performed in circles, or nearly so, let us suppose the sun's apparent motion to be in a circle crossing the equinoctial, at an angle which the above observations will assist us in determining. In fig. 9, Pl. I. let E Q represent a portion of the equinoctial, Q S the meridian on which the sun is, Q S his altitude above the equinoctial, E the place of the star, and E S part of the apparent path of the sun: then, in the spherical triangle E Q S,

right-angled at Q, there are given E Q = 6° 21½′, and Q S = 2° 46′, to find the angle E. By the rules of spherical trigonometry, we have, tangt. of E = $\frac{\text{tangt. of } SQ}{\text{sine of } QE} = \frac{\cdot 0483250}{\cdot 1107463} = \cdot 4364479$ = tangt. of 23° 34′ 43″, the angle E required.

38. To determine how far the conclusion, arrived at in the last article, may be relied upon, we must carefully measure the meridian altitude of the sun when it is at the greatest, about Midsummer, and we shall find it to exceed the complement of the latitude by 23° 28′ very nearly: this differs less than 7′ from the result in the last article, and the difference arises from the observations not having been extended to a sufficient period after the day when the sun was in the equinoctial. For if the calculation had been made with the sun's increased altitude, and his distance from the fixed star, as found on the twentieth day, instead of the seventh, the angle E, as resulting from the calculation, would have exactly agreed with the observations when the sun's meridian altitude was at the greatest.

39. The situation of the circle, in which the sun's apparent motion is performed, being thus ascertained, we may now, with propriety, explain several terms immediately connected with it.

40. The circle itself in which the sun appears to move, is called the *ecliptic*; the angle E in which it crosses the equinoctial, the *obliquity of the ecliptic**,

* The obliquity of the ecliptic is not always equal to 23° 28′. According to the observations of *Pytheas* at *Marseilles*, which were made rather more than 300 years before the Christian æra, the obliquity was then 23° 49½′. *Albategnius*, about the year 880, found it to be 23° 35′, or, by correcting for parallax and refraction, 23° 35′ 40″. *Almæon*, the son of Almansor the Arabian, made it 23° 33′ 30″ in 1140. *Tycho Brabe*, in 1587, found it to be 23° 29′ 30″. Mr. *Flamsteed*, in 1689, 23° 28′ 56″. *Condamine*, at Quito, in 1736, made it 23° 28′ 24″. Dr. *Maskelyne*, in 1769, determined it at 23° 28′ 10″. By a comparison with the most early and accurate observations, it appears that the diminution of

and the two points where it intersects that circle, the *equinoxes.*

41. A portion of the heavens about 16 degrees in breadth, through the middle of which passes the sun in the ecliptic, is called the *zodiac.* Within this portion, or zone, all the planets perform their motions ; as will be shewn hereafter.

42. The distance of the sun, or any of the heavenly bodies, from the equator, measured on an arc of the meridian, is called the *declination* : if the body is situated between the equator and the north pole, it has *north declination* ; if toward the south pole, the declination is *south.* The declinations of the fixed stars are between 0° and 90° ; but the greatest declination of the sun is about 23° 28′ (Art. 38.).

43. Secondaries (Art. 13.) to the celestial equator, are called *circles of declination* : of these, twenty-four, which divide the equator into equal parts, of 15° each, are called *hour circles* ; because the sun in his apparent diurnal motion, passing over 360° of a circle parallel to the equator, goes through $\frac{1}{24}$th of it, or 15°, in the space of an hour.

44. The *right ascension* of a body, is the arc of the equinoctial, intercepted between one of the equinoxes, called the first point of aries (Art. 45.), and a declination circle passing through the body ; it is measured according to the order of the sun's apparent motion, and is called *right* ascension, because in a right sphere the declination circle which passes through the body ascends above the horizon, at the same moment the body does. The *oblique ascension* is an arc of the equator, intercepted between the first point of aries, and that point of the equator which

the obliquity, is at about the rate of 50″ in a century, or half a second in a year. This agrees nearly with the best observations of astronomers of all ages ; and is perfectly consistent with the most satisfactory methods of accounting for the motions and mutual effects of the heavenly bodies upon each other.

rises with any body in an oblique sphere. The difference between the right and oblique ascension is called the *ascensional difference.*

45. The ecliptic is supposed to be divided into twelve signs, reckoning according to the order of the sun's motion; they are generally called *signs of the zodiac*, and each of them is divided into 30°, &c. The signs, beginning at the equinox, where the sun intersects and rises above the equator, have these names*

* Several of the Greek and Roman poets gave ingenious and entertaining, though wild and romantic, fables, respecting the origin of the signs; but, instead of repeating them, we shall here mention it as a very probable conjecture, that the figures and names of the signs are descriptive of the seasons of the year or months in the sun's path. Thus, the first sign aries (the ram) denotes, that about the time when the sun enters that part of the ecliptic, the lambs begin to follow the sheep: the sun's approach to the second sign taurus (the bull) is about the time of the cows bringing forth their young. The third sign, now gemini, was originally two kids, and signified the time of the goats bringing forth their young, which are usually two at a birth, while the former, the sheep and cow, commonly produce only one. The fourth sign cancer (the crab), an animal that goes sideways and backwards, was placed at the point where the sun begins to return back from the north to the southward. The fifth sign leo (the lion) being a very furious animal, was thought to denote the heat and fury of the burning sun when he has quitted cancer. The succeeding sign received the sun at the time of ripening corn and approaching harvest, which was aptly exprest by one of the female reapers with corn in her hand, viz. virgo the maid. The ancients gave to the next sign scorpio, two of the twelve divisions of the zodiac. Autumn which affords fruits in great abundance, affords also the means and causes of diseases, and the succeeding time is the most unhealthy part of the year; expressed by this venomous animal, here spreading out his long claws into the one sign, as threatening mischief, and in the other brandishing his tail, to denote the completion of it. The fall of the leaf was the season of the ancient hunting; for which reason the sun's place at this time was designated by the sign sagittarius, a huntsman with his arrows and club, the weapons of destruction for the large creatures he pursued. The reason of capricornus, the wild goat, being chosen to mark the point where the sun has attained his extreme limit to the south, and begins to mount again to the north, is obvious enough; it being the nature of that animal to be mostly climbing, and ascending some mountain as it

and marks, aries ♈, taurus ♉, gemini ♊, cancer ♋, leo ♌, virgo ♍, libra ♎, scorpio ♏, sagittarius ♐, capricornus ♑, aquarius ♒, pisces ♓. Of these signs, the first six are called *northern*, lying on the north side of the equator; the last six are called *southern*, being situated to the south of the equator. The signs from capricornus to gemini are called *ascending*, the sun approaching or rising to the north pole while it passes through them; and the signs from cancer to sagittarius are called *descending*, the sun, as it moves through them, receding or descending from the north pole.

46. The *longitude* of any of the heavenly bodies is an arc of the ecliptic contained between the first point of aries, and a secondary to the ecliptic or *circle of latitude*, passing through the body; it is always measured according to the order of the signs. If the body be supposed seen from the centre of the earth, it is called *geocentric longitude;* but if it be supposed seen from the centre of the sun, then is the longitude *heliocentric.*

47. The *latitude* of a heavenly body is its distance from the ecliptic, measured upon a secondary to the ecliptic drawn through the body. If the latitude be such as is seen from the earth's centre, it is called *geocentric latitude;* but if it be supposed seen from the centre of the sun, it is *heliocentric.*

48. The *tropics* are two circles parallel to the equinoctial, and touching the ecliptic: that which touches the ecliptic at the beginning of cancer, is called the *tropic of cancer*; the other, touching it at the beginning

browzes. As to the origin of aquarius, the winter being a wet, uncomfortable season, was well expressed by the figure of a man pouring out water from an urn. The last of the signs is pisces, a couple of fishes tied together; the application appeared to be this: the severe season is over, your flocks do not yet yield their store, but the seas and rivers are open, and there you may take fish in abundance.

Hutton's Math. and Phil. Dict. Art. CONSTELLATIONS.

of capricorn, is called the *tropic of capricorn*. When the sun is in either of the points where a tropic touches the ecliptic, he is said to be in one of the *solstices*, because he then appears at a stand with regard to his declination. The sun is in the solstices on June 21, and December 21.

49. *Colures* are two secondaries to the equinoctial: one, passing through the equinoctial points (viz. the points where the ecliptic crosses the equator), is called the *equinoctial colure*; the other, passing through the solstices, is called the *solsticial colure*.

50. That vertical circle (Art. 22.) which intersects the meridian of any place at right angles, is called the *prime vertical*; and the points where it cuts the horizon, are the *east* and *west* points. These points are each 90° distant from the *north* and *south*; and all four are called *cardinal points*.

51. When any heavenly body is referred to the horizon by a secondary thereto, the distance of this secondary (on the horizon) from the north or south points, is the *azimuth* of the body; the distance from the east or west points is the *amplitude*.

52. Having ascertained the situation of the circle in which the sun's annual motion appears to be performed, and explained the necessary terms, we may proceed to shew the reason of *the variation of the length of days and nights*, and *the change of the seasons*: in order to which, we refer to fig. 7, Pl. I. where is represented the position of the chief circles, with respect to Z, the zenith of a spectator in north latitude: E Q denotes the equator, N and S the north and south poles, Æ C the ecliptic crossing the equator at an angle of 23° 28′ (Art. 38.); *a b*, *r* R, &c. parallel circles, in which the heavenly bodies perform their diurnal motions; H R the horizon, N E S Q the meridian, and Z O A perpendicular thereto the prime vertical; R will be the north point of the horizon, H the south, and O will represent the east or west, ac-

cording as the eastern or western hemisphere is represented by the diagram. Now, when the sun is on the equator in March (Art. 73.) he performs his diurnal motion in the equator E Q, and comes to the horizon in O the east point; and as the equator is bisected by the horizon, and the sun's apparent diurnal motion is nearly uniform, he will set in the west point, and will be as long below the horizon as above it, making the days and nights equal; this is called the *vernal equinox*. As the sun's declination increases to the northward, in his passage from the vernal equinox to the summer solstice, a variation will take place in his diurnal motion; this may be illustrated by imagining the sun's declination to be E *a*, his place in the ecliptic to be *w*, and *a b* the parallel which he describes that day. Then will he come to the horizon at the point *o*, between the north and the east; when he arrrives at *i* it will be six o'clock, being half way between *a* and *b* his situations at noon and midnight; at *e* he will be on the prime vertical, or in the east; at *a* he will be on the meridian; afterwards he will proceed in a contrary order along the western hemisphere, and will set between the west and the north. In this situation, the arc *o a*, occupied by the sun in passing from the horizon to the meridian, called the *semidiurnal arc*, being greater than *i a*, the half of *a b*, requires more than six hours to pass through it, and consequently the day is more than twelve hours long. The days will continue lengthening, and the sun rising and setting nearer the north point, as he approaches towards the point Æ, where the north declination is the greatest; and then the sun's diurnal motion will be described in the tropic of cancer, the days will be at the longest, the nights shortest, and this will be the *summer solstice*. From this period the sun's declination will be decreasing, and the days decreasing also, until the sun arrives at the equator, being the first point of libra; then will the length of the day and night be

again equal, and the sun will rise and set in the east and west: this is called the *autumnal equinox*, being about the 23d of September. After this time the declination of the sun will become southerly, and the days will decrease in length, which, with other attending circumstances, may be thus explained: Imagine *m* to be the sun's place in the ecliptic, and *c d* the parallel he describes. Then he arrives at *n* on the prime vertical before he gets above the horizon, and consequently can never be seen to shine from the east or west while his declination is south; nor can he, since *u* is below the horizon, for a similar reason, rise till after six o'clock, during the same period: he rises at *s* between the east and the south, and after proceeding gradually to the meridian, declines as uniformly, and sets between the south and the west. Here the semidiurnal arc *c s* being less than *c u*, the half of *c d* is passed over by the sun in less than six hours, and consequently the day is less than twelve hours in length. The days will continue to decrease until the sun has reached C, the greatest declination south, when his diurnal motion will be described in the tropic of capricorn: he will then rise and set at the farthest from the east and west towards the south, the day will be at the shortest, the night the longest, and this will be the *winter solstice*. The sun's south declination will be decreasing, and the days lengthening, from this time till he again arrives at the equator, which will be at the *vernal equinox*: after which the seasons will continue gradually following each other, in beautiful harmony, in the manner here described *.

* It may be proper to observe, that the different degrees of heat in summer and in winter, do not arise, entirely, from the different times which the sun is above the horizon. The direction and force of the solar rays have also considerable influence. During the short days, the effect of the sun is less, both with respect to the intensity of the rays, their direction, and the time of their conti-

53. The change of seasons to an inhabitant of any place in south latitude, might be elucidated in a similar manner; and it would appear that, to such a person, the seasons would be just opposite to ours: that is, his summer would be our winter, his autumn our spring, and *vice versa*. An inhabitant of the equator has the days and nights all of equal length: but at both the times when the sun is in the equinox he will have summer; for the sun will then be in his zenith at noon: and at both the solstices he will have winter; for then the sun at noon declines 23° 28′ from his zenith: which must be too evident to need farther illustration.

54. Since the regular return of the seasons is intimately connected with *the length of the year*, it will be proper, previous to concluding this chapter, to point out a few methods by which the precise length of the year may be learnt. Astronomers consider the year under two distinctions: first, the *tropical* or *solar year*, properly so called, is the exact space of time in which the sun moves through the twelve signs of the ecliptic; and secondly, the *sidereal* or *astral year*, being the time occupied by the sun in passing from any fixed star till his return to it again. The changes of seasons depending on the first of these, the *tropical* year, we will first describe the manner in which its length may be determined. The latitude of the

nuance: and during the long days it is greater in all these respects. It may be remarked too, that the severest frosts usually take place after the days have begun to lengthen; and the most oppressive heat prevails when the days are decreasing: the reason of which is, that during the summer months, the earth having imbibed more heat than it gave out, is not exhausted of its superabundant warmth till towards the close of the year: in like manner, on account of the waste of the earth's heat being greater in winter than its supply, it continues to get colder and colder till a month or longer after the winter solstice. From a similar cause arises the difference between the spring and autumn; though the position of the sun, in respect of the earth, is in both the same.

place being known, and consequently the elevation of the equator (Art. 26.), observe carefully the sun's meridian altitude when he is nearest the equator (at either the vernal or autumnal equinox); and if the observed meridian altitude on any day is exactly equal to the complement of the latitude, that day at noon is the exact time of the equinox. But as it is not very probable that the sun should be on the equator precisely at noon, take the meridian altitude one day when it is less than the co-latitude, and on the following day when it is greater; then say, as the sum of the defect and excess of these two meridian altitudes (when taken from, and added to, the co-latitude) : the distance in time between the observations : : the defect : the time to be added to the first observation, for the true time of the equinox. After the same manner, may the time of the next vernal or autumnal equinox be found: the difference between these times will be the length of the year. In order to arrive at the greatest accuracy, it will be adviseable to take two such equinoxes as are at the distance of several years from each other: for then, whatever errors may attend the observations, by being divided into many parts, they will become in a manner insensible.

55. *M. Cassini* has given the following method of finding the length of the tropical year: Observe the meridian altitude (a) of the sun on the day nearest to an equinox: then the next year take its meridian altitude on two succeeding days, one when the altitude (m) is less than a, and the next when the altitude (n) is greater than a, and $n-m$ is the change in the sun's declination in twenty-four hours: hence, as the declination near the equinoxes changes uniformly, we have $n-m : a-m :: 24$ hours : the interval from the first of the two days, till the time when the sun's declination is the same as at the observation the year before. This fourth term, therefore, being added to the number of days between

the first two observations, gives the length of the year required. If there be an interval of several years before the second observation is made, and the interval between the times when the declinations were the same be divided by the number of years, the year's length will be determined more exactly.

For example, on March 20, 1672, *M. Cassini*'s father observed the meridian altitude of the sun's upper limb, at the royal observatory at Paris, to be $41^\circ\ 43'$; and on March 20, 1716, Cassini himself found the meridian altitude of the upper limb to be $41^\circ\ 27'\ 10''$, and on the 21st to be $41^\circ\ 51'$. The difference between the two latter altitudes is $23'\ 50''$, and between the two former $15'\ 50''$. Therefore, by the rule, $23'\ 50'' : 15'\ 50'' :: 24$ hours : $15^h\ 56^m\ 39^s$. Hence, on March 20, 1716, at $15^h\ 56^m\ 39^s$, the declination of the sun was the same as on March 20, 1672, at noon. Now the interval between the first two observations was 44 common years, of which 34 consisted of 365 days each, and 10 of 366; therefore the interval in days was 16070, and the whole period between the equal declinations was 16070 days, $15^h\ 56^m\ 39^s$: this divided by 44 gives $365^d\ 5^h\ 49^m\ 0^s\ 53^t$ for the tropical year. This is called by some authors the *apparent* solar year: they apply a minute correction, which depends upon the motion of what is called the sun's apogee (Art. 297.), and thus get $365^d\ 5^h\ 48^m\ 49^s$, for the length of a *mean* solar year.

56. To find the length of a *sidereal year*, *M. Cassini* presents us with this rule: Take the time (t) of a fixed star's transit over the meridian by a clock adjusted to mean solar time; the year following observe the time again on two days, one (m) when the star passes the meridian *before*, and the other (n) *after* the time t; then, $m - n : m - t ::$ twenty-four hours, or more accurately $23^h\ 56^m\ 4^s$, the length of a sidereal day (Art. 114.): the time from m till the difference between the star's and sun's right ascension was the

same as at the first observation: the interval of these two times is the sidereal year.

In the present problem, as well as in the former, it is best to take an interval of several years: by applying the rule to some observations which were made at the distance of nearly one hundred years, the length of the sidereal year is now set down at $365^d\ 6^h\ 9^m\ 12^s$.

57. From comparing the results of the last two articles, it appears that the sun returns to the equinox every year before it returns to the same point of the heavens; and consequently, the equinoctial points have a retrograde motion: this motion, though very small in a year or two, would in the compass of a few centuries amount to something considerable. It therefore did not escape the notice of the early astronomers, although their observations were very inaccurate in comparison with those of the moderns: it is called the *precession of the equinoxes*. *Hipparchus* was, we believe, the first person who noticed this motion; and, by comparing his own observations with some that were made about 155 years previous to his, he judged the motion to be nearly 1° in 100 years. The observations of *Albategnius* in the year 878, compared with those of 1738, give $51''\ 9'''$ for the annual precession. *M. de la Caille's* observations, compared with those given by *Flamsteed*, determine the precession in an 100 years, or the *secular precession*, to be $1°\ 23'\ 45''$, and the *annual precession* $50\frac{1}{4}''$. Thus is this motion determined to as great accuracy as may reasonably be expected; some of its effects will naturally fall under our consideration in the next chapter.

CHAPTER IV.

On determining the relative Situations of the fixed Stars; with Remarks on their Appearances, the Constellations, &c.

ART. 58. THOSE stars which when seen either by the naked eye, or by means of telescopes, keep constantly in the same situation with respect to one another, are called *fixed stars*, as has been already mentioned (Art. 27.). If we contemplate any number of such stars, which to our view form a triangle, a trapezium, or any other figure, regular or irregular; since those stars have continued in the same relative situation ever since we have observed them, and have appeared to astronomers in the same situation with respect to each other as long back as the records of authentic history will carry us, namely, for some thousands of years; we may fairly conclude from analogy, that they have appeared very nearly as they now do ever since the creation of the earth, and will probably continue to appear so as long as the earth shall endure. Some few particular changes must be taken as exceptions from this general inference.

59. The fixed stars appear to us of *different magnitudes*: this may arise either from their different sizes, or from their unequal distances from us; or in some instances both these causes may combine to make the apparent difference. However, be this as it will, astronomers have from their apparent magnitudes distinguished the stars into six orders or classes: the *first* contains those of the largest apparent size, the *second* those which appear next in bigness, and

so on, till we come to stars of the *sixth magnitude*, among which last class are all those that can just be seen without telescopes: those which are only seen with telescopes are called *telescopic* stars. Now although this distribution of the stars is generally received, it is not to be imagined that all the stars of each class are exactly of the same apparent size; for there is almost an innumerable difference either in respect of apparent magnitude, colour, or brilliancy; and in some of these respects many of the stars appear to undergo changes, so that the same star may be reckoned by some astronomers in the first class, while others estimate it as belonging to the second or third. But the general divisions may be looked upon as usually adhered to, since the deviations from them are but few.

60. But, leaving what else is proper to remark respecting their magnitudes and changes to a subsequent article (Art. 683. &c.), we will now proceed to shew the methods of determining the places of any of them, with respect to the most important circles of the sphere: for the accurate determination of the places of the fixed stars will be of considerable utility in enabling us to ascertain the places of the planets, &c. by means of their apparent distances from any of those stars whose situations are known.

61. The declination of any star may be easily found by observing the following rule: Take the meridian altitude of the star, at any place where the latitude is known; the complement of this is the *zenith distance*, and is called north or south, as the star is north or south at the time of observation. Then, 1. When the latitude of the place and zenith distance of the star are of different kinds, namely, one north and the other south, their *difference* will be the declination; and it is of the same kind with the latitude, when that is the greatest of the two, otherwise it is of the contrary kind. 2. If the latitude and the zenith distance are of the same

kind, i. e. both north, or both south, their sum is the declination; and it is of the same kind with the latitude.

To prove the truth of this rule, turn to fig. 7, Pl. I. where Z is the zenith of the place, E Q the equinoctial, and E Z the latitude. 1. Let *r* represent the place of a star on the meridian, and Z *r* the zenith distance, the latitude being greater: then, E *r* (the declination) will be equal to E Z — Z *r* (the zenith distance): again, let *c* be the place of a star in the meridian, when the zenith distance exceeds the latitude; then E *c* (the declination) = Z *c* (the zenith distance) — E Z (the latitude). And it is manifest, that in the former instance Z and *r* are on the same side of the equinoctial; and that in the latter case Z and *c* are on contrary sides. 2dly. Let *y* be the place of a star on the meridian, having its zenith distance Z *y* of the same kind with E Z the latitude of the place: then E *y* (the declination) = E Z + Z *y*; and the declination is of the same kind as the latitude, because Z and *y* are on the same side of the equinoctial. Q. E. D.

For an example, suppose that in north latitude 52° 15′, the meridian altitude of a star is 51° 28′ on the south; then 38° 32′ the zenith distance, being taken from 52° 15′ the latitude, leaves 13° 43′ for the declination of the star *north*.

62. Having, by means like the above, found the declination of a star, it becomes requisite, in the next place, to know *the right ascension* (Art. 44.), as its situation with regard to the equator will then be known. Now the right ascension being estimated from the point where the equator and ecliptic intersect each other in the spring, a point which is marked out by nothing that comes under the cognizance of our senses; some phænomenon, therefore, must be chosen, whose right ascension is either given, or may be readily known, at any time that the right ascensions of other objects may be discovered

by comparison with *it*. For this purpose nothing appears so proper as the sun; because its motion is the most simple, and its right ascension quickly found.

63. For if, in fig. 9, Pl. I. we have given Q S the declination of the sun (which may be easily taken every day at noon by observation), and the angle S E Q the obliquity of the ecliptic—i. e. one leg of a right-angled spherical triangle, and its opposite angle, to find the adjacent leg E Q, the right ascension—it may be done by this proportion; as the tangent of the obliquity of the ecliptic : the tangent of the declination : : radius : the sine of the right ascension reckoned from the nearer equinoctial point.

For example, suppose on the 13th of February, the sun's south declination is found to be 13° 24′, and the obliquity of the ecliptic is 23° 28′; we shall thus find the sun's right ascension:

As tangt. 23° 28′	9·6376106
To tangt. 13° 24′	9·3770030
So is radius . . .	10·0000000
To sine 33° 16′ 58″	9·7393924

Here 33° 16′ 58″ is the sun's distance from ♈; but as the declination is at that time decreasing, and the sun approaching ♈, this must be taken from 360°, and the remainder 326° 43′ 2″ is the right ascension.

In a similar manner may the sun's right ascension be calculated for every day at noon, and arranged in tables for use: for any intermediate time between one day at noon and the following, the right ascension may be determined by proportion.

The longitude E S of the sun, when required, may be readily found by the rules to ascertain the hypothenuse of the same triangle.

64. The apparent diurnal motion of the heavenly bodies being uniform, and performed in circles parallel to the equator, the interval of the times in which two stars pass over any meridian must bear the

same proportion to the period of the diurnal motion, as that arc of the equator intercepted between the two secondaries passing through the stars, does to 360°, as is evident from the nature of the sphere: we may therefore find the right ascension of a star thus: Let an accurate pendulum clock be so regulated that the index may pass over the twenty-four hours during the time in which any fixed star after departing from the meridian will return to it again, which is rather less than twenty four hours. Then let the index of a clock thus regulated be set to twelve o'clock when the sun is on the meridian; and observe the time the index points to, when the fixed star whose right ascension is sought comes to the meridian; which may be most accurately known by means of a transit telescope. Let these hours and parts, as marked by the clock, be converted into degrees, &c. of the equator, by allowing 15° to an hour; and the difference between the right ascensions of the fixed star and the sun will be known: this difference added to the sun's right ascension for that day at noon, gives the right ascension of the fixed star sought.

Or, if a clock whose dial plate is divided into 360°, instead of twelve hours, be ordered in such a manner, that the index may pass round the whole circle in the interval which a star requires to come to the same meridian again, and another index be so managed as to point out the sexagesimal parts: then, when the sun is on the meridian, let the indices of the clock be put to his right ascension at noon that day; and when the ſtar comes to the meridian, its right ascension will be shewn by the clock, without any kind of reduction.

65. The equator being the principal circle which respects the earth, the latitudes and longitudes of terrestrial objects are referred to it; and, for a similar reason (the sun's motion in the ecliptic rendering that the principal of the celestial circles), the situations of heavenly

objects are generally ascertained by their latitudes and longitudes referred to the ecliptic: it is therefore become a useful problem *to find the latitudes and longitudes of the stars, &c. having their declinations, and right ascensions, with the obliquity of the ecliptic, given.* One of the best methods of performing this problem has been thus investigated: Let S be the place of the body (fig. 11, Pl. I.), E C the ecliptic, E Q the equator; and S L and S R being respectively perpendicular to E C and E Q, E R will represent the right ascension, S R the declination, E L the longitude, and S L the latitude; then, by spherics, rad. : sine E R : : co-tang. S R : co-tang. S E R; and S E R∓C E Q=S E L. Also, co-sine S E R : rad. : : tang. E R : tang. E S; and rad. : co-sine S E L : : tang. E S : tang. E L; therefore, co-sine S E R : co-sine S E L : : tang. E R : tang. E L; whence we readily get, $\frac{\text{co-sin. } SEL \times \text{tang. } ER}{\text{co-sine } SER} =$ the tangent of E L, the *longitude.* Then, rad. : sine of E L : : tang. S E L : tang. S L, the latitude.

66. But the same thing may be performed very expeditiously by means of the following excellent rule, given by Dr. *Maskelyne*, the present worthy Astronomer Royal:

1. The sine of the right ascension + co-tang. declination —10= co-tang. of arc A, which call *north*, or *south*, according as the declination is *north* or *south*. 2. Call the obliquity of the ecliptic *south* in the six first signs of right ascension, and *north* in the six last. Let the sum of arc A, and obliquity of ecliptic, according to their titles, = arc B with its proper title. [If one be *north* and the other *south*, the proper title is that which belongs to the greater; and in this case, arc B is their difference.] 3. The arithmetical complement of co-sine arc A+ co-sine arc B× tang. right ascension = tangent of the *longitude*: this is of the same kind as the right ascension, unless

arc B be more than 90°, when the quantity found of the same kind as the right ascension must be subtracted from 12 signs, or 360°. 4. The sine of *longitude* + tang. arc B —10= tang. of the required *latitude*, of the same title as arc B. Note, If the longitude be found near 0° or near 180°, for the sine of longitude, in the last operation, substitute tang. longitude + co-sine longitude —10; and then the last operation will be tang. longitude + co-sine. longitude + tang. arc B —20= tang. latitude. By sine, tang. &c. are meant logarithm sine, log. tang. &c.

This rule may be exemplified by enquiring what are the latitude and longitude of a star whose declination is 12° 59′ north, and right ascension 4^s 29° 38′, the obliquity of the ecliptic being 23° 28′?

Here, sine of right ascension	4^s 29° 38′		9·7037486
Co-tang. of declination	12 59		10·6372126
Co-tang. of arc A, north	24 31		10·34069612
Obliquity of ecliptic, south	23 28		
Arc B, north - -	1 3	cos.	9·9999271
Arith. comp. of co-sine arc A			0·0410347
Tangent of right ascension			9·7678344
Tangent of longitude	147° 13′ 26″		9·8087962

Or 4^s 27° 13′ 26″, answering to 27° 13′ 26″ of Leo.

Then, sine of longitude	- -	9·7334843
Tangent of arc B	- -	8·2631153
Tang. of latitude, north, 34′ 6″		7·9965996

67. By the method just described the longitude and latitude of any heavenly body may be learnt, after the right ascension and declination are carefully ascertained; either according to the mode here pointed out, or in other ways made use of by practical astronomers: but it may frequently happen that the longitudes and latitudes of ſtars, &c. may be required, when there are no opportunities of finding their right ascensions and declinations. In these

cases, if we know the accurate situations of a certain number of the most noted fixed stars, we may find the place of any star by measuring its distance from any two stars whose positions are determined. Thus, in fig. 10, Pl. I. let E C represent the ecliptic, and P its pole; A and B two stars whose latitudes and longitudes are given, and S the star whose latitude and longitude are required. Let the circles of latitude P E, P D, and P C, be drawn; then the arc E C measures the angle E P C, the difference of the given longitudes. In the triangle A P B, the sides A P, B P, the co-latitudes, being given, and the included angle P, we thence find A B, and the angle A B P. Then, in the triangle A S B are given all the sides (viz. A S and S B the two distances from the known stars, and A B just found), to find the angle A B S, which taken from A B P leaves S B P. Lastly, in the triangle S B P, two sides, S B, B P, are known, together with their included angle S B P; whence we find P S and S P B. The complement of P S is D S, the latitude of the star; and the angle B P S is the difference of longitude of the stars B and S; this difference is to be added to the longitude of the star B, if S be to the east of it, or subtracted, if S be to the west; the sum or difference will be the longitude of S required. The solutions of other cases of this problem, according to the different positions of B and S with respect to A, may be readily traced out in a similar manner.

Or, if the situations of A and B, with regard to any other circle, the equator for inſtance, were known; that is, if their right ascensions and declinations were given, together with the distance of a third from both; the right ascension and declination of the third star would be found by the same kind of operation.

68. As it would be absolutely impossible to furnish names for all the fixed stars (or even for such

as are visible to the naked eye), and retain those names in the memory; it became necessary not only to ascertain their exact relative situations, but to invent some method by which the principal part of the stars which can be seen may be known, without having a separate name for each. The ancients accomplished this by portioning out the firmament into imaginary figures of birds, beasts, fishes, &c. called *asterisms* or *constellations*; and then pointing out the place of the star by its place in the constellation; as the bull's eye, the lion's heart, the dog's nose, &c. This division of the heavens into constellations is obviously very ancient; for some of them are mentioned by the oldest heathen poets, *Hesiod* and *Homer*, both of whom probably flourished nearly 1000 years before Jesus Christ. *Arcturus*, *Orion*, and the *Pleiades*, are mentioned twice in the book of *Job*: and in the prophecy of *Amos*, composed about 800 years before the christian æra, we find the following sublime exhortation:——"Ye who turn "judgment into wormwood, and leave off righteous-"ness in the earth, seek Him who maketh the "*seven stars* and *Orion*, and turneth the shadow of "death into morning, and maketh the day dark "with night: that calleth for the waters of the sea, "and poureth them out upon the face of the earth: "The LORD is his name." Here the Pleiades and Orion are spoken of as well known, both by Amos, who was an herdsman, and the common people, to whom this prophecy was addressed: and consequently the invention of constellations was not new, but had been of some standing at that period.

69. As the knowledge of the stars became more extensive, the number of constellations was increased; and at the same time more stars were introduced into each constellation, as their positions became ascertained. *Ptolemy* set down the longitudes and latitudes of rather more than 1000 stars: such of them

as were not comprehended under any constellations, were then called *informes, sporades*, or unformed stars. The modern astronomers have reduced not these *informes* only, but many other stars, into new figures; and it is probable that other constellations will still continue to be invented. *Tycho Brahe*, a noble Dane, was the first who determined with *exactness* the situations of the fixed stars. After Tycho, we must mention *Bayer* as the author of a very considerable improvement in the catalogues of stars. Besides accurately distinguishing the relative size and situation of each star, he marked the stars in each constellation with the letters of the Greek and Roman alphabets, setting the first Greek letter α to the first or principal star in each constellation, β to the second in order; then, when the Greek alphabet was gone over, he passed to *a*, *b*, *c*, of the Roman; and so on. This useful method of noting and describing the stars has been adopted by all astronomers since the time of Bayer; and they have farther enlarged it, by adding the ordinal numbers 1, 2, 3, &c. when any constellation contains more stars than can be marked by the two alphabets. Besides these literal and numeral marks, many of the stars have their peculiar names, as *Arcturus, Aldebaran, Procyon, Sirius, Capella, Regulus, &c.*

70. The *Via lactea*, *Galaxy*, or *Milky Way*, may also be reckoned under the head of constellations; for it is now known to be a collection of innumerable little fixed stars, too small to be observed without telescopes of considerable magnifying powers. This galaxy is a lightish, milk-coloured zone, varying in breadth from 4° to 20°, or on the average about 10°. It passes through cassiopeia, perseus, auriga, the feet of gemini, orion's club, part of monoceros, the tail of canis major, through argo navis, robur carolinum, crux, and the feet of the centaur: after which, it is divided into two parts; its eastern

part passes through ara, the tail of scorpio, the eastern foot of serpentarius, the bow of sagittarius, scutum sobiescianum, the feet of antinous and cygnus; but the western part passes through the upper part of the tail of scorpio, the right of serpentarius and cygnus, and ends its course in cassiopeia.

71. The following tables exhibit the names of the constellations, and the number of stars in each, including all those to the sixth magnitude. They also contain the names of some of the most remarkable stars, standing against the constellations in which they are included.

I. *Constellations in the Zodiac.*

Constellations.	*No. of Stars.*	*Chief Stars.*	
Aries	66		
Taurus	140	Aldebaran	1
Gemini	85	Castor and Pollux	1·2
Cancer	83		
Leo	95	Regulus	1
Virgo	110	Spica Virginis	1
Libra	51	Zubenich Mali	2
Scorpio	44	Antares	1
Sagittarius	69		
Capricornus	51		
Aquarius	108	Scheat	3
Pisces	112		

II. Constellations northward of the Zodiac.

Constellations.	*No. of Stars.*	*Chief Stars.*	
Ursa Minor	24	Pole Star	2
Ursa Major	87	Dubhe	1
Perseus	59	Algenib	2
Auriga	56	Capella	1
* Bootes	54	Arcturus	1
Draco	60	Rastaber	3
* Cepheus	35	Alderamin	3
* Canes Venatici scil. Asterian et Chara	25		
* Car Caroli	3		
* Triangulum	10		
Triangulum minus	5		
* Musca	6		
* Lynx	44		
* Leo Minor	24		
* Coma Berenices	40		
* Camelopardalus	58		
* Mons Menelaus	11		
Corona Borealis	21		
Serpeus	50		
Scutum Sobieski	8		
Herculus, cum Ramo et Cerbero	113	Ras Algiatha	3
* Serpentarius, sive Ophiuchus	67	Ras Alhagus	3
* Taurus Poniatowski	7		
Lyra	22	Vega	1
* Vulpecula and Anser	37		
Sagitta	18		
Aquila	40	Altair	1
Delphinus	18		
* Cygnus	73	Deneb Adige	1
* Equuleus	10		
* Lacerta	16		
* Pegasus	85	Markab	2
* Andromeda	66	Almaac	2

III. Constellations southward of the Zodiac.

Constellations.	*No. of Stars.*	*Chief Stars.*	
* Phœnix	13		
* Officina sculptoria	12		
Eridanus	76	Achernar	1
* Hydrus	10		
* Cetus	80	Menkar	2
* Fornax Chemica	14		
* Horologium	12		
* Reticulus Rhomboidalis	10		
* Xiphias	7		
* Celapraxitellis	16		
* Lepus	19		
* Columba Noachi	10		
Orion	78	Betelguese	1
Argo Navis	50	Canopus	1
Canis Major	30	Sirius	1
* Equuleus Pictorius	8		
* Monoceros	31		
Canis Minor	14	Procyon	1
* Chameleon	10		
* Pyxis Nautica	4		
* Piscis Volans	8		
Hydra	60	Cor Hydræ	1
* Sextans	4		
* Robur Carolinum	12		
* Machina pneumatica	3		
* Crater	11	Alkes	3
* Corvus	9	Algorab	3
* Crosiers	6		
* Musca	4		
* Apis indica	11		
* Circinus	4		
Centaurus	36		
* Lupus	24		
* Quadra Euclidis	12		
* Triangulum australe	5		
Ara	9		

Conclusion of Table III.

Constellations.	*No. of Stars.*	*Chief Stars.*	
* Telescopium	9		
* Corona Australis	12		
* Pavo	14		
* Indus	12		
* Microscopium	10		
* Octans Hadleianus	43		
* Grus	14		
* Toucan	9		
Piscis Australis	20	Fomalhaut	1

In these tables, the constellations which have the mark * against them, are modern ones, the others, ancient. The figures placed against the principal stars denote their magnitudes.

IV. *Number of Stars of each Magnitude.*

		Magnitudes.						
		1st	2d	3d	4th	5th	6th	Total.
Constellations in the Zodiac	12	5	16	44	120	183	646	1014
In the Northern Hemisphere	33	6	24	95	200	291	635	1251
In the Southern Hemisphere	45	9	36	84	190	221	323	865
		20	76	223	512	695	1604	3130

72. It may be asked, by what means are we to know the stars and constellations in the heavens? In answer to this enquiry various methods might be pointed out; but perhaps one of the best is, to fix a good celestial globe, on which the constellations and stars are accurately depicted, in such a direction that the

north, south, and other points on its horizon, may correspond with the points of the compass; then by the observer imagining himself at the centre of the globe, and supposing the lines of direction from himself to the various stars on the surface extended to the heavens, he might speedily find the places of most of the stars.

Or, to be a little more particular, suppose he would know the stars in the constellation Taurus; going out on some fair night, and fixing the globe near him, to compare with the heavens, he would readily find the pleiades, since they are known to every one by the name of the *seven stars.* Then, noticing the globe, he would remark a large star somewhat to the left-hand of the pleiades; and looking to the firmament, and observing the same position, he would find this large star, which is aldebaran, or the bull's south eye. Having found this large star, and looking again to the figure of the constellation, he would see five stars placed almost like a roman V, one of them being aldebaran just found; then directing his view to the heavens, and considering aldebaran as one of the tops of the V, he would behold the stars called the hyades. By imagining lines drawn straight from the bottom of the V along each side, and extended forwards, they would go a little on the outside of both the north and south horns of the bull. If he supposed a line drawn from aldebaran through the pleiades, and continued nearly double that distance farther, it would reach the triangle over the constellation aries; whence, by referring to the globe, the stars in that constellation would soon be known. Or, by imagining lines drawn from the hyades, by the north and south horns of the bull, and continued about double that distance, he would perceive they included two bright stars, called castor and pollux; and then he might soon learn the stars in gemini. And thus

he would be able, by proceeding gradually, to learn the names of the principal stars in each constellation.

73. It must be acknowledged that there is one method preferable to the above, which is, to have a hollow globe constructed of a size sufficiently large for observers to stand on a frame, near its centre: then, if the stars and constellations were painted on the inner surface, in their proper situations, and the globe turned on the poles, elevated according to the latitude, the observers would behold the motions and aspects of the stars as they really are in the heavens, and would sooner obtain a correct idea of the whole than by any other instrument we know, or ever heard of. But as a globe of this kind could not be erected without considerable expence as well as ingenuity, there will be but few able to learn the motions and appearances of the heavenly bodies by such means*.

* Large astronomical machines, on the plan above recommended, have been made at *Gottorp*, at the expence of *Frederick III. duke of Holstein*; and at *Paris*, by the direction of *Cardinal d'Etrees*: but these were far inferior in size to one erected at *Pembroke-college, Cambridge*, under the direction of the late Dr. *Long*. The description of this machine is here added in the doctor's own words:

"I have, in a room lately built in Pembroke-hall, erected a "sphere of eighteen feet diameter, wherein thirty persons may "sit conveniently; the entrance into it is over the south pole by "six steps: the frame of the sphere consists of a number of "iron meridians, not complete semicircles, the northern ends "of which are skrewed to a large round plate of brass, with a "hole in the centre of it; through his hole, from a beam in "the ceiling, comes the north pole, a round iron rod, about "three inches long, and supports the upper parts of the sphere "to its proper elevation for the latitude of Cambridge: the "lower part of the sphere, so much of it as is invisible in England, is cut off; and the lower or southern ends of the meridians, or truncated semicircles, terminate on, and are skrewed down to, a strong circle of oak, of about thirteen feet "diameter, which when the sphere is put into motion runs upon "large rollers of lignum vitæ, in the manner that the tops of

74. There have been changes in some of the fixed stars, at different times: some have totally disappeared, as two of the second magnitude, which were formerly in the stern of argo navis, but are not now to be seen. Some new ones, as one in the breast of cygnus, have appeared: so long ago as the time of *Hipparchus*, a new fixt star was noticed; and this, as *Pliny* informs us, "was the reason why "Hipparchus set about numbering the fixt stars for "the use of posterity, and reducing them to some "rule." There are also some stars which appear and disappear alternately; of which kind is a remarkable one, called *stella mira*, in the neck of cetus. Others are found to vary with respect to

"some windmills are made to turn round. Upon the iron meri-"dians is fixed a zodiac of tin, painted blue, whereon the eclip-"tic and heliocentric orbits of the planets are drawn, and the "constellations and stars traced: the great and little bear, and "draco, are already painted in their places round the north-pole; "the rest of the constellations are proposed to follow: the whole "is turned round with a small winch, with as little labour as it "takes to wind up a jack, though the weight of the iron, tin, and "wooden circle, is about a thousand pounds. When it is made "use of, a planetarium will be placed in the middle thereof. "The whole, with the floor, is well supported by a frame of "large timbers." Since this was written, in 1758, the constellations and chief stars visible at Cambridge have been painted in their proper places upon plates of iron joined together, which form one concave surface.

I cannot conclude this note without expressing the grief and disappointment I felt, on seeing this sphere in the beginning of the present year 1801. Instead of beholding the new constellations painted thereon, and tracing out many improvements since the time of Dr. Long, as I naturally expected to do; I could hardly find any thing but strong tokens of long neglect, and speedy decay. The whole was exposed to the effects of every change in the atmosphere, by reason of a large window being constantly left open, and the glass in the other windows being broken in several places: some of the constellations could scarcely be discerned, for dust and cobwebs; the planetarium had but few vestiges remaining, by which one might ascertain whether it ever existed or not; and the wires about the zodiac were, in many places, entirely corroded through with rust!!

their apparent magnitude, and brilliancy; as, one of the stars in hydra, one in cygnus, one in medusa's head, and the star *algol*, the latter of which in particular goes through its variations in a period of 2^d 21^h. It varies from the second magnitude to the fourth in about $3\frac{1}{2}$ hours, and back again in the same time, and retains its greatest brightness for the remainder of its period. It is also found, from an attentive examination of the stars with good telescopes, that many, which appear single to the naked eye, consist of two, three, or more stars. Dr. *Herschel*, who has observed the heavens with great constancy and care, finds this to be the case with several stars: thus, α *herculis* is a double star, and so are δ *lyræ*, α *geminorum*, γ *andromedæ*, μ *cygni*, and many others; ν *lyræ* is a treble star; and ε *lyræ*, β *lyræ*, λ *orionis*, ζ *libræ*, are either quadruple, or double-double, as the doctor calls them. But it will be sufficient, in the present part of our enquiry, if we merely allude to these particulars.

75. When the situations of the stars are carefully determined, it becomes necessary, in order to know their situations at any future period, to take into account the effects of their *proper motion*, as it is sometimes called, namely, the apparent motion produced by the *precession of the equinoctial points*. (Art. 57.) By this motion the stars appear as though they moved backwards from west to east, exceeding slowly, at the rate of about 1° in $71\frac{1}{2}$ years; and in consequence of this motion it is, that the constellations of the zodiac, which, we have reason to suppose, about the time of their invention coincided with the first points of the respective signs of the ecliptic to which they belong, have now so far changed their positions as to be found more than a sign advanced: thus we find the constellation aries 3° or 4° within the sign taurus; taurus, within the sign gemini, &c. This motion causes the longitudes of the stars to be always

increasing: and if we reckon the precession at $50\frac{1}{4}''$ annually, when the longitude of a star is once known, it may be soon found for any antecedent or subsequent period: for instance, the longitude of *sirius* in 1690 was 9° 49′ 1″; if we deduct 90 times $50\frac{1}{4}''$, or 1° 15′ 22″, it will leave 8° 33′ 39″ for the longitude of that star in 1600; and by adding 110 times $50\frac{1}{4}''$, or 1° 32′ 7″, the sum will give 11° 21′ 8″ for the longitude of sirius for the year 1800.

76. Some have supposed that the latitudes of the fixt stars are invariable; but such supposition is erroneous, for it assumes in the first place that the *obliquity of the ecliptic* (Art. 40.) is always the same, which is known to be contrary to fact: and it takes for granted also, that the stars are so fixt as to keep their places immoveably; but this is by no means certain; nay the observations of *Dr. Halley*, and others of the best practical astronomers, lead us to adopt a contrary opinion.

77. It is manifest that a variation in the longitudes of the stars must produce a corresponding change in the right ascensions; and although changes in the latitudes, arising from the first of the causes suggested in the last article, will make no difference in the declinations, yet, as the other cause alluded to may, it is thought proper, in the most correct tables of the fixed stars, to appropriate columns for the annual variations in right ascension and declination.

CHAPTER V.

On Parallax, Refraction, and the Equation of Time.

ART. 78. HAVING in the foregoing chapter explained some of the methods which are made use of in determining the relative positions of the fixed stars, we might now proceed to enquire into the distances, sizes, apparent and real motions, of the sun, moon, and other heavenly bodies; but previous to this it is necessary to shew the nature of some corrections which must be applied to our observations, before they will be of use in our enquiries. Of these corrections the first we shall notice is that arising from

PARALLAX.

79. HITHERTO we have considered the various phænomena of the celestial bodies as though they were viewed from the centre of the earth, whereas in reality they were observed from the surface; and this we have done, by conceiving the earth to be, as it were, a point, or that the distance between the centre and surface is *insensible* when compared with the vast distance of the heavens. This is so nearly true with respect to the fixed stars, as to produce no assignable error; but with respect to the sun, moon, and planets, the admission of this idea of the earth's exceeding minuteness would involve some mistakes which may be prevented by duly attending to what follows.

80. In fig. 12, Pl. I. let A be the place of an observer on the earth's surface, A B D; H, G, F, and E, the real places of four heavenly bodies; then the apparent places of these bodies in the supposed concave sphere of heaven, as seen from A, would be O, M, K, and Z, and their places with respect to the centre of the earth would be N, L, I, and Z: now the lines which mark out the apparent places as seen from the surface and from the centre respectively, intersect at the real places of the body, and form angles, as A H C, A G C, A F C; one of these angles, as A G C, is called the parallax of the body G in that *situation*, A F C and A H C the parallaxes of the bodies F and H: that is, the *parallax* of a heavenly body relative to the observer, is *the inclination of two visual rays passing from the body, one to the earth's centre, the other to the point where the observer is placed on its surface.*

81. The parallax of a heavenly body depends upon two circumstances, the distance of that body from the centre of the earth, and its position with respect to the zenith. The separate and joint effects of these two circumstances we shall now exhibit.

82. *The sines of the parallaxes* of two bodies *at unequal distances* from the centre of the earth, but at equal apparent distances from the zenith, *are reciprocally as the distances from the earth's centre.* For, if *f* and *h* are the places of the bodies, it is manifest from the principles of plane trigonometry, that the sine of the angle A *h* C is to the sine of the angle *h f* C or of its supplement A *f* C; that is, that the sine of the parallax of *h*, is to the sine of the parallax at *f*, as *f* C to *h* C; that is, reciprocally as the distances of *h* and *f* from the centre C.

83. Of bodies *at equal distances* from the centre of the earth, *the sines of the parallaxes* are always *as the sines of the apparent distances from the zenith*, or as the co-sines of the apparent altitudes. For, if

E F H be a vertical circle, in the triangle A F C, A C : C F : : sine A F C : sine F A C = sine E A F : and, in the triangle A G C, A C : C G or C F : : sine A G C : sine G A C = sine E A G. Therefore, the antecedents of both proportions being equal, the consequents are proportional, and sine A F C : sine E A F : : sine A G C : sine E A G.

From this it is obvious that the parallax of a body is greatest when it is in the horizon; this is called the *horizontal parallax* : it is also manifest, that an object in the zenith of an observer has no parallax, for the two lines which form the parallactic angle then coincide.

84. *The sine of the parallax of one body, is to the sine of the parallax of another body, in a ratio compounded of the inverse ratio of the distances from the earth's centre, and the direct ratio of the sines of the apparent distances from the zenith.* For, when the distance from the zenith is given, the sine of the parallax is reciprocally as the distance from the earth's centre (Art. 82.); and when the distance from the earth's centre is given, the sine of the parallax is as the sine of the apparent zenith distance (Art. 83.); consequently, when neither is given, the sine of the parallax is conjunctly, as the distance from the earth's centre reciprocally, and as the sine of the zenith distance directly.

85. The general effect of the parallax of a star is to make it appear *nearer the horizon than it really is* : and as the position of the stars is determined by their right ascension and declination, that is, by the period of their passage over the meridian and their meridian altitude; it follows that, *in the meridian*, which is a vertical circle, the parallax of a star is *all in declination*, and *nothing in right ascension.*

86. The parallax of the stars makes them appear *farther from the meridian* than they really are. For, since all the verticals concur at the zenith, and

recede gradually from one another as they approach the horizon, and since the parallax of stars does not remove them from their verticals, but causes them to appear nearer the horizon than they really are, they consequently appear farther from the meridian than they are in reality. Hence the stars appear *to rise later* and *set sooner*, on account of parallax.

87. Parallax causing the stars to appear in other points of the heavens than those they are actually in, must also change the *latitudes*, *longitudes*, *declinations*, and *right ascensions*. The difference of the longitude observed from the centre, called the *true longitude*, and that seen from the observer's place, called the *apparent longitude*, is called the *parallax of longitude*. The same might be remarked of the *parallax of latitude*, of *right ascension*, of *declination*, and of *altitude*.

88. When any three of the five following things are given, namely, the observer's distance from the star, the star's distance from the centre of the earth, the earth's semi-diameter, the true or apparent altitude of the star, and the parallax;—the other two may be determined: for then in the right-lined parallactic triangle, three things are known, whence the other two may be found. Hence it appears that an observer situated on the earth, may know the real distance of any heavenly body from the earth's centre, if he can find its *parallax*, and particularly its *horizontal parallax*, which being largest is most fit for the purpose. It may be proper therefore to explain a few of the methods by which the parallax of a celestial object is discovered.

89. The parallax of a body has been sometimes found, by measuring the apparent distance of the body from a fixed star when they are both on the same vertical; then observing when the body and star are at equal altitudes above the horizon, at which time the distance between the body and star must be

again measured: the difference of these distances will be very nearly the parallax required.

90. But the following method, when practicable, is both more certain in practice and more accurate in theory. Two observers must be placed on the same terrestrial meridian, each about 50 or 60 degrees from the equator, one towards the north, the other towards the south: each person must determine, by a very accurate instrument, the distance of the body from his zenith at the instant it transits the meridian, and the zenith distance of some fixed star whose parallel is nearly known, and differing but little from that of the body. Then on each side comparing the body's zenith distance with that of the fixed star, the distance of the body's apparent parallel from that of the star would be known. If this comparison gave the body the same parallel relatively to each observer, it would have no parallax: but if the comparison shewed that the body appeared in different parallels to the two observers, the distance of these parallels would shew the body's horizontal parallax by this analogy: *as the sum of the sines of the two distances of the body from the zenith is to radius, so is the distance of the two apparent parallels of the body to its horizontal parallax.* In the two last terms of this analogy, the distance of the parallels and the parallax are put instead of their sines, because, on account of their minuteness, they may safely be substituted the one for the other.

When the two observers are on the same side of the equator, use the difference of the sines of the zenith distances instead of their sum in the above analogy. If the two observers were not under the same meridian, but distant from one another by a known quantity, regard must be paid to the motion of the body in declination between the times of its passage over the two meridians.

Example. On Oct. 5, 1751, *M. de la Caille*, at the *Cape of Good Hope*, observed the planet mars to be 1′ 25″·8 below the parallel of λ in aquarius, and at 25° distance from the zenith. On the same day at *Stockholm* (the longitude of which does not differ more than $\frac{1}{4}$ of a degree from that of the Cape), mars was observed to be 1′ 57″·7 below the parallel of λ aquarius, and at 68° 14′ zenith distance. Here 1′ 57″·7—1′ 25″·8=31″·9, the distance of the apparent parallels; and the sines of the zenith distances are ·4226183 and ·9287017. Hence 1·35132 : 1 :: 31″·9 : 23″·6, the horizontal parallax of mars.

91. Another method, which may sometimes be more convenient than the above, as it requires but one observer, is here added: Observe the right ascension of the planet when it passes the meridian, at which time there is no parallax in right ascension (Art. 85.), and compare it with that of a fixed star: about six hours after, take the difference of their right ascensions again, and remark how much the difference *(d)* between the apparent right ascensions of the planet and fixed star has changed in that time. Then observe the right ascension of the planet for three or four days, in order to obtain its true motion in right ascension; and if the proportional motion in right ascension, for the interval elapsed between the taking of the right ascensions of the planet and star on the meridian, and at a distance from it, be equal to *d*, the planet has no parallax in right ascension: but if it be not equal to *d*, the difference is the parallax in right ascension; and the horizontal parallax is found *by dividing the product of the parallax in right ascension into the co-sine of the planet's declination, by the product of the co-sine of the latitude of the place of observation, into the sine of the hour angle from the meridian of the planet.*

To demonstrate this rule, let E Q (fig. 14, Pl. I.) represent the equator, P its pole, Z the zenith, v the true place of the body, and r the apparent place as depressed by parallax; draw the secondaries to the equator P v a, P r b, and $r s$ will be the parallax in declination, and $a b$ in right ascension. Now we have $v r : v s :: 1$ (rad.) : sine $v r s$, or Z v P, and $v s : a b ::$ co-sine $v a$: 1, fromthe nature of parallel circles; hence $v r : a b ::$ co-sine $v a$: sine Z v P; and consequently by multiplying means and extremes we get $ab = \frac{vr \times \text{sine } ZvP}{\text{co-sine } va}$. But it follows from Art. 83, that $v r =$ horizontal parallax $\times$ sine v Z, and it is manifest that sine v Z : sine Z P :: sine Z P v : sine $ZvP = \frac{\text{sine } ZP \times \text{sine } ZPv}{\text{sine } vZ}$; whence by substitution we find $ab = \frac{\textit{hor. par.} \times \text{sine } ZP \times \text{sine } ZPv}{\text{co-sine } va}$; and the horizontal parallax $= \frac{ab \times \text{co-sine } va}{\text{sine } ZP \times \text{sine } ZPv}$. Q. E. D. Note: In the eastern hemisphere, the apparent place is to the east of the true place, and of course the right ascension is diminished by parallax; but in the western hemisphere the right ascension will be increased by parallax. If the right ascension be taken both before and after the meridian, the whole change of parallax in right ascension between the two observations is the sum (s) of the two parts before and after the meridian, and is consequently $= \frac{vr}{\text{co-sine } va} \times$ sum (S) of the sines of the two hour angles: the horizontal parallax will then be $= \frac{s \times \text{co-sine } va}{\text{sine } ZP \times S}$.

The following example will illustrate this rule: On August 15, 1719, *mars* was very near a star in the eastern shoulder of aquarius: at 9^h 18^m in the evening, mars followed the star in 10^m 17^s of time; and on Aug. 16, at 4^h 21^m in the morning, the planet followed the star in 10^m 1^s: consequently in that interval the apparent right ascension of Mars

had increased 16^s in time. Now the planet was on the meridian 3^h 17^m after the first observation, and 3^h 46^m before the second; and his declination when on the meridian was 15°; the latitude of *Paris*, the place of observation, being 48° 50′. According to the time of mars coming to the meridian for some days after, it appeared that in the interval between the two observations the right ascension had increased but 14^s. Therefore 2^s in time, or 30″ in motion, is the real parallax in right ascension; 41° 10′ is the co-latitude; and 49° 15′ and 56° 30′ are the hour angles from the planet's meridian: consequently $\frac{30'' \times \text{co-sine } 15°}{\text{sine } 41° \; 10' \times \text{sine } 49° \; 15' + \text{sine } 56° \; 30'} = \frac{30'' \times \cdot 9659258}{\cdot 6582516 \times 1 \cdot 5914508} = 27'' \cdot 6$ nearly, horizontal parallax of mars; differing from the conclusion drawn from the former method about 4″.

92. If the parallax in altitude of a planet is known, and its position in respect of the ecliptic and circles of longitude, we may readily find the parallax of latitude and longitude: or, if we know the position relative to the equator and circles of declination, we may determine the parallax of right ascension and declination.

For, 1. Let H R (fig. 14, Pl. I.) represent the horizon, E Q the ecliptic, P its pole, Z the zenith, *r* the apparent place of the body, Z *r* K a vertical circle drawn through it, *v* the true place, P*va*, P*rb*, circles of longitude: then, *v r* being the parallax of altitude, *r s* is the parallax of latitude, and *a b* the parallax of longitude. Then, on account of the minuteness of the triangle *v r s*, we may reckon it right-lined, and shall have given *v r*, the angle *v r s*, and *s* a right angle; whence we obtain *r s* and *v s*: and in the similar triangles P *v s*, P *a b*, it will be as sine P *v* : radius : : *v s* : *a b*; whether this is to be added to, or subtracted from, the longitude at *v*, will ap-

pear from considering the positions of E Q, Z K, and P *a*.

2. If E Q represent the ecliptic, P its pole, and H R the horizon; then *r s* will be the parallax of declination, and *a b* that of right ascension, as observed in Art. 91.; and the method of finding them is manifest from what has already been said.

If P *r* and Z *r* coincide, then $v\,r = s\,r$, and $v\,s = 0$. That is, if the vertical circle coincide with the circle of longitude or of declination, the parallax of longitude or of right ascension is nothing, and the parallax of latitude or of declination is equal to the parallax of altitude. This agrees with what was observed in Art. 85.

But if E Q and Z K coincide, then $v\,r = a\,b$, and $r\,s = 0$. That is, if the vertical circle coincide with the ecliptic or the equator, then the parallax of longitude or of right ascension is equal to the parallax of altitude, and there is no parallax of either latitude or declination, according to the respective cases.

98. From the enquiry which has been made into the nature and effects of parallax, it appears, that in all observations upon the heavenly bodies it is necessary to know how much they are depressed by parallax, in order that we may add so much to the apparent, to obtain the true, altitude: however, in most of them, except the moon (whose horizontal parallax varies from about 61′ 32″ to 53′ 52″), the parallax is so minute as to demand our attention but in few cases: the mean horizontal parallax of the sun is not more than about 8″·75, as will be shewn in a subsequent chapter (Chap. XX.); and of course its parallax at the altitude of 50° or 60° will be very inconsiderable; it may be allowed for, at any altitude, by the rule given in Art. 83. or by Table V. at the end of the volume.

But the great use of parallaxes is in finding the distances of the heavenly bodies from the earth, as

mentioned in Art. 88. On this account they are of great importance in astronomy, and, as we shall find, assist us very materially in determining not only the distances, but the dimensions of the planets and their orbits *.

REFRACTION.

94. It is an established principle in the theory of Dioptrics, that as soon as rays of light enter obliquely into a medium, the density of which differs from that of the medium whence they came, they are bent from their rectilinear direction: if the medium into which the rays enter is equally dense throughout, they are only bent at their entrance; but if its density increases in proportion to its depth, the rays of light will be more and more curved, their curvature following a law that is correspondent to that proportion.

This takes place in the rays of light passing from the heavenly bodies to the eye; for the atmosphere, through which they must pass before they arrive at the eye, being unequally dense, viz. rarest in the upper regions, and increasing gradually in density unto the parts contiguous to the earth, where it is most dense, causes the rays to be bent, and arrive at the eye in a different direction from that in which they would come to it, were it not for the effect of the intervening medium: the difference between the real and apparent places of the heavenly bodies, as

* We have considered the effects of parallax upon the supposition that the earth is a sphere; a supposition which will in no case respecting parallax produce a material error, unless in the more nice and accurate calculations relative to the moon, whose parallax is larger than that of any other heavenly body. Those who wish to see the computations in this case, considering the earth as a spheroid, are referred to Chap. VI. of *Mr. Vince's Complete System of Astronomy.*

affected by the passage of the rays of light through the atmosphere, is by astronomers called *refraction;* and is what we now propose to consider.

95. In fig. 16, Pl. I. B O D is designed to represent the surface of the earth, on which the observer stands at O, and F G H is the surrounding atmosphere: now rays of light proceeding from a body Z in the zenith, falling *directly* on the atmosphere, are not refracted thereby, but continue in the same direction till they reach the eye of the observer. But if the rays proceed from a body not in the zenith, as A, they fall on the atmosphere *obliquely*, and are bent or refracted into a curve whose concavity is turned towards the earth's centre C, and thus reach the observer at O in the direction O *a*: the observer perceiving objects only by the impression which rays proceeding from them make on his eye, concludes that those objects are in the direction of the impression his eye received; and therefore he supposes that the body A is at the place *a*. Thus also, if the rays from a star E pass on to the eye by a curve-line H O, the observer judges that star to be in the direction of the side of this curve terminating at the eye; that is, he conceives it to be at *e*, in the direction O *e*, touching the curve at the point O, where it enters the eye. The length of the curve, and its degree of curvature, depends upon the space it runs through in crossing the atmosphere, on the density of the different lamina of the atmosphere through which the rays pass, and on their obliquity in entering these lamina.

96. From the above it naturally follows, *that an object at the zenith is not liable to refraction;* but that, as the distance from the zenith increases, the refraction becomes perceptible, and is proportionally greater as the body observed is farther from the zenith, until it reaches *the horizon, where the refraction is greatest.*

97. Also, that by the effect of refraction the heavenly bodies *appear more elevated above the horizon than they really are;* and that, in order to compensate for this, something must be subtracted from all the apparent altitudes of those bodies: in consequence of this, it likewise happens that the stars, sun, &c. are actually below the horizon, when they appear to be above it; and hence they appear to rise sooner and set later than they would otherwise do.

98. It follows also, that all the heavenly bodies are equally affected by refraction at equal apparent altitudes; and therefore it differs essentially from parallax in two respects: parallax causes the apparent places of the heavenly bodies to be lower than their real ones, refraction causes them to be higher; the former depends both upon the apparent altitude of a body and its distance, the latter depends upon the apparent altitude alone, without any regard to its distance.

99. Refraction likewise occasions two celestial objects to appear nearer each other than they are in reality: thus, let the star A (fig. 15, Pl. I.) be in the vertical A Z, the star B in the vertical B Z, their true distance is A B, an arc of a great circle; but if A be raised by refraction to *a*, and B by the same cause be apparently elevated to *b*, their apparent distance will be *a b*, which must be less than A B, because the two verticals Z H, Z R, converge so as to meet in the zenith Z. If the apparent altitude of each star be the same, and we know the refraction at that altitude, by deducting it we get the true altitude of each star: and then it will be, as sine of Z *a*, the apparent zenith distance, : sine of Z A, the true zenith distance, : : *a b*, the apparent distance between the stars, : A B, their true distance. But if the apparent altitudes of the two stars are different, then in the spherical triangle Z *a b*, we know Z *a*, and Z *b*, the complements of the apparent altitudes,

and *a b*, the apparent distance, whence we find the angle Z; and after adding the refractions, we have given in the triangle Z A B, the two sides, Z A and Z B, with the included angle, whence we discover A B the true distance.

100. The horizontal refraction being the greatest, is the cause of the sun and full-moon appearing of an oval form when rising and setting; for the lower edge being more refracted than the upper one, they are brought nearer together in appearance, and the vertical diameter shortened: but refraction does by no means shorten the horizontal diameter so much; —hence the body appears oval, but not completely elliptical.

101. Refraction will be of various denominations, as it respects different circles on the earth or heavens: these being very similar to the different kinds of parallax, will be sufficiently explained by referring to fig. 13, Pl. I. Thus, if E Q be the equator, P its pole, C L the ecliptic, *p* its pole, Z R a vertical circle, A the real place of an heavenly body, B its apparent place; then A B is the *refraction of altitude*, *d c* is the *refraction of right-ascension*, B *o* the *refraction of declination*, A *u* the *refraction of latitude*, and *i b* the arc on the ecliptic by which the longitude is increased or diminished by refraction, the *refraction of longitude*. The mode of deducing one of these from the other, is so much like that which was illustrated in Art. 92. as to need no farther remarks.

102. The refraction being occasioned by the atmosphere, must be inconstant, and liable to changes in some degree proportional to the variations happening in the state of the air: for when the air is condensed, refraction is increased; and when it is rarefied, refraction is diminished. Refraction is therefore greater in cold weather, or in cold countries, than

in hot; it is greater also in the morning than in the evening, because the air is rarefied by the heat of the sun in the day, and condensed by the coolness of the night. Experience teaches us, that the horizontal refraction varies from 31′ to 35′ or 36′; and the refraction to the height of 10° or 12° varies proportionally. On these accounts no *exact* method can be laid down for deducing the refractions of heavenly bodies, either immediately from celestial observations, or by means of any physical hypothesis. Yet, as a proper combination of experiment and theory will enable us to determine the refractions at all altitudes with sufficient accuracy for every practical purpose, one or two of the best methods are here explained.

103. The refraction may be ascertained, at some particular altitudes, thus: Take the altitude of the sun, or of some star whose right ascension and declination are known, and observe the time by a good clock; then remark carefully the interval which elapses from the moment of the observation to the sun or star's transitting the meridian, from which the hour angle will be known, by reckoning 15° to an hour. Then (fig. 14, Pl. I.) in the triangle Z P *v*, there are known P Z the complement of the latitude, P *v* the complement of the sun's or star's declination, and Z P *v* the hour angle from the meridian; whence we find Z *v* the complement of the altitude. The difference between the altitude obtained by observation and that resulting from this calculation, is obviously equal to the refraction at that altitude.

This rule has been thus put in practice by *Cassini*: he observed the altitude of the sun's centre in latitude 48° 50′ 10″, on May 1, 1738, at $5^h\ 20^m$ in the morning, and found to be 5° 0′ 14″. The sun's declination was 15° 0′ 25″, the distance from the meridian $6^h\ 40^m$, which gives 100° for the hour angle Z P *v*; the co-latitude P Z was 41° 9′ 50″, and the co-decli-

nation $Pv = 74° 59' 35''$, from which the calculation gave $Zv = 85° 10' 8''$ for the co-altitude; consequently $4° 49' 52''$ was the true altitude. Add $8''$ for parallax, to $5° 0' 14''$ apparent altitude of the centre; the sum $5° 0' 22''$ is the apparent altitude corrected for parallax: therefore $5° 0' 22'' - 4° 49' 52'' = 10' 30''$, the refraction at the apparent altitude $5° 0' 14''$.

104. *Another method.* Take the greatest and least altitudes of some circumpolar star (Art. 31.) which at its greatest altitude is either at or near the zenith, and of course then free from refraction: then, having the latitude of the place, the apparent distance of the star from the pole will be known at each observation (Art. 61); the less of these distances taken from the greater, will leave the refraction at the least altitude.

Example. M. de la *Caille* observed a star to pass the meridian of Paris within 6′ of the zenith, and consequently at the distance of 41° 3′ 50″ from the pole, the latitude being 48° 50′ 10″, as above stated: he observed its meridian altitude when under the pole to be 7° 52′ 25″; but the altitude as deduced from the polar distance should be 7° 46′ 20″; the difference between these, viz. 6′ 5″, is the refraction at the apparent altitude, 7° 52′ 25″; on the supposition that the observations were accurately made.

105. The refraction at some one or more apparent altitudes being found, it then becomes necessary to enquire by what law it varies; so that when it is determined in a few instances, it may be ascertained in every possible case: in order to this, let ANi, in Fig. 2, Pl. II. represent the angle of incidence, ANa the angle of refraction, and, of course, iNa the quantity of refraction. Then AT is the tangent of Aa, Ba its sine, and Ci the sine of Ai: draw ae parallel to AN, and because the refraction in the case now under consideration is very small, aei may

be considered as a rectilinear triangle; therefore, by similar triangles, we have $BN : Na :: ei : ai = \frac{Na \times ei}{BN}$. Here, as Na is constant, and the ratio of $Ba : Ci$, the sines of refraction and incidence, is constant also, by the laws of refraction, their difference ei must vary as Ba; and consequently ai will vary as $\frac{Ba}{BN}$. But AT being $= \frac{aN \times Ba}{BN}$, which, because aN is constant, must also vary, as $\frac{Ba}{BN}$; therefore ai the refraction, will be as AT the tangent of the apparent zenith distance of the celestial body: for the angle of refraction ANa is equal to the apparent zenith distance, in this kind of refraction.

This rule will deviate in very small apparent altitudes, where the rays coming to the atmosphere in a direction nearly parallel to the horizon, have a very large portion of it to pass through, which enlarges the refraction so much, that aei cannot without error be considered as a rectilinear angle. On this account, Dr. *Bradley*, adopting the principle, that the force with which a ray is attracted in passing through the atmosphere is uniform, has deduced a very simple and general rule for the refraction r at any altitude a whatever; viz. as radius 1 : cotang. $\overline{a + 3r}$ :: $57''$: r, the refraction in seconds: this rule is found to agree exceedingly well with observations made at the mean states of the barometer and thermometer. He has also given formulæ, by which the refraction may be found, at any other states of the atmosphere, to as great exactness as need be looked for in such enquiries.

106. The methods given in articles 103. and 104. are both liable to objection; for each of them requires a knowledge of the latitude of the place of observation, which cannot be obtained without an allowance for refraction; an allowance that certainly cannot be made with accuracy until the refraction is

determined. An ingenious method, invented by *Boscovich*, is not liable to this objection, and is therefore now given, as it requires the admission of no other principles than those exhibited in the last article. He proposes to find the refractions by the variation of the zenith distances of the circumpolar stars, thus: Let A and a be the apparent zenith distances of a star on the meridian below and above the pole, X and x the respective refractions, B and b the apparent meridian zenith distances of another star below and above the pole, Z and z the corresponding refractions: then the true distances from the pole will be $A + X$, $a + x$, and $B + Z$, $b + z$; and as the distance of the pole from the zenith is equal to half the sum of the greatest and least true zenith distances, we have $A + X + a + x = B + Z + b + z$; consequently (I) $X + x - Z - z = B + b - A - a$. Now, supposing (Art. 105.) the refractions to be as the tangents of the zenith distances, we have, tang. A : tang. a : : X : $x = \frac{X.\ \text{tang.}\ a}{\text{tang.}\ A}$; for a similar reason $Z = \frac{X.\ \text{tang.}\ B}{\text{tang.}\ A}$, and $z = \frac{X.\ \text{tang.}\ b}{\text{tang.}\ A}$. By substituting these in the equation (I) we obtain $X = \frac{\overline{B + b - A - a} \times \text{tang.}\ A}{\text{tang.}\ A + \text{tang.}\ a - \text{tang.}\ B - \text{tang.}\ b}$; whence the other refractions become known. Or, if we adopt Dr. *Bradley*'s theorem, and suppose the refractions to vary as the tangent of the zenith distance diminished by three times the refraction; then, by putting $A - 3X = M$, $a - 3x = m$, $B - 3Z = N$, and $b - 3z = n$, we have by reduction, $X = \frac{\overline{B + b - A - a} \times \text{tang.}\ M.}{\text{tang.}\ M + \text{tang.}\ m - \text{tang.}\ N - \text{tang.}\ n}$, for the true refraction at the apparent zenith distance A; and from this we get the other refractions, by the equations $x = \frac{X.\ \text{tang.}\ m}{\text{tang.}\ M}$, $Z = \frac{X.\ \text{tang.}\ N}{\text{tang.}\ M.}$, and $z = \frac{X.\ \text{tang.}\ n}{\text{tang.}\ M}$; or more generally by Dr. *Bradley*'s rule.

In this manner the refraction of all altitudes may be found by means of a few accurate observations, and reduced into a table for general use, as given in table VIII. at the end of this volume.

107. The atmosphere also causes a particular kind of phenomenon, besides the astronomical refractions, which is the morning and evening *twilight*: it is called twilight, as being between or partaking of two lights, that of the sun and that of the stars; the latin word *crepusculum* arises from its being doubtful whether it be day or night. The morning twilight begins when the sun is not more than about 18° below our horizon, for then his rays first reach the eastern parts of the air so as to be brought, in part by refraction, and part by reflection, within our horizon; as the sun grows nearer rising, his light diffuses itself farther round, and enlightens a larger portion of air, which thus becomes more and more illuminated, till the sun rises; in like manner, after sun-set the light gradually decreases, till the sun is gotten so low that none of his rays can reach the western parts of the atmosphere so as to be brought by reflection and refraction without our visible horizon. There is some little difference in the duration of twilight, arising from the different density of the atmosphere, and other causes; thus, when the vapours and other particles which reflect the solar rays are carried to a greater height than at other times, the sun may be depressed more than 18°, and yet produce twilight; when these particles are lower in the atmosphere, the sun cannot produce twilight when it is depressed 18°. On these accounts the evening twilight is longer than that in the morning, and the twilight is longer in hot weather than in cold, other circumstances being the same. There are also differences in the time of twilight, owing to the different situations of places upon the earth, or to the change of the sun's apparent place in the heavens: thus, if a place be situated in a *parallel sphere*, or

nearly so, the apparent motion of the sun being either quite or nearly parallel, he will be carried round for some months at a less depression than 18°, during which there will, of course, be no real night. In a *right sphere*, the twilight is shortest, because the sun rising and setting at right angles to the horizon, he is depressed 18° below the horizon in the shortest period. In any place in an *oblique sphere*, the nearer it is to one of the poles, the longer the twilight; and, consequently, the nearer the equator, the shorter the twilight. As to the different places of the sun; the twilight is longest in all places in north latitude, when that luminary is in the tropic of cancer; in south latitudes, when the sun is in the tropic of capricorn. The time of the shortest twilight is different in different latitudes: in England it is shortest about the eleventh or twelfth of October, and the second or third of March, as will be shewn by calculation in the next chapter (Art. 141.).

Equation of Time.

108. It would be far from an easy task to give a precise definition of *duration* and *time*; but it is not necessary to attempt it, since from the succession of our own ideas, and from the successive variations of external objects, we easily acquire the ideas of them and of their measures. We conceive true or absolute time to flow uniformly in an unchangeable course, which alone serves to measure with exactness the changes of all other things; and unless we apply to the vulgar measures of time, which are gross and inaccurate, some proper corrections, the conclusions are always found erroneous. Yet however various the flux of time may appear to different intellectual beings, it cannot for a moment be thought to depend upon, or be regulated by, the ideas of any created being whatever: it therefore becomes requisite to choose

some object as a proper and adequate measure of time. For this purpose we fix upon either some natural object, whose motion is conspicuous and nearly uniform, or some artificial contrivance invented for the purpose of determining this measure: of the first kind the *sun* has been chosen, and of the latter a *pendulum clock*; but as the motion of the sun is not exactly uniform, and a clock is liable to variation in its motions, as it is affected by the changes in the atmosphere with respect to heat or cold, dryness or moisture, some mode of comparison must be contrived which will enable us to correct the *measure of time* deduced from the one, by means of that which is gained from the other. This is best done by tracing out the nature and extent of the irregularities in the sun's apparent motions.

109. The *astronomical day*, at any place, begins when the sun's centre is on the meridian of that place; it is divided into 24 hours, reckoning in a numerical succession from 1 to 24: the first 12 are sometimes distinguished by the mark P.M. signifying *post meridiem*, or afternoon; and the latter 12 are marked A.M. signifying *ante meridiem*, or before noon. But astronomers generally reckon through the 24 hours, from noon to noon; and what are by the civil or common way of reckoning called morning hours, are by astronomers reckoned in the succession from 12, or midnight, to 24 hours. Thus 9 o'clock in the morningof February 14th, is, by astronomers, called February the 13th at 21 hours.

110. *The sun's daily motion in longitude* is the arc of the ecliptic run through in a day; his *daily motion in right ascension* is the corresponding arc of the equator: the *mean* daily motion in either circle is measured by 59′ 8″ nearly. For $365^d : 1^d :: 360° : 59' 8''\cdot 2$.

111. The interval of time between two successive transits of the sun's centre over the same meridian,

is called a *solar*, or, as above, an *astronomical day;* it is measured by the sum of the whole equator (360°), and an arc of it equal to the daily motion in right ascension. For at the end of a diurnal rotation, which is known by observation to be uniform, the meridian is against the same star, or point of the ecliptic, it was against at the preceding noon (setting aside the very minute difference arising from the precession of the equinoxes); but the sun, during this period, has removed from that star to another, which has a greater right ascension: therefore, before the sun can be again on the same meridian, such an additional arch must be described as is equal to the daily motion in right ascension.

112. A *sidereal day* is the interval between two successive returns of the same fixed star to the same meridian; it is less than the solar day, for it is measured by 360°, whereas the mean solar day is measured by 360° 59′ 8″ nearly.

113. *True or mean time*, is that shewn by a clock whose 24 hours measure the time which the sun takes to describe an equatorial arc equal to 360° 59′ 8″: *apparent time*, is that shewn by the sun or a dial, where 24 hours are measured by the sum of 360°, and that day's motion in right ascension. The *equation of time*, is the difference between *mean* and *apparent* time, or between the *mean* and *apparent* noons, or between the times shewn by a *clock* and a *sun-dial*. If this *equation* can be accurately known at all periods, it will always enable us to reduce apparent time to that which is *absolute* or *true*.

114. The difference between the measures of a mean solar day, and a sidereal day, viz. 59′ 8″, reduced to time, at the rate of 24 hours to 360°, gives $3^m\ 56^s$; from which we learn that a star which was on the meridian with the sun on one noon, will return to that meridian $3^m\ 56^s$ previous to the next noon; therefore a clock, which measures mean days

by 24 hours, will give $23^h\ 56^m\ 4^s$ for the length of a sidereal day.

115. The mean and apparent solar days are never equal, except when the sun's daily motion in right-ascension is $59'\ 8''$; this is nearly the case about April 15th, June 15th, September 1st, and December 24th: on these days the equation is nothing, or nearly so; it is at the greatest about November 1st, when it is $16^m\ 14^s$.

116. The *equation of time* is calculated by tracing out the effects of three combined causes; *the obliquity of the ecliptic, the sun's unequal apparent motion therein,* and *the precession of the equinoctial points:* in consequence of the first of these, in the first and third quadrants of the ecliptic from aries, that is, between aries and cancer, and between libra and capricorn, the right ascension being less than the mean longitude, the point of right ascension is to the west, and therefore the apparent noon precedes the mean noon; but in the second and fourth quadrants, namely between cancer and libra, or capricorn and aries, the right ascension being greater than the longitude, or the mean motion taken in the equator, the mean noon is westward, and therefore precedes the apparent noon. But, even if the plane of the ecliptic coincided with that of the equator, there would be a correction necessary; for the apparent annual motion of the sun being not quite uniform, a longer arc would be described in some days than others; that is, since the right-ascension and longitude would in this case be the same, the daily increments of right ascension would be unequal.

117. Dr. *Maskelyne* has invented a rule for computing the *equation of time*, in which all the three causes are considered; it was investigated in the following manner: Let A P L Q (fig. 3, Pl. II.) be the ecliptic, A L Q the equator, A the first point of

aries, P the point where the sun's apparent motion is slowest, S any place of the sun; draw Sv perpendicular to the equator, and take An = AP. When the sun begins to move from P, suppose a star to begin to move from n with the sun's mean motion in right-ascension or longitude, viz. (Art. 110.) at the rate of 59′ 8″ in a day, and when n passes the meridian let the clock be adjusted to 12. Take nm = Ps, and when the star comes to m, if the sun moved uniformly with his mean motion, he would be found at s; but at that time let S be the place of the sun. Let the sun S, and consequently v, be on the meridian; and then as m is the place of the imaginary star at that instant, mv must be the equation of time. The sun's mean place is at s, and as An = AP, and nm = Ps, we have Am = APs, consequently mv = Av — Am = Av — APs. Let a be the mean equinox, or the point where it would have been if it had moved with its mean velocity, and draw az perpendicular to AQ; then Am = Az + xm = Aa × co-sine zAa + zm: or because the co-sine of zAa the obliquity of the ecliptic, 23° 28′, is = $\frac{11}{12}$ very nearly, Am = $\frac{11}{12}$ Aa + xm: hence mv = Av — zm — $\frac{11}{12}$ Aa. Here Av is the sun's true right ascension; zm the mean right ascension, or mean longitude; and $\frac{11}{12}$ Aa (viz. Ax) is the equation of the equinoxes in right ascension; therefore the equation of time is equal to *the difference of the sun's true right ascension and his mean longitude corrected by the equation of the equinoxes in right ascension*. . . . . . When Am is *less* than Av, *mean* or *true* time precedes *apparent*; when it is greater, *apparent* time precedes *mean*. That is, when the sun's true right ascension is *greater* than his mean longitude corrected as above shewn, we must *add* the

equation of time to the apparent to obtain the mean time; and when it is *less*, we must *subtract*. To convert mean time into apparent, we must *subtract* in the *former* case, and *add* in the *latter*.

Tables of the equation of time are computed by this rule, for the use of astronomers: they are either calculated for the noon of each day, as given in the *Nautical* and some other *almanacs*; or for every degree of the sun's place in the ecliptic, as is done in Table VI. at the end of the volume. But a table of this kind will not answer accurately for many years, on account of the precession and other causes, which render a frequent revisal of the calculations necessary.

118. It must be evident from the nature of the chief subjects treated on in this chapter, viz. Parallax, Refraction, and the Equation of Time, that the respective corrections for each of them must be carefully attended to in our observations upon the heavenly bodies, in order that the conclusions resulting from them may be relied upon in point of accuracy. The ancients, though not entirely ignorant of the nature of these corrections, conceived the necessity of attending to them to be much les than it really is, and consequently but seldom regarded them: on this account we cannot place so much dependence upon some of their observations as might be wished; though, on the whole, we have much greater reason to admire them for their skill, than to complain of their want of exactness.

CHAPTER VI.

On determining the Times of the Rising, Culminating, Setting, &c. of the Sun and fixed Stars.

ART. 119. WHEN we enquire on what part of the day the sun is in different positions, with respect to the horizon, meridian, prime vertical, &c. we always consider either the sun's declination, longitude, or right ascension, as known: for the nature of the sun's apparent motion is now so well ascertained, that his declination for every day at noon is determined with great precision; and this being known, together with the obliquity of the ecliptic, we readily find either the longitude or right ascension, by a common and easy rule in right-angled spherical triangles. Nay, in the *Nautical Almanac* (which, together with the *Requisite Tables*, cannot be too ſtrongly recommended to the young astronomer), not only the declination, but the longitude and right ascension of the sun are given for every day at noon; and either of them may be found for any intermediate time, by proportion. Thus, suppose the declination on April 15th at noon is 9° 41′ 50″ N., and on the 16th at noon 10° 3′ 14″ N.; and the declination for 8 o'clock P.M. be required; say, as $24^h : 8^h :: 21' \ 24''$ (the difference between the two declinations) : 7′ 8″; which added to 9° 41′ 50″, gives 9° 48′ 58″ N. for the declination sought.

120. It will be proper to remark in this place, that the declinations, longitudes, or right ascensions, as given in the *Nautical Almanac*, will not answer exactly for any other meridian than that of Green-

wich, until an addition or subtraction is made correspondent to the difference of longitude of the place from Greenwich. For since the sun, in his apparent diurnal motion, is on the meridian of places eastward of Greenwich sooner than on the meridian of Greenwich, and comes to the meridian of places westward of Greenwich after he has quitted the meridian of Greenwich, and the difference of time in either case is proportional to one hour for 15° of longitude; we must, when the declination, &c. is required for any place in *east* longitude, *lessen* the known declination at Greenwich proportionally to the difference of longitude, if the declination at that time be *increasing*, or *add* to it proportionally, if the declination be *decreasing*: a contrary mode of procedure must be adopted if the place be in *west* longitude. Thus, suppose the sun's declination was required at eight o'clock P.M. April 15th, in 60° *west* longitude: then say, as 360° : 60° : : 21′ 24″ (difference of declinations as found in Art. 119.): 3′ 34″; which, as the declination is *increasing*, and the longitude *west*, must be added to 9° 48′ 58″ N. (the declination at eight o'clock P.M. at Greenwich); gives 9° 52′ 32″ for the declination sought. These corrections may be made, by means of the *Requisite Tables*, without any calculation. . . . . A due regard to these preliminary observations will be of considerable utility in solving accurately the problems we now enter upon.

121. PROB. I. *To determine the times on which the sun rises and sets, having the latitude of the place, and the sun's declination.*

Ex. At Cambridge, latitude 52° 12′ 35″ N. when does the sun rise and set on May 4th, 1801?

In fig. 4, Pl. II. where H R is the horizon, E Q the equator, Æ C the ecliptic, and *m* the sun's place therein on May 4th, the declination being north; we shall have *a b* for the parallel of declination nearly in which the sun will move on the whole of that day,

and if the figure represent the eastern hemisphere, i will be the point on which the sun will rise. Then, in the triangle Z N i are known Z N $=37° \; 47' \; 25''$ the co-latitude, N$i=74° \; 5' \; 35''$ the co-declination, and Z $i=90°$ the distance from the zenith to the horizon; to find Z N i the hour angle from the meridian: which converted into time, at the rate of 15° to an hour, will give the time from apparent noon. Now, one side, Z i, of the triangle being $=90°$, it may be solved by *Napier*'s rule of the circular parts, taking the parts adjacent to Z i, and the complements of the other three parts, for the circular parts. Therefore, as rad. : co-tangent N i : : co-tangent Z N : co-sine Z N i; or as rad. : tang. declin. : : tang. latitude : co-sine of hour angle from noon. Hence, in the present case, rad. . . 10·0000000

present case, rad.	10·0000000
tang. 15° 54′ 25″	9·4548275
tang. 52 12 35	10·1104697
Co-sine of 68° 26′ 11″ or of its suppt. 111° 33′ 49″*	9·5652972

This converted into time, gives $7^h \; 26^m \; 14^s$ from noon, for the time of the sun's setting; and taken from 12^h, leaves $4^h \; 33^m \; 46^s$ for the time on which the sun rises, May 4th, at Cambridge.

122. But in the above solution we have not taken into account either the effects of refraction and parallax, or the change of declination in the interval between the sun's rising and setting: yet, as the solution cannot be accurate whilst these are neglected, we will make a fresh calculation, in which they will be attended to. Here we shall, by proceeding as

* It will be evident from a slight inspection of the figure, that when the latitude of the place and the sun's declination are of the same kind, viz. both *north*, or both *south*, then Z N i, the hour angle, being greater than Z N O, will exceed 90°: and that when the latitude and declination are of different kinds, the hour angle will be less than 90°.

directed in Art. 119. find the declination of the sun for the estimate times of rising and setting (say $7\frac{1}{2}$ hours from noon) to be 15° 49′ 19″ and 15° 59′ 31″; therefore, N *i* = 74° 10′ 41″, or 74° 0′ 29″: also, Z *i* = 90° × mean horizontal refraction — horizontal parallax = 90° × 33′ — 8″ = 90° 32′ 52″; and Z N = 37° 47′ 25″: then there are given the three sides of an oblique spherical triangle, to find the angle Z N *i*, which may be thus determined. First, for the sun rising:

Z N =	37°	47′	25″
Z *i* =	90	32	52
N *i* =	74	10	41
Sum	202	30	58
From ½ sum	101	15	29
Take Z *i*	90	32	52
Difference	10	42	37

Arith. comp. sine N *i*	0·0167737
Do. sine Z N	0·2127005
Sine of ½ sum	9·9915618
Sine of difference	9·2691569
	2)19·4901929
Co-sine of ½ Z N *i* or co-sine of ½ 112° 26′ 12″	9·7450964

Hence the time from noon is $7^h\ 29^m\ 45^s$, or $4^h\ 30^m\ 15^s$, the time when the sun rises.

2dly. For the time of sunsetting:

$ZN =$	37°	47′	25″
$Zi =$	90	32	52
$Ni =$	74	0	29
Sum	202	20	46
From ½ sum	101	10	23
Take Zi	90	32	52
Difference	10	37	31

Arith. comp. sine Ni	0·0171406
Do. sine ZN	0·2127005
Sine of ½ sum	9·9916895
Sine of difference	9·2657254
	2)19·4872560
Co-sine of 56° 20′ 52″ or co-sine of ½ ZNi	9·7436280

Therefore twice 56° 20′ 52″, that is 112° 41′ 44″ is the hour angle from noon, and this reduced to time, gives $7^h\ 30^m\ 47^s$ the moment when the sun sets.

123. From a due consideration of the last article we may infer, that when the latitude and declination are of the *same kind*, and the declination is *increasing*, the interval from sun-rising to noon will be *less* than from noon to sun-set; but when the declination is *decreasing*, from sun-rising to noon the interval will be *greater* than from noon to sun-set: on the contrary, when the declination and latitude are of *different kinds*, if the declination is *increasing*, the interval from sun-rising to noon will be *greater* than from noon to sun-set; but if the declination be *decreasing*, the former interval will be *less* than the latter.

124. Were the times at which the sun rises and sets, at any place on the same parallel of lati-

tude as Cambridge, to be determined, we must proportion the change of declination to the difference of longitude, as directed in article 120, and we shall find that if the sun have *north* declination *increasing*, the days will be *longer* in *west* longitude, and *shorter* in *east* longitude (in the same parallel), than at Cambridge; if the north declination be *decreasing*, the days will be *shorter* in *west* longitude, and *longer* in *east*: but if the declination be *south* and *increasing*, the days will be *shorter* in *west*, and *longer* in *east* longitude; and contrariwise if the south declination be *decreasing*. Similar remarks to these will apply in any other situation in north latitude; and just the reverse in south latitude. But they are too obvious to need stating more at large.

125. We have still paid no regard to the equation of time; nor have we made an allowance for the apparent semidiameter (Art. 304. or Tab. IV.) of the sun: to bring the latter of these into the calculation, let it be added to the side Z i of the triangle, and the work performed by the rule used in Art. 122. But this is left for the exercise of the learner.

126. PROB. II. *To find the sun's azimuth from the north or south when rising or setting, the latitude and declination being known.*

EX. On May 4, 1801, what is the sun's azimuth from the north, when rising at Cambridge?

Referring to the same figure, we have given $ZN = 37^\circ\ 47'\ 25''$, $Ni = 74^\circ\ 5'\ 35''$, and $Zi = 90^\circ$, to find the angle iZN, which is measured by the arc iR. From the circular parts we deduce this analogy; sine ZN : rad. : : co-sine N i : sine iZN; or, as co-sine of latitude : rad. : : sine of declination : co-sine of azimuth from the north. That is, in the instance before us:

Arith. Comp. co-sine	52° 12′ 35″	0·2127004
Rad.		10·0000000
Sine	15 54 25	9·4378706
Co-sine 63° 25′ 53″ Azimuth from the north		9·6505710

In this problem, as well as the last, if we wish to allow for refraction, parallax, and the change of declination, we must have recourse to the rules for oblique spherical triangles; for we shall have $Z\,i = 90° + 33' - 8''$, $Z\,N =$ the co-latitude, and $N\,i =$ the co-declination reduced to the time of the sun's rising or setting, to find the angle $N\,Z\,i$.

127. PROB. III. *Given the latitude of the place, and the sun's declination; to find the time when he is on the prime vertical, and his altitude at that time.*

EX. What are the times when the sun is on the east or west, and what his altitude at those times, at Cambridge, on May 4, 1801?

Here are given in the triangle $Z\,N\,w$, $Z\,N$ the co-latitude, $N\,w$ the co-declination, and the angle $w\,Z\,N = 90°$; to find $Z\,N\,w$ the hour angle from apparent noon, and $Z\,w$ the co-altitude. And here we are furnished with the following analogies; viz. rad. : co-tang. $N\,w$: : tang. $Z\,N$: co-sine $Z\,N\,w$; or, as rad. : tang. declination : : co-tang. latitude : co-sine of the hour angle from noon; also, co-sine $Z\,N$: rad. : : co-sine $w\,N$: cosine $Z\,w$; or, as sine latitude : rad. : : sine declination : sine of the altitude. Hence, for this example:

Radius		10·0000000
Tangent	15° 54′ 25″	9·4548275
Co-tang.	52 12 35	9·8895736
Co-sine 77° 13′ 55″ Hour angle from noon		9·3444011

Arith. Comp. sine	52° 12′ 35″	0·1022306
Radius		10·0000000
Sine	15° 54′ 25″	9·4378706
Alt. sine	20° 17′ 34″	9·5401012

The above hour angle converted into time, gives $5^h\ 8^m\ 56^s$ for the time past noon when the sun is on the prime vertical to the *west*; or taken from 12, leaves $6^h\ 51^m\ 4^s$ in the morning, when the sun is on the prime vertical to the *east*.

128. From this problem we learn, by attending to the figure, that when the latitude of the place and the sun's declination have the *same* name, then the greater the declination and latitude, the *greater* the *altitude*, and the *less* the time from noon: and having *contrary* names, the *greater* the *depression*, and the *greater* the time from noon. In this, as well as most of the subsequent problems in this chapter, the corrections for refraction, parallax, and change of declination, are not noticed, as they may easily be supplied.

129. Prob. IV. *Given the latitude of the place, and the sun's declination; required the sun's altitude and azimuth at six o'clock.*

Ex. What are the sun's altitude and azimuth, at six o'clock, on May 4, 1801, at Cambridge?

Referring still to the same figure, we have given, in the triangle Z N *e*, Z N the co-latitude, N *e* the co-declination, and Z N *e* = 90°; to find Z *e* the co-altitude, and *e* Z N measured by the azimuth: which may be done by these analogies; as rad. : co-sine Z N : : co-sine N *e* : co-sine Z *e*; or as rad. : sine latitude : : sine declination : sine altitude; also, as rad. : sine Z N : : co-tang. N *e* : co-tang. *e* Z N; or, as rad. : co-sine latitude : : tang. declination : co-tang. azimuth. Hence the operations below:

Radius		10·0000000
Sine	52° 12′ 35″	9·8977694
Sine	15 54 25	9·4378706
Sine 12° 30′ 32″ Altitude at six		9·3356400

Radius		10·0000000
Co-sine	52° 12′ 35″	9·7872996
Tangent	15 54 25	9·4548275
Co-tang. 80° 5′ 39″ Azimuth from north		9·2421271

130. From a due consideration of this problem we may learn, that as the declination of the *same* name as the latitude *increases*, the *altitude* at six o'clock *increases*, and the azimuth *lessens*; the contrary happens while the declination is *diminishing*: if the declination be *different* from the latitude, the *depression* of the sun at six, and the azimuth from the *opposite* point H of the horizon, will be affected in a similar manner, by the increase or decrease of the declination.

131. Prob. V. *Given the latitude of the place, the sun's declination, and altitude, to find his azimuth and the time.*

Ex. At Cambridge, on May 4, 1801, what are the sun's azimuth and the time from noon, when the altitude is 40°?

In this problem are given Z N the co-latitude, Z *m* the co-altitude, and N *m* the co-declination, to find Z N *m* the hour angle, and *m* Z N the azimuth: these may be found by the rule which was used in Art. 122.*: thus, for the hour angle:

* *Bishop Horsley*, at p. 215 of his *Elementary Treatises*, has pointed out a process for the determination of any angle of a spherical triangle when the sides are given, which, as it may be fre-

Z N =	37°	47′	25″
Z *m* =	50	0	0
N *m* =	74	5	35
Sum	161	53	0
From ½ sum	80	56	30
Take Z *m*	50	0	0
Difference	30	56	30

Arith. comp. sine Z N	0·2127004
Do sine N *m*	0·0169567
Sine of ½ sum	9·9945496
Sine of difference	9·7108917
	2)19·9350984
Co-sine of 21° 52′ 28″ or co-sine of ½ Z N *m*	9·9675492

Therefore the hour angle is 43° 44′ 56″, and the time before or after noon 2h 55m. Then for the azimuth:

½ Sum, as before,	80°	56′	30″
Take N *m*	74	5	35
Difference	6	50	55

quently applied with advantage, is here given: Suppose in the triangle Z N *m* (fig. 4, Pl. II.), of which all the sides are known, it is required to determine the angle Z N *m*: in an indefinite right line take *a f* (fig. 4, *a*, Pl. II.) equal to the difference of the co-sines of the given arcs Z N, N *m*. Through one extremity of that line, as *f*, draw *f b*, at right angles with *a f*; and from the other extremity *a*, to *f b*, draw down *a b* equal to the chord of the arc Z *m*. Then upon the base *f b* construct a triangle whose sides *b g*, *f g*, shall be equal to the sines of the given arcs *m* N, Z N, respectively. In this triangle, the angle *f g b*, opposite to the side *f b*, is equal to the spherical angle Z N *m*. The resolution, therefore, of these two right-lined triangles *a b f*, *b g f*, solves the spherical problem.

Arith.	Comp. sine Z N	0·2127004
	Do. sine Z m	0·1157460
	Sine of $\frac{1}{2}$ sum	9·9945496
	Sine of difference	9·0764452
	2)	19·3994412
	Co-sine of 59° 56′ 34″ or co-sine of $\frac{1}{2}$ m Z N	9.6997206

Consequently 119° 53′ 8″ is the azimuth from the north, or 60° 6′ 52″ from the south.

132. In this problem we have an excellent method of finding the time at any hour of the day, by means of the sun's altitude; but that the time may be determined accurately, the altitude must be taken with great care, and the proper allowances must be made for refraction and parallax. We may also by assuming any errour in finding the altitude, ascertain the corresponding errour in time: for, let $r\,s$ (in the same figure) represent the errour in altitude, and let $m\,s$ be drawn parallel to the horizon, then, as we suppose the sun's declination to continue the same during so short an interval, m and r will be the apparent and real places of the sun on the parallel of declination, and the arc $m\,r$, or angle m N r, will measure the corresponding error in time. Now the angle at s is 90°, and the triangle $m\,s\,r$ being so exceedingly small, may safely be supposed rectilinear: hence

$s\,r : r\,m :: \text{sine } s\,m\,r : \text{radius},$

and $m\,r : p\,q ::$ co-sine $q\,r$: radius; therefore, by multiplying the corresponding terms, $s\,r : p\,q ::$ sine $s\,m\,r \times$ co-sine $q\,r$: radius squared; and consequently the arch $p\,q = s\,r \times \frac{\text{Radius}^2}{\text{sine } s\,m\,r \times \text{co-sine } q\,r}$. But Z$r$N

$= s\,m\,r$, $s\,r\,m$ being the complement of each; and we have sine $Z\,r\,N$ (or sine $s\,m\,r$) : sine $Z\,N$ :: sine $r\,Z\,N$: sine $N\,r$ or co-sine $q\,r$; consequently, sine $s\,m\,r \times$ co-sine $q\,r =$ sine $Z\,N \times$ sine $r\,Z\,N$; and therefore $p\,q = s\,r \times \frac{\text{Radius}^2}{\text{sine } Z\,N \times \text{sine } r\,Z\,N} = r\,s \times \frac{\text{Radius}^2}{\text{co-sine lat.} \times \text{sin. azimuth}}$. From which it appears, that since the sine of the azimuth is *a maximum*, when the sun is on the prime vertical, the errour in time will then be *a minimum*, provided the refraction be properly allowed for. It is best therefore to deduce the time from an altitude taken when the sun is on or near the prime vertical.

Suppose now that, in the example to this problem, the errour in the altitude is 1′, then $p\,q$ the errour in time $= 1' \times \frac{\text{Radius}^2}{\text{Co-sine } 52^\circ\ 12'\ 35'' \times \text{sine } 119^\circ\ 53'\ 8''} = \frac{1}{\cdot 6128 \times \cdot 8670} = 1' \cdot 882$ of a degree, or $7^s \cdot 528$ in time.

133. It may be proper to remark, that the work of this problem might have been shortened, by first finding the azimuth, and then the hour angle by means of the proportion between the sines of opposite sides and angles. Had the azimuth been given instead of the altitude, the time and the altitude might easily have been determined, by attending to the parts given and required, in the triangle Z N *m*.

134. Prob. VI. *Given the latitude of the place, less than the sun's declination, also the declination and azimuth; required his altitude and the time of observation.*

Ex. In the latitude of 13° 30′ N, when the sun has 23° 28′ N. declination; what are the times of the day, and the sun's altitude, when he is seen on the east-north-east azimuth, or 67° 30′ from the north?

Let the primitive circle H Z R Q, in fig. 5, Pl. II. represent the meridian of the place, H R the hori-

zon, Z *a* the prime vertical, N and S the poles, N R = E Z the latitude, *n m* the parallel of 23° 28′ N. declination, Z D a verticle circle, which will either touch the parallel *n m*, or cut it in two points A and B; in the present case it intersects, therefore let N A, N B, part of two hour circles, be drawn. Then, in the spheric triangle N Z A or N Z B, there are known, the co-latitude N Z = 76° 30′, the co-declination N A or N B = 66° 32′, and the azimuth N Z D = 67° 30′; to find Z A or Z B the co-altitudes, and the angles Z N A, Z N B.

Here, radius : co sine azimuth : : cotang. lat. : tang. of a fourth arc M = 57° 54′; and sine lat. : sine declin. : : co-sine M : co-sine of a fifth arc N = 24° 56′. Then M + N, or 57° 54′ + 24° 56′ = 82° 50′ = Z A, comp. of least altitude; and M — N, or 57° 54′ — 24° 56′ = 32° 58′ = Z B, comp. greatest altitude. Consequently, when the sun has 7° 10′ or 57° 2′ of altitude, he is on the given azimuth.

Again, co-sine declin. : sine azimuth : : co-sine least alt. : sine angle from noon 87° 55′; and, co-sine declin. : sine azimuth : : co-sine great. alt. : sine angle from noon 33° 14′. These, reduced to time, give $5^h\ 51^m\ 40^s$ and $2^h\ 12^m\ 56^s$, the respective times from noon; therefore the sun will be seen on the E N E. azimuth at $6^h\ 8^m\ 20^s$, and again at $9^h\ 47^m\ 4^s$, both in the morning; also, in the afternoon he will be on the W N W. azimuth at $2^h\ 12\ 56^s$ and $5^h\ 51^m\ 40^s$.

135. But suppose it were required to find at what time, and at what altitude, the azimuth of the sun would be greatest, at the same place and day: the azimuth circle in this case will touch the parallel *n m* in the point C, and there will be given in the spherical triangle Z N C, right-angled at C, the co-latitude N Z = 76° 30′, and the co-declination N C = 66° 32′, to find N Z C the azimuth, Z N C the hour angle, and Z C the co-altitude: these, by the well known propor-

tions of right-angled triangles, are soon found to be, 70° 37′ the greatest azimuth, 56° 26′ the hour angle, or $3^h\ 45^m\ 44^s$ the hour from noon, and 54° 8′ the co-altitude, or 35° 52′ the altitude.

Hence it appears that the azimuth is altered only 3° 7′ in $2^h\ 6^m$, and consequently, as the alteration is so small, the variation of the compass may be observed with more certainty in the torrid zone than elsewhere.

136. Prob. VII. *Given the latitude of the place and the sun's declination, required the time when the twilight begins and ends.*

Ex. At what time does the twilight begin and end, at Cambridge, on May 4, 1801?

It was observed in Art. 107. that, at a medium, the twilight begins in the morning, and ends in the evening, when the sun is 18° below the horizon: therefore if $t\ l$ (in fig. 4, Pl. II.) represent a parallel to the horizon drawn 18° below it, or, as it is sometimes called, the *crepusculum circle*, $a\ b$ the parallel in which the sun moves; then will o be the sun's place when the twilight begins and ends: hence there are given, in the triangle Z N o, Z N the co-latitude, N o the co-declination, and Z o = 90° + 18° = 108°; to find Z N o the hour angle from noon; which may be done by the rule used in Art. 122. thus:

ZN =	37°	47′	25″
N o =	74	5	35
Z o =	108	0	0
Sum	219	53	0
From ½ sum	109	56	30
Take Z o	108	0	0
Difference	1	56	30

Arith.	Comp. sine Z N	0·2127004
	Do. sine N *o*	0·0169567
	Sine $\frac{1}{2}$ sum	9·9731515
	Sine of difference	8·5299659
		2)18·7327745
	Co-sine of 76° 33′ 25″ or co-sine of $\frac{1}{2}$ Z N *o*	9·3663872

Consequently 153° 6′ 50″ is the hour angle, and $10^h\ 12^m\ 27^s$ the time from noon: that is, $1^h\ 47^m\ 33^s$ is the time when day breaks, and $10^h\ 12^m\ 27^s$ in the evening is the time when twilight ends. If the time on which the sun rises and sets be taken from these times, we shall have $2^h\ 46^m\ 13^s$ for the duration of either morning or evening twilight. If the change of the sun's declination had been taken into the account, remarks would have been deduced similar to those in Art. 123.

137. When the declination is equal to the difference between the co-latitude and 18°, then the parallel of declination *a b* will just touch the *crepusculum circle t l*, and the twilight will just continue all night: when the declination becomes greater than the difference between the co-latitude and 18° (the latitude and declination being of the same kind), then *a b* will neither cut nor touch *t l*, and there will be no night at that place, but the twilight will continue from sun-setting to sun-rising: on this account, there is no total darkness at London, the latitude of which is 51° 32′, from May 22d to July 21st, the sun's declination during all that interval being greater than 20° 28′, which is the difference between 18° and 38° 28′, the complement of the latitude.

138. PROB. VIII. *Given the latitude of the place, to*

find the time of the shortest twilight, and its duration.

Let $a\ b$ (fig. 8, Pl. II.) be the parallel of the sun's declination at the time required; draw $c\ d$ indefinitely near and parallel to it, and T W a parallel to the horizon 18° below it; then v P w, s P t measure the twilight on each parallel of declination, and when the twilight is shortest, the increment of the hour angle being $= o$, these must be equal; hence v P $r = w$ P z, therefore $v\ r = w\ z$; and as $r\ s = t\ z$, and the angles r and z are right ones, $r\ v\ s = z\ w\ t$; but P $v\ r =$ 90° $=$ Z $v\ s$; take Z $v\ r$ from each, and there remains P v Z $= r\ v\ s$: for the same reason P w Z $= z\ w\ t$; hence P v Z $=$ P w Z. Take $v\ e = w$ Z $=$ 90°, and join P e; then as P $v =$ P w, $v\ e = w$Z, and P $v\ e =$ P w Z, we have P $e =$ P Z; and if P y be perpendicular to e Z, then will Z $y = y\ e$. Now, by trig. co-sine P v : co-sine P e or P Z : : co-sine $v\ y$: co-sine $e\ y$; that is, sine declination : sine latitude : : sine $e\ y$: co-sine $e\ y$: : tang. $e\ y =$ 9° : radius; or, *radius : sine latitude : : tang.* 9° *: sine of the sun's declination at the time of shortest twilight.* Because P Z is always less than 90°, and Z $y =$ 9°, therefore P y is always less than 90°, and consequently its co-sine is *positive*; also $v\ y$ is always greater than 90°, therefore its co-sine is *negative*; hence co-sine P v ($=$ co-sine P y × co-sine $v\ y$) is *negative*, consequently P v is greater than 90°, and therefore the declination and latitude are of contrary kinds.

139. To find the length of the shortest twilight: since w P Z $= v$ P e, therefore Z P $e = v$ P w measuring the time sought. Now sine P Z or co-sine latitude : radius : : sine Z $y =$ 9° : sine Z P y, which doubled, gives Z P e or v P w; and this, being converted into time, gives the duration of the shortest twilight.

140. The same analogies for the solution of this pro-

blem, have been deduced from a *stereographic projection*, in the following manner:

PROJECT. Having described, about the centres P, *m* and *n* (fig. 6, Pl. II.), the primitive circle Æ H Q representing the equator, *z* O H the horizon, and *p* S *l* a parallel thereto, at 18° distance below it, describe from the centres P, *n*, with the radii P *m*, and *n* S — *m* O, two arcs intersecting each other in C; then producing the right line *n* C, joining those points, it will meet the parallel of depression *p* S *l* in *a*, the point through which the parallel of the sun's declination will pass on the day the proposed phenomenon happens, and *a* P, the right line joining those points, will be the distance of the parallel from the visible pole.

DEMONST. Describe the circle of perpetual apparition, R B O D, and the arcs *t w*, *a g*, *p o*, *x z*, &c. of the parallels of declination intercepted between *p* S *l* (the parallel of depression) and *z* O H (the horizon); also about the centre C, with the radius C *a* (= *m* O), let the circle K M be described, intersecting the parallels of declination in *u*, *e*, *a*, *i*, *y*, &c. and it will touch the parallel circle *p l* in *a*, by 11 Euc. III.; it will also touch the circle R B O D, because C P = *m* P, by construction; and it will make an angle *a c h* with the equator = O *h e*, the angle which the horizon makes with the same; and hence the parallel arcs *u w*, *e h*, *a g*, *i o*, *y z*, &c. intercepted by them, will be similar, and consequently passed over by the sun in the same time, which will manifestly be less than the time in which the corresponding twilight arcs *t w*, *f h*, *p o*, *x z*, &c. will be passed over, except the twilight arc *a g*, which will be the same, and will therefore be the shortest.

CALCULA. Describe the vertical circle Z *a*, cutting the horizon and equator in *r* and *d* respectively, and the meridional arc P *g*, intersecting the equator in *k*;

then the right-angled spherical triangles $e\,a\,d$, $h\,r\,d$, being equiangular, will be mutually equilateral, and consequently $a\,d = d\,r = 9°$; whence, since $h\,r = e\,a = h\,g$, we have radius : tang. $d\,r$: : co-sine $g\,h\,k$ (= sine latitude of the place) : sine of $g\,k$, the required declination. Also, in the spherical triangle $a\,e\,d$, we have, sine $a\,e\,d$ (or co-sine of latitude) : sine of $a\,d$ (= 9°, half the depression) : : radius : sine of $e\,d$, the arc of the equator measuring half the duration of the shortest twilight required *.

141. *Ex.* Required the time of the year when the twilight is shortest at Cambridge, also the duration of the twilight at that time?

* It is rather surprising, that, although this problem has been considered, during the last century, by some of the most eminent mathematicians in Europe, and almost all of those who have considered it have deduced from their investigations the analogies given above, yet some persons have taken upon them to call in question both the principles on which the investigations have proceeded, and the conclusions arrived at. The grand objection of most of these censurers, is against the *method of fluxions*, which has been applied with so much ingenuity and success to the solution of this problem: on this account, although for my own part I think most of the fluxionary solutions admirable, I have given two investigations on principles totally independent of each other, and each unconnected with fluxions: the first is taken from Mr. *Vince's Principles of Astronomy*, and is an improvement upon M. *Cagnoli's* method; the other is extracted from the *Ladies' Diary* for 1767. One can hardly help expecting that those who endeavour to amend the solution of this problem, should have some little knowledge of the subject on which they presume to decide, contrary to the opinion of the most learned and competent judges: yet in this expectation I have been disappointed; for in No. VIII. of that useful work the *Mathematical Repository*, a gentleman has given what he calls "*A new Solution to an old Problem*," the conclusion of which involves this absurdity, that in all latitudes greater than 10°, the declination of the sun at the time of shortest twilight must be such *that its sine shall considerably exceed radius*, that is, *the part shall be far greater than the whole!!*

Radius	10·0000000
Sine 52° 12′ 35″	9·8977694
Tang. 9°	9·1997125
Sine 7° 11′ 25″ decl.	9·0974819

Co-sine 52° 12′ 35″, A.C.	0·2127004
Sine 9°	9·1943324
Radius	10·0000000
Sine 14° 47′ 26″	9.4070328

Here the declination of the sun answers nearly to March 2d, and October 12th: and twice 14° 47′ 26″ converted into time, gives $1^h\ 58^m\ 19^s$ for the duration of the shortest twilight, on the supposition that 18° is the proper allowance for the sun's depression.

142. PROB. IX. *Given the latitude of the place, and the sun's declination, to find the time of the day when the hour-angle from noon, and the sun's azimuth from the south, are equal to each other.*

Ex. At what time of the day, on February 19th, is the hour-angle from noon equal to the sun's azimuth from the south, in latitude 51° 32′ N.?

Let H Z O N (fig. 7, Pl. II.) be the meridian of the place, P B S the required hour circle, and Z B N the required azimuth, or vertical circle; from their intersection, B, let the perpendicular B C be demitted upon the meridian. Then the angles C Z B and C P B being equal, from the nature of the problem, and C P B being, by spherics, equal to C S B, therefore C Z B = C S B, and the triangles Z B C and S B C are equal in all respects. Hence we have, in the triangle S C B right-angled at C, S C = $\frac{1}{2}$ S Z = $\frac{1}{2}$ latitude + $\frac{90°}{2}$ = 25° 46′ + 45° = 70° 46′, and S B = the co-declination = 78° 39′; whence the angle C S B will be found = 65° 26″, and con-

sequently the time is either $7^h\ 38^m\ 16^s$ in the morning, or $4^h\ 21^m\ 44^s$ in the afternoon.

143. Since the triangles Z C B, and S C B are mutually equal, Z B the co-altitude is equal to S B the co-declination, and the altitude and declination are equal: when, therefore, the sun's declination and the latitude are of different kinds, and the time is required when the sun's altitude is equal to his declination, it may be found, whenever it is possible (that is, when the sun's meridian altitude is equal to, or greater than, twice the declination), by means of the triangle C S B, as in this problem. This will be much easier than it would be to apply the general method adopted in Problem V. to this particular case.

144. PROB. X. *Given the latitude of the place and the declination of the same name, to find the beginning and end of the hour during which the shadow of an object will be increased to double its length.*

Ex. The latitude of the place is 52° N. and the sun's declination 20° N.; required the times which will answer the conditions of the problem?

In fig. 9, Pl. II. which is an orthographic projection of the sphere, P is the north pole, Z the zenith, H O the horizon, E Q the equator, R D the parallel of declination in which the sun moves, a and c the places of the sun at the beginning and end of the hour. Put s and n for the sine and co-sine of Z P (= 38°) the co-latitude, c and m for the sine and co-sine of P a or P c (= 70°) the co-declination, d and p for the sine and co-sine of a P c = 15°; also x and y for the sine and co-sine of Z P a, the hour angle from noon to the beginning of the required hour: then, by trigonometry, $px + dy$; and $py - dx$ will be the sine and co-sine of Z P c. By Theor. 3. page 19, *Turner's Projection of the Sphere*, we have $scy + nm$ for the sine of aq, and $scpy - scdx + nm$

sine of cv: therefore, as the lengths of the shadows of an object on a plane are as the co-tangents of the sun's altitudes, we have $\frac{\sqrt{1-\overline{scy+nm}^2}}{scy+nm} = \frac{\sqrt{1-\overline{scpy-csdx+nm}^2}}{scpy+nm-scdx}$; from which equation y may be readily exterminated, and x found $= \cdot 99785$, sine of $86^\circ\ 15'$ nearly; this gives $5^h\ 45^m$ and $6^h\ 45^m$ for the times required.

145. Or, this problem may be solved by trial and errour, thus: Since the lengths of the shadows, or the cotangents of the sun's altitudes, at the beginning and end of the required hour, are in the ratio of 1 to 2, it is evident that the arcs themselves must also be nearly in the ratio of 1 to 2, and therefore the required hour must end about one hour before the sun sets: the time on which the sun sets, as found by Prob. I. is about $7^h\ 52^s$; therefore we may suppose the times to be $5^h\ 50^m$ and $6^h\ 50^m$. The tangents of the sun's altitudes at these times (found by the converse of Prob. V.) are ·3085957 and ·1458842, or as 2·115 to 1, and these are inversely as the shadows or cotangents, therefore the errour is ·115. In order to correct this assumption, let the tangents of the altitudes at the times $5^h\ 40^m$ and $6^h\ 40^m$ be found, these will be ·3466068 and ·1706338, or in the ratio of 2·031 to 1, therefore the errour here is ·031; hence we have ·084, the difference of the errours : ·031 the least :: 10^m, the difference of the corresponding times, : $3^m\ 41^s$, which added to the last assumption, will give $5^h\ 43^m\ 41^s$, for the beginning of the hour. This, corrected by another similar operation, gives $5^h\ 44^m\ 45^s$ and $6^h\ 44^m\ 45^s$, for the times required; agreeing very nearly with the result of the first solution.

146. Prob. XI. *Given the latitude of the place; to find the sun's declination, when the shadow of a perpendicular object shall, after a certain number of*

hours, be projected towards the point of the horizon at which the sun rose.

Ex. In the latitude of 51° 30′ N. I observed the sun to rise on a certain point of the compass, and eleven hours after the sun had arisen I perceived my shadow to be projected towards the same point; what was the sun's declination at the time of these observations?

Let E S W N (fig. 10, Pl. II.) represent the plane of the horizon, E, S, W, N, the east, south, west, and north points, Z the zenith, and P the north pole; R the point where the sun rises, and O his place when the shadow is projected towards R. Let the arches P R and P O be drawn, then will the angle O P R be equal to 15°×11 or 165°; put m and n for the sine and co-sine of 82° 30′, half this angle; s for the sine of 51° 30′, the given latitude; and x for the complement of the sun's declination at rising: then, supposing the declination to continue invariable for eleven hours, or P R = P O, we shall have $1 : x :: m : mx$, the sine of half O R; its co-sine therefore is $\sqrt{1 - m^2 x^2}$. Hence $2mx \times \sqrt{1 - m^2 x^2}$ will be the sine of O R, as is shewn by the writers on trigonometry: also, $2mx\sqrt{1 - m^2 x^2} : 2mn :: x : \frac{n}{\sqrt{1 - m^2 x^2}}$, the sine of P R O; and $x : 1 :: s : \frac{s}{x}$, the sine of P R N. But these angles P R O and P R N are the complements of each other; and therefore as the sum of the sines of an angle, and of its complement, is equal to radius, we have, by squaring, $\frac{s^2}{x^2} + \frac{n^2}{1 - m^2 x^2} = 1$: this equation reduced (by substituting m^2 for $1 - n^2$), gives $x^4 - \overline{1 + s^2} \times x^2 = -\frac{s^2}{m^2}$; whence, by completing the square, &c. we get $x = \sqrt{\frac{1+s^2}{2} \pm \sqrt{\left(\frac{1+s^2}{2}\right)^2 - \frac{s^2}{m^2}}}$

$=\cdot 985077$, the natural sine of $80^\circ\ 5'\ 19''$. Consequently the sun's declination was $9^\circ\ 54'\ 41''$, and the days April 15 and August 28.

It must be evident that the declination is of the *same* kind as the latitude; for when the declination is of a *different* kind, the sun cannot be on two opposite azimuths during the time he is above the horizon.

147. PROB. XII. *Given the latitudes of two places on the same meridian, and on the same side of the equator; to find the time when the altitude of the sun will be equal at each place.*

Ex. In the latitudes of 30° and 50° north, on the same meridian, on the 21st of June in the morning, it is required to determine the exact instant when the sun's altitude will be equal, if observed at both places; also the altitude at that time?

Put $a = \cdot 642787$, natural sine of the complement of the greater latitude, and $n = \cdot 766044$, the co-sine; $d = \cdot 917292$, sine of $66^\circ\ 32'$, the co-declination of the sun, and $e = \cdot 398215$, the co-sine; also $s = \cdot 866025$, the sine of the complement of the less latitude, $c = \cdot 5$, its co-sine, and $x =$ co-sine of the hour angle sought. Then, by a well-known theorem in spherics, $sdx + ce =$ sine of the sun's altitude at the less latitude, and $adx + ne =$ sine of the sun's altitude at the greater latitude: but these, by the question, are equal; therefore $sdx + ce = adx + ne$: this equation reduced gives $x = \frac{n-c}{s-a} \times \frac{e}{d} = \cdot 51777 =$ co-sine of $58^\circ\ 49'$, the hour angle, which reduces to $3^h\ 55^m\ 16^s$ from noon, or $8^h\ 4^m\ 44^s$ in the morning.

The altitude may be ascertained by either of the two original equations, x being determined: thus, by the first, $sdx + ce = \cdot 866025 \times \cdot 917292 \times \cdot 51777 + \cdot 398215 \times \cdot 5 = \cdot 6104228 =$ sine of

37° 37′ 17″, the altitude of the sun at both latitudes, on the time just found.

148. On a slight consideration, it might appear as though this problem were impossible; for as the sun's meridian altitude is greater at the latitude of 30° than at the latitude of 50°, by the difference of latitudes, it might be concluded that the sun's altitude could at no one instant be the same at both places. But it should be recollected, that the parallel in which the sun moves, is much nearer a perpendicular to the horizon in the less latitude than in the greater, and of course the sun's apparent motion is quicker in the former than in the latter. Hence the sun rises latest at the less latitude, but as he ascends his altitude increases faster than in the greater latitude, till it is equal at both places; after which the altitude at the less latitude continues to increase faster than at the greater, until the sun arrives at the meridian, when the difference of the altitudes is a *maximum.*

149. PROB. XIII. *Given the latitude of the place and the sun's declination; to find to what height a person must be raised perpendicularly so as just to see the sun at midnight.*

EX. To what height must a person be raised above the city of London, on the 21st of June at midnight, so that he may see the sun's upper limb; reckoning 3980 miles for the radius of the earth?

On the 21st of June the sun describes the parallel of 23° 28′ north declination, corresponding to the parallel of latitude *a b* (fig. 11, Pl. II.): H R representing the horizon, and E L the equator, L R will be the complement of the latitude: also, *b* being the point directly under the place of the sun at midnight, *b* R corresponding to the midnight depression, will be equal to L R (the co-latitude) — L *b* (the declination) = 38° 28′ — 23° 28′ = 15°. From this subtract the sun's apparent semi-diameter 15′ 46″

(Tab. IV.); the remainder, corrected for parallax, gives $14^\circ\ 44'\ 22''$ for the apparent depression of the sun's upper limb. Draw S P a tangent to the surface of the earth, making an angle P S B with a parallel S B to the horizon $= 14^\circ\ 44'\ 22''$, and meeting the vertical line C R, produced in S; then S will be the point to which the person must be raised, so that he may just see the sun's upper limb, in the direction S P of the tangent to the earth's surface. Hence, in the right-angled triangle C S P, will be known C P = 3980, and the angle S C P = B S P (each being the complement of C S P), to find C S: thus, as co-sine $14^\circ\ 44'\ 22''$: C P = 3980 : : radius : S C = 4115·4283 miles, from which C R = C P being taken, leaves 135·4283 miles = R S, the height required.

In this solution no regard has been paid to the effect of the atmosphere in refracting and reflecting the rays; indeed it would be far from an easy task to reduce this to calculation in the present instance.

150. Prob. XIV. *To determine the time the sun is passing the meridian, or the horizontal, or perpendicular wire of a telescope.*

Here we again refer to fig. 4, Pl. II.; in which suppose $m\,r$ to be the apparent diameter, d, of the sun, estimated in seconds of a great circle; then, since the seconds in $m\,r$, considered as part of a small circle, must be greater in proportion as the radius is less (because when the length of the arc is given, the angle from the centre is inversely as the radius), we have sine N r, or co-sine declination $q\,r$: radius : : seconds d in $m\,r$ of a great circle : the seconds in $m\,r$ of the small circle $a\,b$, which are equal to the seconds in $p\,q$, equal to the angle p N q. Consequently the angle p N $q = d$ divided by co-sine of declination, radius being unity; or, since the secant is universally a third proportional to the co-sine and radius, or the secant inversely as the co-sine,

$pNq = d \times$ secant declination. But this angle measures the time the sun is passing over the meridian; and as the motion is performed at the rate of $15''$ of a great circle to 1^s of time, we have $\frac{d \times \text{secant dec.}}{15''}$ for the time in which the diameter transits the meridian.

151. If the sun's diameter pq in right ascension were required, it would be found by multiplying the seconds in the apparent diameter by the secant of the declination: thus, suppose $31' = 1860''$, the sun's diameter, and 18° the declination; then $1860'' \times 1{\cdot}0514 = 32'\ 35''{\cdot}6$, the diameter in right ascension.

152. The time in which the sun passes across the horizontal wire of a telescope, is manifestly the same as the time in which he ascends through a perpendicular space equal to his diameter. Suppose $rs = d$, the sun's diameter, then because $pq = rs \times \frac{\text{radius}^2}{\text{co-sine lat.} \times \text{sine azim.}}$ (Art. 132.); it it also $= d \times \frac{\text{radius}^2}{\text{co-sine lat.} \times \text{sine azim.}}$, and therefore the time will be expressed by $\frac{d}{15''} \times \frac{\text{radius}^2}{\text{co-sine lat.} \times \text{sine azim.}}$. If d, instead of representing the diameter of the sun, be put for $33' = 1980''$, the horizontal refraction, the same expression would enable us to determine the time in which the sun, as affected by refraction, rises earlier than it would otherwise do; and here, as $\frac{1980''}{15''} = 132''$, we have $132'' \times \frac{\text{radius}^2}{\text{co-sine lat.} \times \text{sine azim.}}$ for the interval sought. This formula will assist us in finding the correct time of the sun's rising and setting, in a rather easier manner than that described in Art. 122.; provided we take no notice of parallax and the change of declination.

153. The time occupied by the sun in passing across the vertical wire of a telescope, is the same as

the time in which his horizontal motion is equal to his diameter; therefore, in this case, we suppose $ms = d$. Now, the triangle mrs, on account of its minuteness, being supposed rectilinear, and s being a right angle, we have, sine srm : sine smr :: $ms : sr = ms \times \frac{\text{sine } smr}{\text{sine } srm}$; therefore (Art. 132.), $pq = ms \times \frac{\text{sine } smr}{\text{sine } srm} \times \frac{\text{radius}^2}{\text{sine } smr \times \text{co-sine } qr} = ms \times \frac{\text{radius}^2}{\text{sine } srm \times \text{co-sine } qr}$; and, consequently, $\frac{d}{15''} \times \frac{\text{radius}^2}{\text{co-sine } ZrN \times \text{co-sine dec.}}$ expresses the time in which the sun would pass the vertical wire of a telescope.

154. PROB. XV. *Given the latitude, the sun's place in the ecliptic, and the hour of the day; to find the angle made by the horizon and ecliptic, or (as it is called) the height of the nonagesimal degree; the point of the ecliptic ascending, the point of the nonagesimal degree; the azimuth of the ascending point, the point culminating, its altitude, &c.*

Let H Z R C (fig. 12, Pl. II.) be the meridian, H R the horizon, Z the zenith, D Q the equinoctial, P its pole, C A M the ecliptic, E the first point of aries, S the place of the sun, P S B an hour circle, A the point of the ecliptic intersecting the horizon, called the *descending point*; N, at the distance of 90° from A, the highest point of the ecliptic, and Z N G a vertical circle.

The place of the sun at S being given, his right ascension E B is readily found, either by calculation or by the tables. And the hour being known, the arch of the equinoctial B Q, measuring the hour angle B P Q, will be given, and consequently the complement B O. Therefore the difference of E B and B O is given, that is, E O. Consequently, in the oblique spherical triangle A E O, there are given the angle A E O the obliquity of the ecliptic, A O E the complement of the latitude, and the side E O,

whence the other angle and sides may be determined. Therefore we shall have,

155. The angle EAO, and its supplement SAO, which is the angle made by the horizon and the ecliptic, and is measured by the arc NG, *the height of the nonagesimal degree* *.

* When we know the latitude of a place, the apparent time at that place, and the obliquity of the ecliptic (23° 28′), the altitude of the *nonagesimal degree*, and its longitude, may in general be found very readily by observing the following precepts:

1st. *To find the altitude of the nonagesimal degree.* Find the right ascension of the mid-heaven, by adding to the given time the sun's right ascension at that time, allowing for the difference of meridians between the place and Greenwich, and convert it into degrees and minutes: then,

If the right ascension of the mid-heaven is between	0° and	90°	the said right ascen. of the mid-heav. in deg. and min.	increased by 90°
	90	180		subt. from 270
	180	270		subt. from 270
	270	360		lessen'd by 270

will give an arc or angle, which call A; then to twice the logarithmic sine of half the arc A, add the constant logarithm 9·90115, and the logarithmic co-sine of the latitude of the place reduced to the earth's centre; find the natural number corresponding to this logarithm, and take it from the natural co-sine of the difference between 66° 32′ (the complement of 23° 28′) and the reduced latitude of the place, the remainder is the natural co-sine of the altitude of the nonagesimal degree.

When the index of the sum of the four logarithms is 39, 38, 37, 36, &c. the number of places in the corresponding natural number is 5, 4, 3, 2, &c.

The latitude of the place may be reduced to the earth's centre, by a table for this purpose in *Mayer's Tables*; or, very readily, by Mr. *Evans*'s rule given in a note to Art. 659. of this work.

2dly. *To find the longitude of the nonagesimal degree.* To the arith. comp. of logarithmic sine of the altitude of the nonagesimal degree, add the logarithmic sine of the arc A, and the logarithmic co-sine of the reduced latitude of the place; the sum, deducting 10 from the index, is the logarithmic sine of an arc, which call B: and

When the right ascen. of the mid-heaven is between	0° and	90°	the arc B	subt. from 90°	is
	90	180	the arc B	added to 90	
	180	270	the arc B	subt. from 270	
	270	360	the arc B	added to 270	

the longitude of the nonagesimal degree.

156. The side A E will be known, and therefore the point A of the ecliptic descending will be found, and consequently the opposite point is determined, namely, *the ascending point of the ecliptic.*

157. The side A O will also be known; this is the *azimuth* of the *descending point* in the *west*, or of the *ascending point* in the *east*.

158. By adding A N, or 90°, to E A, we get the arch E N, which gives the *longitude* of the highest point, or of the nonagesimal degree.

159. A G and O R are both quadrants, therefore G R = A O: but G R is the *azimuth of the nonagesimal degree* from the south.

160. To find the *culminating point*, or that point which is on the meridian, we must add the quadrant O R to A O; then, in the right-angled spherical triangle A R M, there are given A R and the angle M A R found before, to determine the hypothenuse A M; this added to A E, already known, gives E M, the distance of the culminating point M of the ecliptic, from E the first point of aries.

161. The side R M of the same triangle M A R is easily found, and is the *altitude of the culminating point* of the ecliptic: and the angle A M R of the same triangle is the intersection of the ecliptic with the meridian, or the *meridian angle.*

162. The angle Z S M, which the vertical circle makes with the ecliptic, is sometimes called the *parallactic angle*, and is thus found: In the right-angled triangle Z S N, the sides Z N, N S, are given; for Z N is the complement of N G, and N S is the complement of AS, or of E S—E A; consequently the angle Z S N may soon be found. Or it might be found by the triangle Z M S, where Z M and angle Z M S are given, and M S = M A—A S; to determine Z S M.

163. In the same triangle the side Z S may be found, and is manifestly the sun's zenith distance.

164. The angle which the vertical circle Z S makes with a *less circle* F I K, at a given point I, may thus be found: Let the vertical circle cut the horizon in T, and let M S A be the ecliptic, or any *great circle* parallel to F I K. Through the given point I draw I L, perpendicular to M S A; then the angle Z S N is known, by the last article, and, of course, in the right-angled triangle I S L, are given the angle I S L, and the side I L, the distance of the parallel from its great circle; whence the angle S I L may be found, and this is the complement of K I S, or Z I F, the required angle.

165. Or, if S A T and A T are known; then, in the triangle A S T, we have co-sine A T : co-sine S. : : rad. : sine A $= \text{rad.} \times \frac{\text{co-sine S}}{\text{co-sine A T}}$; also, in the triangle S I L, co-sine I L : co-sine S : : rad. : sine I $= \text{rad.} \times \frac{\text{co-sine S.}}{\text{co-sine I L}}$. Therefore, $\frac{\text{co-sine S.}}{\text{co-sine A T}}$: $\frac{\text{co-sine S}}{\text{co-sine I L}}$: : sine A : sine I; or, as co-sine I L : co-sine A T : : sine S A T : co-sine K I S.

166. PROB. XVI. *Given the time and place, to erect a scheme or figure of the heavens.*

Let H M R C (fig. 13, Pl. II.) be the meridian, H R the horizon, D Q the equator, M A C the ecliptic. Find the point M of the ecliptic that culminates at the given time (Art. 140.), and D its right ascension: then to the right ascension D add 30°, 60°, 90°, &c. respectively, or in the equinoctial D Q, make D B, B I, I O, O T, &c. each equal to 30°; and thus determine the points B, I, O, T, V, &c. Through these points describe the circles of position H B R, H I R, H T R, &c.; the points N, K, A, L, S, &c. where these circles intersect the ecliptic are called *cusps*. A is the cusp of the first house, L that of the second, S of the third, C of the fourth; the fifth, sixth, seventh, eighth, and ninth, fall in

the opposite hemisphere; M is the cusp of the tenth, N of the eleventh, K of the twelfth house. Then, to find these points of the ecliptic, we must proceed as follows: In the right-angled spherical triangle H D B, are known D H the complement of the latitude, D B an arc of 30°; to find the angle D B H, or angle of position with the equator: also, in the several right-angled triangles H D I, H D O, H D T, &c. the side H D common to each is known, and the sides D I, D O, D T, &c.; to find the angles of position at I, O, T, &c. Again, in the oblique triangle E B N, are given E B, the angle E B N = D B H, and B E N, the obliquity of the ecliptic; to find E N, the distance from the equinoctial point E. In like manner, in the several oblique triangles E I K, E O A, E T L, E V S, &c. there are given respectively the sides E I, E O, E T, E V, &c. the angles at I, O, T, V, &c. and in each of them the angle E: whence, the sides E K, E L, E A, E S, &c. will be determined; and therefore the points of the ecliptic in which the cusps of the several houses are placed, will be known: for, the 7th and 1st, the 8th and 2d, the 9th and 3d, the 10th and 4th, the 11th and 5th, and the 6th and 12th, are diametrically opposite to each other. The first is in the horizon on the eastern side, and is called the *ascendant* or *horoscope*, on the *orient*; the fourth is in the meridian below; the seventh is on the horizon towards the west; and the tenth is in the meridian above. The positions of the twelve houses thus determined, only answer for the instant of time for which they are calculated, or for the expiration of every sidereal day, reckoning from that instant: at all intermediate parts of the day, different points of the ecliptic coincide with the horizon; and therefore the calculation must be repeated on the same principles.

167. Most of the terms explained in this problem belong peculiarly to *Astrology;* an art which was

made a subsidiary branch of the science of *Astronomy* from the time of *Ptolemy* till *Kepler*, or rather later: but as the principles on which astrology is founded are trifling, absurd, fanciful, and unsatisfactory, while those on which the fabric of astronomy is erected are solid, permanent, harmonious, and consistent with the soundest reasoning, the unnatural union between them is now completely broken. Still, however, it was thought expedient to insert this problem, as it may be sometimes useful, though what it assists us in determining may be sought under other names.

PROBLEMS *relating to the* STARS.

168. IN the solution of the following problems respecting the fixed stars, we suppose their right ascensions and declinations to be known; for they may readily be found for any given period, by means of those tables of the stars, in which not only the right ascensions and declinations, but the annual variations are given. But it sometimes may be necessary to find the right ascensions and declinations, having the longitudes and latitudes given: in which case we may proceed exactly as in Art. 65. only considering E Q as the ecliptic (fig. 11, Pl. I.) and E C the equator. Or we may here also adopt with advantage another valuable rule, given by Dr. *Maskelyne*, which will enable us to find the right ascension and declination, having the latitude, longitude, and obliquity of the ecliptic.

1. Sine longitude + co-tang. latitude — 10 = co-tang. arc A, which call *north* or *south*, according as the latitude is *north* or *south*. 2. Call the obliquity of the ecliptic *north* in the first semicircle of longitude, and *south* in the second. Let the sum of arc A, and obliquity ecliptic, according to their titles, = arc B with its proper title. 3. The arithmetical comp.

of co-sine arc A + co sine arc B + tang. long. — 10 = tang. of *right ascension*, of the same kind as the longitude, unless arc B be more than 90°, in which case the last quantity found of the same kind as the longitude must be subtracted from twelve signs, or 360°. 4. The sine of right ascension + tang. arc B — 10 = tang. of the required *declination*, of the same title as arc B. N.B. If the right ascension be found near 0°, or near 180°, for its sine in the last operation, substitute its tangent + its co-sine — 10; or the last operation will be tang. A R + co-sine A R + tang. arc B — 20 = tang. declination: where A R is put for right ascension.

169. When the declination of a star is known, its meridian altitude may readily be found at any given latitude; for, if the latitude and declination be both *north*, if it comes on the meridian to the *south*, the meridian altitude will be equal to the sum of the declination and the comp. of the latitude of the place: or, if it be a *circumpolar star*, its meridian altitude towards the *north* will be equal to either the sum or the difference of the latitude, and co-declination, or polar distance: when the declination is *south* and the latitude *north*, the declination taken from the co-latitude, leaves the meridian altitude; if the declination be greater than the co-altitude, the star never appears above the horizon. This must, after what has been said in Art. 61. be too evident to need any formal proof.

170. The distances of any two stars may be found from their latitudes and longitudes; or from their right ascensions and declinations, by the method explained at Art. 67. We may therefore, without farther observations, enter upon the problems.

171. PROB. XVII. *To find when any known fixed star will culminate, or be on the meridian, on a given day.*

This problem comprehends two cases, which may

be separately considered. *First*, when the place is on, or near, the meridian of Greenwich: subtract the sun's right ascension, for the proposed day at noon, from the right ascension of the given star; the difference will be the time of the star's culminating, *nearly*. Then say, as twenty-four hours are to the daily change of the sun's right ascension, so is the approximate time of culminating, to a fourth number, which being subtracted from the approximate time, will give the *true time*. If this time be less than twelve hours, the star culminates in the afternoon; but, if it be greater than twelve, the excess above twelve will shew the time on the next morning. This made less by the daily change of the sun's right ascension, will give the time of culminating on the given morning. N.B. When the sun's right ascension is greatest, twenty-four hours must be added to the star's right ascension.

Ex. 1. On the 8th of April 1801, at what hour will the star called *virgin's spike* be on the meridian of London?

Here the right ascension of virgin's spike on January 1, 1800, is $13^h\ 14^m\ 40^s{\cdot}11$, and the annual variation is $3^s{\cdot}137$, whence we get $3^s{\cdot}98$ for the correction, to April 8: therefore

Virgin's spike's right ascen. April 8	13^h	14^m	44^s
Sun's right ascension, noon do. .	1	7	0
Approximate time of culminating	12	7	44
Then $24^h : 3^m\ 40^s$(diff. R A) :: $12^h\ 8^m$:	0	1	51
True time of culm. $5^m\ 53^s$ after midnight	12	5	53

This made less by $3^m\ 40^s$, gives $2^m\ 13^s$ A.M. for the accurate time of coming to the meridian on April 8, 1801.

Ex. 2. Find the time when *virgin's spike* will be on the meridian at London, on October 8th, 1801.

Virgin's spike, right ascen. Oct. 8	$13^h\ 14^m\ 45\frac{1}{2}^s$
Sun's right ascension, noon, do.	12 54 11
Time of culminating	0 20 $34\frac{1}{2}$

172. From these examples we may draw this inference; those stars which are on the meridian at any part of the year, *near midnight*, will in the opposite part of the year (i. e. after the interval of six months) be on the meridian, *near noon*, and will consequently be under the horizon in the evening, unless they are circumpolar stars.

173. In the *second case* of this problem, the time of the star's culminating is required at a place which is not on or near the meridian of Greenwich: then, if the longitude be *west*, *add* the longitude in time to the approximate time of culminating; or, if the longitude be *east*, *subtract* it; and use the sum or difference, instead of the approximate time of culminating: observing only, in the latter case, that if the longitude in time be greater than the approximate time of culminating, then the minutes and seconds resulting from the proportions must be *added* to the approximate time, instead of being *subtracted* from it.

Or, find when the star is on the meridian of Greenwich, and allow at the rate of 10 seconds of time for every 15° of longitude; which must be *subtracted* from the time at Greenwich for places in *west* longitude, and *added* for places in *east* longitude. For, as the sun's right ascension varies about $3^m\ 56^s$, or nearly four minutes, in twenty-four hours; 4^m divided by 24 gives 10^s for the variation in one hour, or 15° of longitude.

Ex. At what times on April 8, 1801, will the star *virgin's spike* be on the meridians of Philadelphia, longitude 75° 20′ W., and Pekin, longitude 116° 21′ E?

	h. m. s.
Time of culminat. at London or Greenwich	12 5 53
Then $15^\circ : 10' :: 75^\circ\ 20'' :$	0 0 50
Remainder, is the time at Philadelphia	12 5 3

	h. m. s.
Again : time of culminating at Greenwich	12 5 53
$15^\circ : 10^s :: 116^\circ\ 21' : 1^m\ 18^s$, add	0 1 18
	12 7 11

Therefore at $5^m\ 3^s$ after midnight, at Philadelphia, virgin's spike will be on the meridian there; and it will be on the meridian at Pekin $7^m\ 11^s$ after midnight, April 8th or 9th, according to the civil account.

174. Hence it appears, that the same fixed star is on the meridian of every place in any kingdom of moderate extent, nearly at the same instant. For since there are only 10 seconds of time between the instants when a fixed star is on the meridians of two places differing 15° in longitude, any less difference of longitude, as five or six degrees, will cause but a very inconsiderable variation in the times of culminating. We may also take it for granted, that any fixed star is on the meridian at nearly the same hour, on the same day of the month, for many successive years: for the annual variation in a star's right ascension is seldom greater than three or four seconds, and this in forty years will not cause a difference in time of more than two or three minutes.

175. Prob. XVIII. *To find what fixed star will be on the meridian at a given time.*

Add the sun's right ascension, corrected for the day and hour, to the time of the day given, the sum is the right ascension of the *mid-heaven* (note, Art. 155.);

if the sum be more than 24 hours, deduct 24; the remainder being sought among the right ascensions of the fixed stars, will ascertain what star will be on or near the meridian at the time proposed.

Ex. 1. What star will be on the meridian of Cambridge at 9 o'clock, P.M. February 20, 1801?

	h.	m.	s.
Given time, Feb. 20	9	0	0
Sun's right ascension : noon .	22	13	54
Variation in 9 h.	0	1	49
	31	15	43

The sum lessened by 24 hours, gives $7^h\ 15^m\ 43^s$, for the star's A R. This corresponds nearly with the A R of β *canis minoris*, a star of the third magnitude; which will therefore be on the meridian about the given time.

Ex. 2. What fixed star will be on the meridian of Ningpo in China, longitude 120° 23′ E. on April 14, 1801, when the clock there is at 10^h P.M.?

	h.	m.	s.
Sun's right ascension, Apr. 14	1	29	2
Given time	10	0	0
Longitude in time, add . .	8	1	32
	19	30	34

This right ascension corresponds nearly with that of ϑ *cygni*, a star of the 4th magnitude.

176. Had it been required to find on what day of the year any fixed star will culminate on a given hour; the hour given must then be taken from the star's right ascension, the remainder would be the right ascension of the sun on the day required.

Ex. On what day of the year 1801 will the star *aldebaran* be on the meridian of Greenwich at $8^h\ 14^m\ 30^s$ P.M.?

	h.	m.	s.
Aldebaran's A.R. 1801, increased by 24,	28	24	31
Subtract the given time	8	14	30
Remainder is the sun's A.R. . . .	20	10	1

This agrees very nearly with the sun's A.R. at the given hour on January 20, which is therefore the day sought.

177. PROB. XIX. *To find the time on which a known fixed star rises and sets, also its azimuth at rising and setting, at any place of known latitude, on a given day.*

The latitude of the place and the star's declination being known, the hour angle from rising to culminating may be found, as in Prob. 1, and the azimuth, as in Prob. 2. The time on which the star culminates may be found by Prob. 17, from which the time of rising and setting may readily be obtained.

Ex. On January 1, 1801, at what times did *castor* rise and set at Cambridge, and what was the azimuth at rising and setting?

	h.	m.	s.
Castor's A.R. Jan. 1, 1801	7	21	52
Sun's A.R. noon, ditto	18	46	31
Approx. time culmin.	12	35	21
$24^h : 4^m\ 22^s :: 12\frac{1}{2}^h : 2^m\ 16^s$ add	0	2	16
True time of culminating . . .	12	33	5

Castor's declination is 32° 19′ N. therefore, Prob. 1, and 2,

Radius	10·0000000
Tang. lat. 52° 12′ 35″	10·1104697
Tang. dec. 32° 19′	9·8011161
Co-sine 35° 19′ 59″, or of its sup. 144° 49′ 1″	9·9115858

Arith. comp. co-sine lat.	0·2127004
Radius	10·0000000
Sine declination	9·7280275
Co-sine 29° 15′ 30″ azim. from north .	9·9407279

In the present instance, the declination being of the *same* kind as the latitude, the hour angle from culminating to rising or setting, or the *semi-diurnal* arc, is greater than 90°: the calculation gives 144° 49′ 1″, or 9ʰ 39ᵐ 16ˢ in time; this subtracted from the time of culminating, gives 2ʰ 53ᵐ 49ˢ P.M. for the time of castor's rising; and, added to the time of culminating, gives 10ʰ 12ᵐ 21ˢ A.M. Jan. 2, for the time of setting.

178. Prob. XX. *Given the latitude and day of the month; to find when any known star will be on the prime vertical, and its altitude at that time.*

In this problem, the declination of the star and the latitude of the place must evidently be of the same name: then the time of culminating may be found by Prob. 17; the hour angle from culminating to being on the prime vertical, and the altitude on the prime vertical, by the rule given in Prob. 3.

Ex. At what times, on Jan. 1, 1801, was *castor* due east and west at Cambridge, and what was the altitude at those times?

Radius	10·0000000
Tang. dec. 32° 19′ 0″	9·8011161
Co-tang. lat. 52° 12′ 35″ . . .	9·8895736
Co-sine of 60° 37′ 22″ hour ang. from culm.	9·6906897
Arith. comp. sine lat.	0·1022306
Radius	10·0000000
Sine dec.	9·7280275
Sine alt. 42° 34′ 10″	9·8302581

The hour angle converted into time gives $4^h\ 2^m\ 29^s$, which taken from $12^h\ 33^m\ 5^s$, the time of culminating, leaves $8_h\ 30^m\ 36^s$, time when castor is due east; and added to $12^h\ 33^m\ 5^s$, gives $4^h\ 35^m\ 34^s$ A.M. Jan. 2, time when that star is due west.

179. Prob. XXI. *Given the time of the year, the latitude of the place, and the altitude or azimuth of a known fixed star; to find the hour when the observation was made.*

In fig. 4, Pl. II. where HR represents the horizon, Z the zenith, N the pole, and *m* the place of the star; if the altitude be given, we have in the oblique spherical triangle Z N *m*, Z N the co-latitude, Z *m* the co-altitude, and *m* N either the co-declination or 90° + declination, according as the latitude and declination are of like or different kinds, to find Z N *m*, the hour angle from the meridian. Or, if the azimuth is given, we have the sides Z N and *m* N (the co-latitude and co-declination), and the angle *m* Z N (the azimuth), opposite to one of the given sides, to find the angle between the two given sides; for which there are various rules given by the writers on trigonometry.

Ex. At Cambridge, on January 1, 1801, in the evening, the altitude of the star *castor* was found to be 45°; what was the time of the observation?

Here Z N is $= 37°\ 47'\ 25''$, $Zm = 45°$, and $mN = 57°\ 41'$; whence Z N *m*, the hour angle, is found to be $56°\ 39'\ 28''$, or, in time, $3^h\ 46^m\ 38^s$, which taken from $12^h\ 33^m\ 5^s$, the time of culminating, leaves $8^h\ 46_m\ 27^s$ P.M., time when the observation was made.

180. Prob. XXII. *Given the latitude of the place, and the day of the month; to find the time when the apparent motion of a known fixed star is perpendicular to the horizon, also the altitude and azimuth of the star, at that time.*

All those stars whose declinations are greater than the latitude of the place, and of the same name with it (for to such only this problem is applicable), perform some part of the motion in their respective semi-diurnal arcs, in a vertical direction; which may be thus ascertained: Let *y z* (fig. 7, Pl. I.) be the parallel described by such a star, draw the vertical circle Z V to touch it at *p*, and when the star arrives at *p*, as *y z* and Z V then nearly coincide during a small part of the motion of the star, its motion will appear perpendicular to the horizon. Here the angle Z *p* N will be 90°, and therefore radius : tang. *p* N : : co-tang. Z N : co-sine Z N *p*; or, radius : co-tang. declination : : tang. latitude : co-sine, hour angle from culminating. Then, by the proportions of opposite sides and angles, we find Z *p* the co-altitude, and *p* Z N the azimuth.

Ex. At Peterborough, latitude 52° 32′ N. on February 21, 1801, at what time will the apparent motion of *dubhe*, or *α ursa majoris*, be vertical; and what will be the altitude and azimuth of the star at that time?

The right ascension of *dubhe* is $10^h\ 51^m\ 19^s$, and on February 21, at noon, the sun's right ascension is $22^h\ 17^m\ 44^s$, and the diurnal variation $3^m\ 49^s$; whence, by Prob. 17, the true time of the star's coming to the meridian, is $12^h\ 31^m$. Then,

Radius	10·0000000
Co-tang. star's dec. 62° 50′	9·7102824
Tang. lat. 52° 32′	10·1155428
Co-sine of 47° 57′ 46″ hour ang. from mer.	9·8258252

Radius	10·0000000
S. Z N. 37° 28″	9·7841177
S. Z N *p*. 47° 57′ 46″	9·8708193
Sine of 26° 51′ 31″, or co-si. alt. 63° 8′ 29″	9·6549370

And, as sine Z N : radius :: sine N p : sine p Z N $=48°\ 38'\ 28''$. Therefore $63°\ 8'\ 29''$ is the altitude of *dubhe* when its motion is vertical; $48°\ 38'\ 28''$ its azimuth from the north; and, by taking $3^h\ 11^m\ 51^s$, the hour angle in time, from the instant of culminating, we have $9^h\ 19^m\ 9^s$ P.M. the time required.

181. This problem is, in some measure, analogous to Prob. 6; and by comparing the two together, these conclusions will be deduced:—Every star whose declination is of the same name as the latitude, and greater than it, may be twice on the same azimuth, in the interval between its rising and culminating; and it may be twice on another azimuth, between its culminating and setting:—when such a star is at its greatest azimuth, its apparent motion is vertical:—also, when the sun's declination exceeds the latitude of the place, and is of the same name, its apparent motion is vertical when its azimuth is *a maximum.*

182. Prob. XXIII. *Given the latitude, and day of the month; to find when two known stars are on the same azimuth.*

Ex. At London, on the 1st of January, 1800, what was the time of night when the stars *aldebaran* and *rigel* were on the same azimuth?

In fig. 14, Pl. II. which is a stereographic projection on the plane of the equinoctial, A and B are the supposed situations of *aldebaran*, on the same azimuth circle B A Q: a circle described about P the pole, at the distance of the co-latitude of London, will intersect the azimuth circle B A Q in Z or z, the nearest of which to the stars is the zenith; for as both the stars are above the horizon, the zenith distance of each is less than 90 degrees.

We have given, sun's right ascension $18^h\ 47^m\ 36^s$; latitude of London $51°\ 32'$; aldebaran's declination $16°\ 5'$ N. A.R. $4^h\ 24^m\ 27^s$, rigel's declination $8°\ 27'$ S. A.R. $5^h\ 4^m\ 55^s$: difference of the stars' right ascensions $0^h\ 40^m\ 28^s$, or $10°\ 8'$. Hence, in

the triangle APB, are known PB=98° 27′, rigel's co-declination; PA=73° 55′, aldebaran's co-declination; and APB = 10° 8′, difference of right ascensions: whence BAP is found 156° 54′. Then, in the triangle APZ are given AP, as before, PZ = 38° 28′, co-latitude of London, and PAZ = supplement of BAP; to find APZ = 24°, aldebaran's hour angle from the meridian, which reduced to time is $1^h\ 36^m$. Now, aldebaran culminates at $9^h\ 35^m\ 10^s$; from which $1^h\ 36^m$ being taken, we have $7^h\ 59^m\ 10^s$, or 8 o'clock P.M. nearly, for the time required.

183. PROB. XXIV. *Given the latitude and day of the month; to find the time when two known stars are at the same altitude.*

Ex. At London, on the 1st of January, 1800, the stars *betelguese* and *pollux* were observed to have the same altitude; what was the time of the evening?

In fig. 15, Pl. II. P is the pole of the equinoctial, A is the place of betelguese, and B of pollux; Z the zenith, being equally distant from each. The right ascension of betelguese is $5^h\ 44^m\ 20^s$, its declination 7° 21′ N.; pollux's right ascension is $7^h\ 33^m\ 3^s$, its declination 28° 30′: the difference of their right ascensions is $1^h\ 48^m\ 43^s$, or 27° 10′ nearly. The sun's R. A. and the latitude of the place, as in the last problem. Then, in the triangle ABP, we know PA = 82° 39′ betelguese's co-declination, PB=61° 30′ pollux's co-declination, and APB = 27° 10′ difference of right-ascensions; whence we get BAP=47°, and AB = 33° 14′. Again, ZA and ZB being equal to each other, a great circle *cz*, passing perpendicularly through AB, will bisect it in C: wherefore, in the triangle ACD are given DCA = 90°, BAP = 47°, and AC = 16° 37′; to find CDA = 45° 30′, and AD = 23° 38′. Lastly, in the triangle PDZ, are known PZ = 38° 28′ the co-latitude, PD = PA — AD = 59° 1′, and PDZ = CDA = 45° 30′; whence is

found ZPD = 48° 55′, the hour angle of betelguese from the meridian. But betelguese culminates at $10^h\ 54^m\ 45^s$; and consequently as the stars are on the east side of the meridian, $3^h\ 15^m\ 40^s$ the hour angle in time, taken from $10^h\ 54^m\ 45^s$, leaves $7^h\ 39^m\ 5^s$ P. M. for the time of observation.

184. PROB. XXV. *Given the latitude of a place, to find when a known fixed star rises cosmically, or with the sun, and likewise when it sets cosmically, that is, when the sun rises.*

The first thing to be determined is the *ascensional difference* (Art. 44.), which may be thus found: Let *a b* (fig. 4, Pl. 2.) be the parallel in which the star appears to move, E Q the equinoctial, N *z* a meridian passing through the star when rising at *i*; then O *z* is the ascensional difference: in the right-angled spherical triangle O *i z*, are known the angle O = the co-latitude, and side *i z* = the declination of the star, to find O *z*; for which we have this proportion; as radius : co-tang. O, or tang. latitude : : tang. declin. *i z* : sine O *z*, ascensional difference: this must be either added to, or taken from, the right ascension of the star, according as its declination is of the same name as the latitude, or a different name; the sum or difference is the *oblique ascension.*

Let H R (fig. 13, Pl. 2.) be the horizon, H D R the meridian, D Q the equinoctial, M A C the ecliptic, *s* the star: then in the oblique triangle E O A, there are given the oblique ascension E O, the angle A E O the obliquity of the ecliptic, and E O A equal to either the co-latitude or its supplement, according as E is below or above the horizon; to find the side E A, and consequently the point A of the ecliptic which rises with the star. Find on what day the sun is in that point of the ecliptic; for that is the day on which the star *rises cosmically.*

Again, add or subtract the ascensional difference,

to or from the right ascension, according as the star's declination and the latitude of the place are of the same or different kinds; the sum or difference is the *oblique descension*. Then there are given the same parts of the triangle E O A as before, to find E A; and A in this case is the point setting with the star: consequently the point opposite to A in the ecliptic, is the place of the sun when he rises just as the star sets; and by finding the time the sun is in that part of the ecliptic we know when the star *sets cosmically*.

Ex. On what days in the year 1801, does the star *arcturus* rise and set cosmically, at Dover, latitude 51° 7′ N.?

Answer: rises cosmically, September 26; sets cosmically, June 22.

N.B. The calculations of this and the next problem, are left for the exercise of the young student.

185. Prob. XXVI. *Given the latitude of the place, to find when a known fixed star rises or sets achronically, that is, at sun-set.*

Find the point of the ecliptic in which the sun is, when the star rises cosmically, by the directions in the last problem; then it is obvious that when the sun is in the opposite point of the ecliptic, he sets when the star rises: therefore, find the day in which he is in such opposite point; for on that day he *rises achronically*. Again, find by means of the last problem, the place the sun is in when he sets with the star; for the day corresponding to that place is that on which the star *sets achronically*.

Ex. On what days in the year 1801, does the star *procyon*, in *canis-minor*, rise and set achronically, at Elsineur in Denmark, latitude 56° N.?

Answer: rises achronically, January 28; sets achronically, June 13.

186. Prob. XXVII. *Given the latitude of the*

place, to find when a known star rises or sets heliacally.

The term *heliacal* has been used by astronomers according to rather different acceptations; but in every acceptation it relates to the sun. A star may be said to *rise heliacally* when it rises in the morning just so long before the sun, as not to be rendered invisible by the strength of his rays; and a star *sets heliacally* when it sets in the evening, when the sun has just sunk so far below the horizon as not to obscure the setting star by his beams: thus are the terms to be understood in this problem.

187. It is found by experience, that the number of degrees which the sun must be depressed below the horizon, that the stars may just be visible above it, are as follow: for the first magnitude 12°; for the second, 13°; the third, 14°; the fourth, 15°; the fifth, 16°; and the 6th, 17°.

188. Let S (fig. 13, Pl. 2.) be the place of the sun below the horizon when the star at *s* rises heliacally: through the zenith Z suppose the vertical circle Z G S to be drawn; then in the right-angled triangle A G S, G S is known, being 12°, 13°, &c. according as the star is of the first, second, &c. magnitude, and G A S may be found by Art. 174; hence A S will be had: and as E A, the distance from the equinoctial point to the horizon, may be determined by Prob. 16, we readily get the sun's distance from the equinoctial point: wherefore the sun's place becomes known, and, of course, the day of the month.

In like manner, if S be the place of the sun when the star *s sets heliacally*, it may be found by an operation nearly similar. But as the problem is not of great utility, and may be performed with tolerable accuracy by a good *celestial globe*, there are probably but few who will undertake the calculation.

189. When *sirius*, the *dog-star*, rose heliacally, the ancients imagined it caused the sultry weather usually felt in the latter part of the summer. Traces of this opinion may be found in the works of *Homer*, *Hippocrates*, *Virgil*, and *Pliny*. Some ancient authors say, that on the day this star thus rises in the morning, the sea boils, wine turns sour, dogs begin to grow mad, the bile increases and irritates, all animals become languid, and burning fevers with phrensies rage amongst men. All this was ascribed to the dog-star; and about twenty days both before and after the star's rising in this manner were called *canicular*, or *dog-days*: a brown dog was, in consistence with this opinion, sacrificed to *sirius* yearly, to appease his rage*. But the idea that the star's rising thus was the cause of the hot weather, and the diseases of the season, could not prevail very long; for the star not only varies in its heliacal rising, in any one year, as the latitude varies, but, on account of the precession of the equinoxes, it is later and later every year in all latitudes; so that in process of time, the star might, by a similar rule, be charged with producing frost and snow, instead of scorching heats and burning fevers.

The *dog-days* in our almanacs are now put earlier by more than a month, than they would be if they

* It is worthy of observation, that customs and opinions, which at first sight appear very absurd and ridiculous, may frequently be traced up to something neither unreasonable nor useless. Thus, in the present instance, the Egyptians, as an ingenious foreigner remarks, having observed that the return of the periodical inundation answered constantly, for a series of years, to the heliacal appearance of a beautiful star towards the source of the Nile, which seemed to warn the husbandman against being surprised by the waters, were naturally led to compare this circumstance with the conduct of the domestic animal, which by *barking* gives notice of danger, and therefore called this star *the dog*, or the *barker*. From which opinion, in itself both reasonable and ingenious, in process of time sprung the absurdities above mentioned.

were still regulated by the rising of sirius before the sun: and thus they are made to correspond with the hot weather, as they did formerly.

190. Prob. XXVIII. *Given the altitudes of two known fixed stars, and the altitude of another star or planet, when in the same azimuth with one of those stars; to find the place of that star or planet.*

Ex. Observed the *moon* and *cor leonis* on the same azimuth, when the *moon's* zenith distance was 36° 37′; *cor leonis'* zenith distance 45° 0′; and at the same time the zenith distance of *cor hydræ* was 49° 16′. Quere, the place of the moon?

The declination of *cor leonis* is 12° 57′ N. its right ascension $9^h\ 57^m\ 42^s$; the declination of *cor hydræ*, 7° 47′ S. its right ascension $9^h\ 17^m\ 45^s$; the difference of their right ascensions $39^m\ 57^s$, or in motion 9° 58′. Hence, the problem might be constructed thus: On the plane of the equinoctial, the pole of which is P (fig. 17, Pl. 2.), draw the colures ♈ ♎, ♑ ♋, and in the solstitial take E for the pole of the ecliptic; put the given stars at B and A by their declinations and right ascensions: about B and A as poles, with their respective zenith distances, describe circles intersecting in Z the zenith; through Z and B describe an azimuth circle, and making Z *m* equal to the moon's zenith distance, it gives her place: then describe the great circles Z A, A B, E *m*; and the arc E *m* will be the co-latitude, and PE *m* the longitude from the first point of cancer. From this we deduce the following method of computation: In the triangle A B P are known P A = 97° 47′ cor hydra's co-declination, P B = 77° 3^m cor leonis' co-declination, and B P A = 9° 58′ difference of their right ascensions: whence are found A B P = 153° 57′, and A B = 23°, distance of the stars. Again, in the triangle A Z B are given Z A = 49° 16′, Z B = 45°, the zenith distances of the two stars, and A B just found; to determine A B Z = 89° 38′. Also,

in the triangle B *m* P, we know B P as before, $Bm = BZ - Zm = 8^\circ\ 23'$, and $PBm = ZBP = ABP - ABZ = 64^\circ\ 19'$; to find $Pm = 73^\circ\ 26'$ and $BPm = 7^\circ\ 53'$; the first of these is the moon's co-declination, the last her right ascension from B. Then $90^\circ +$ ♎ $PB + BPm = 128^\circ\ 27\frac{1}{2}' = EPm$. Lastly, in the triangle P *m* E, we have $PE = 23^\circ\ 28'$ obliquity of the ecliptic, $Pm = 73^\circ\ 26'$, and $EPm = 128^\circ\ 27\frac{1}{2}'$; whence are found $mE = 88^\circ\ 40'$ the moon's co-latitude, and $PEm = 48^\circ\ 41'$ moon's longitude from ♋; her absolute longitude is, therefore, $18^\circ\ 41'$ of leo. Or, when the right ascension and declination were determined, the longitude might have been found by the rule in Art. 66.

191. Prob. XXIX. *Observed a planet, or other celestial body, in the point of intersection of two lines connecting four known fixed stars; to find the right ascension and declination of that celestial body.*

Suppose A, B, C, D, the places of the four fixed stars, the right ascension and declination of each of which are known; also S (fig. 16, Pl. II.) the apparent place of the heavenly body. Then in the triangle A P B, where P is the pole of the equator, are known P A and P B, the co-declinations of the stars A and B, and the angle A P B the difference of their right ascensions; to find the angle P A B. In the triangle P A C, we have A P, P C, and the angle A P C; whence are found A C, P A C, and P C A. Then $PAC - PAB = CAB$. Again, in the triangle C P D are known, C P, D P, and the angle C P D, to find P C D; which added to A C P gives A C D. Also, in the triangle A C S we have A C, A C S, and C A S; whence we get A S. And lastly, in the triangle A P S, are known A P, A S, and P A S, to find A P S and P S; the former of which is the difference of right ascension of A and S; and the latter the complement of the declination of S. Had the longitudes and latitudes of the four stars

been known, then P being the pole of the ecliptic, a similar process would have determined the longitude and latitude of S.

192. Or this may be done with tolerable accuracy, in a mechanical manner, by means of a good celestial globe, on the surface of which the stars are very carefully placed. Thus, take a thread and extend on the globe, between the stars represented by A and B; and let another be extended from C to D; the intersection of these two threads will point out the place of the body. If at the same hour every evening, for nine or ten successive days, the place of the body be thus observed at the intersecting points between any four known stars, we shall manifestly obtain the apparent course of the body in the heavens, by drawing a line on the globe through the various intersecting points. And this will be a very easy, practicable way of finding the path of any new celestial phenomenon.

193. PROB. XXX. *Given the altitude of a known star, to find the altitude of the culminating point of the ecliptic, at any place the latitude of which is given.*

In this problem, we first, from the declination of the star, its altitude, and the latitude of the place, find the hour angle from the star's time of culminating, as in Art. 179. Then either add or subtract this hour angle, to or from the right ascension of the star in degrees, according as the observation was made after or before the star was on the meridian; the sum or difference is the right ascension of the culminating point of the ecliptic. This right ascension, or its supplement if it be greater than 90°, is one leg of a right-angled spherical triangle, and 23° 28′, the obliquity of the ecliptic, is the acute angle opposite the required leg, which when found is the distance of the ecliptic from the equator, at the culminating point, or the

declination of that point. Finally, if the right ascension of the culminating point be less than 180°, add its declination just found to the complement of the latitude (supposing it to be north); or if the A. R. be greater than 180°, subtract the declination from the co-latitude; the sum or difference respectively, will be the altitude of that point, as required. All which will appear very evident after comparing it with what was done in Problem 15.

CHAPTER VII.

On the most noted Systems invented to explain the Planetary Motions.

ART. 194. THE preceding Chapters contain that part of astronomy relating to the sun and fixed stars, most of which would be obvious even to such observers as did not examine the heavens with much minuteness and attention: but it is time to direct our consideration to the moon, and those other bodies which, though the least remarkable objects in the heavens to a careless and cursory observer, have occupied much the greater share of the industry and ingenuity of astronomers, and have led to the noblest inventions and discoveries which have been enrolled in the annals of science. The bodies we now allude to are the *planets*, or wandering stars, as their name imports; and indeed their appearances, with respect to an inhabitant of the earth, deviate so far from all our ideas of regularity and order, as to render the epithet *wandering* strictly applicable, and lead us to infer, did not the language of experience contradict the inference, that the reduction of their motions to any fixed and regular rules, would bid defiance to the utmost efforts of human sagacity. But almost insuperable obstacles vanish when approached with constant assiduity and skill; and the history of astronomy furnishes us with indubitable evidence of the astonishing success which has uniformly followed in the train of persevering diligence and ingenuity.

195. The names of the planets, with their astronomical characters, are thus given: *mercury* ☿, *venus* ♀, *mars* ♂, *jupiter* ♃, and *saturn* ♄; these were known to the ancient astronomers. The

indefatigable attention and care of Dr. *Herschel* have added another to the number of planets, called by him *georgium sidus*, or the *georgian planet*: by some astronomers it is called *Herschel*, in honour of the discoverer; though among almost all foreigners, it has acquired the name of *uranus*, which it is likely to retain. Its astronomical character is ♅ *.

196. In our enquiry into the natures, distances, magnitudes, and motions, of these bodies, we shall first state the most usual and striking appearances they exhibit; then describe the most celebrated schemes or systems which have been invented to account for these appearances; and assign the reasons which have induced the modern astronomers to reject all these systems, except what is now distinguished by the title *Newtonian*, and those which are manifestly included in it, or might lead to its invention.

197. With respect to the *moon*, we naturally conclude that she is constantly at nearly the same distance from the earth; for, since there is no reason to suppose her magnitude varies, as her apparent diameter at the same altitude never varies considerably (being at the greatest 33′, and at the least 29′ 30″), we may fairly infer that her distance from us is not subject to very great changes. As to her *shape*, we readily conceive it to be nearly spherical; for, although her appearance to a common observer is that of a flat circle, called the *disc*, yet, the assistance of telescopes, in union with just analogical reasoning, would induce us at once to reject the opinion of her being flat, should our hasty observations lead us to adopt it. If an opake globe be at a moderate distance from the eye, the picture of it upon the retina

* Since this was written, it is just reported, that another new primary planet has been discovered, viz. by M. Piazzi an Italian astronomer, on Jan. 1, 1801, or the first day of the 19th century. It appears only as a star of the 8th magnitude, and moves in an orbit between mars and jupiter, at $2\frac{3}{5}$ times the earth's distance from the sun.

is a circle properly diversified with light and shade; so as to excite in the mind the idea of a globe; if a globe be at a great distance from the eye, the difference between the lights and shades, which form the picture of a globe, will be imperceptible, and then the body will appear like a circular plane. A luminous globe, though but at a moderate distance from the eye, appears like a luminous plane, as will be evident to any one who makes the experiment with a globe of iron heated red-hot. So that whether the moon be luminous or opake, her distance from us causes her to appear flat, though she be in reality nearly globular.

198. The moon comes to the meridian later every day than the preceding, and the differences of the times of her culminating are very unequal; she also has very different altitudes when on the meridian. She, besides this, is very mutable in her figure, constantly putting on different phases. Sometimes she increases, from little more than an arc of a circle, to a full circular form; then she wanes and diminishes as it were in old age; sometimes she is bent into horns, then she appears like a half circle; then she becomes gibbous or hunchbacked, and next assumes a full round face; after this she gradually declines and loses all her lustre. Sometimes she enlightens us the whole night; at other times she does not appear till late at night, or early in the morning. Sometimes she keeps in the southern regions of the heavens; at others she rises higher and higher, and visits the northern hemisphere. Changes like these occurring so frequently, the whole of them being gone through in less than thirty days, naturally attracted attention; and thus the periods of these mutations soon became known; though *Endymion*, the first among the Greeks who carefully studied her motions, by his regular attention, gave rise to the fable that he had fallen in love with her.

199. The planet *Venus*, when viewed through a telescope, appears to undergo changes in her phases, similar to those of the moon; sometimes her full disc appears, at other times she appears gibbous, at others horned. Her apparent diameter also, as measured at the same altitude, is subject to considerable variation, being at some periods five or six times greater than at others. She sometimes is found to come to the meridian with the sun, then to precede that luminary in coming to the meridian, and so have an apparent motion from east to west, until her time of arriving at the meridian precedes that of the sun by about $3^h\ 10^m$, when she seems to be stationary for a short time; then her coming to the meridian precedes that of the sun by less and less degrees, until they are again on the meridian together. From this period she culminates later than the sun, and continues her apparent motion from west to east in a retrograde order, till her culminating is about $3^h\ 10^m$ after that of the sun; at this period she again appears stationary for a short time, after which her apparent motion is from east to west, as it was previous to her being stationary in the former instance. Her declination too is found to vary much, being sometimes more than 27° north, and occupying all the intermediate degrees, till it becomes as great toward the south. All the time when venus appears easterly of the sun she sets after him, and is called the *evening star;* when she is westward of that luminary, she sets before him in the evening, but shines before he rises in the morning, and is then called the *morning star*.

200. Observations can but seldom be made on the planet *Mercury*; but it has been observed sufficiently to determine that his motions, appearances, and mutations, are very similar to those of venus, with the exceptions that they are performed in a far less space of time, and that the distance of mercury

from the sun is never greater than about $1^h\ 50^m$ in time, or nearly $27\frac{1}{2}°$.

201. *Mars* is found sometimes to come to the meridian with the sun, sometimes to precede him, at others to follow him, and at some to be directly opposite to that luminary, so as to rise when he sets, and set when he rises. When mars is thus opposite to the sun, his apparent diameter is the greatest: being about five times as large as at those periods when he comes to the meridian with the sun. The apparent motion of this planet, like that of mercury and venus, is sometimes direct, or from east to west; at others retrograde, or from west to east; and sometimes the appearance is stationary. Mars also has different phases, appearing, when viewed through a telescope, either full and round, or gibbous, but never horned.

202. The motions of *Jupiter* and *Saturn*, compared with those of the sun, are found to be at times direct, at others retrograde, and at others stationary, like those of the other planets. These two bodies are also sometimes in opposition to the sun, as above mentioned respecting mars; and they, as well as the other planets, are sometimes to the north of the ecliptic, and sometimes to the south, but very seldom upon it.

203. To explain and account for these apparent motions of the moon and planets, together with those of the sun, various hypotheses or systems have been invented, some complex and confused, others on simple and general principles. "Systems," as Dr. *Adam Smith* ingeniously observes, "in many "respects resemble machines. A machine is a little "system created to perform, as well as to connect "together in reality, those different movements and "effects which the artist has occasion for. A system "is an imaginary machine invented to connect to- "gether in the fancy those different movements and

"effects which are already in reality performed. "The machines that are first invented to perform "any particular movement are always the most complex; and succeeding artists generally discover "that with fewer wheels, with fewer principles of "motion than had originally been employed, the "same effects may be more easily produced. The "first systems, in the same manner, are always the "most complex, and a particular connecting chain "or principle is generally thought necessary to unite "every two seemingly disjointed appearances; but "it often happens, that one great connecting principle "is afterwards found to be sufficient to bind together "all the discordant phenomena that occur in a whole "species of things." Nay, with respect to the motions and appearances in astronomy, a system has at length been invented which corresponds so accurately with the phenomena of nature, and explains them so satisfactorily, that even Dr. *Smith* himself, whilst endeavouring to represent all philosophical systems as *mere* inventions of the imagination, was insensibly drawn in "to make use of language expressing "the connecting principles of this one, as if they "were the real chains which Nature makes use of "to bind together her several operations."

204. That such has been the progressive improvement of the systems of astronomers, the following brief account of the chief of them will serve in some measure to shew. *Ptolemy*, an Egyptian philosopher, who lived in the time of the Roman emperor Adrian, was the author of an hypothesis in which it was attempted to reconcile the appearances of the heavenly bodies with the prevailing philosophy of the times. He supposed that the earth was fixed immoveably in the centre of the universe, and that the moon, mercury, venus, the sun, mars, jupiter, and saturn, revolved about it in the order here mentioned. Above these he supposed the firmament of the fixed

stars, the crystalline orbs, the *primum mobile* and *cœlum empyrium*, or heaven of heavens, were situated. All these immense orbs were imagined to move round the earth once in twenty-four hours; and to move also in certain periodical times, agreeably to their annual changes and appearances. Every star was supposed to be fixed in a solid transparent sphere, like crystal; and to account for their different motions, he was obliged to conceive a number of circles called eccentrics and epicycles, like little wheels attached to the circumference of larger, and intersecting one another in almost every possible direction: so that every motion which became newly discovered by the industry of observers, required the addition of a new wheel or epicycle to carry the centre of its eccentric round the centre of the earth. In consequence, therefore, of the increasing accuracy of the observations, which called for fresh epicycles and eccentrics, this system became at length so much clogged with incumbrance after incumbrance, that the imagination could no longer sustain it with tranquillity and satisfaction. And, besides this, after the lapse of a few centuries, it was found that this hypothesis was far from consistent with all the phenomena of the heavenly bodies; and the tables of Ptolemy gave the places of those bodies altogether different from what their real situations were found to be; so that the world became persuaded that this long-established system needed a thorough revisal, and then it required no painful efforts to induce them to reject it entirely.

205. The confusion in which the above hypothesis represented the planetary motions, suggested to *Copernicus*, a native of Thorn in Prussia, the idea of forming a new system, that, as himself tells us, "the noblest works of Nature might no longer ap-"pear devoid of that harmony and proportion which "discover themselves in her meanest productions."

He found in Plutarch that some old Pythagoreans had represented the earth as *revolving* in the centre of the universe, like a wheel round its own axis; and that others of the same sect had removed it from the centre, and represented it as revolving in the ecliptic, like a star round the central fire. The authority of these old philosophers might not originally suggest to him his system, but it seems to have confirmed him in it. He concluded "that if the earth was supposed to revolve every day round its axis, from west to east, all the heavenly bodies would appear to revolve in a contrary direction, from east to west. The diurnal revolution of the heavens, upon this hypothesis, might be only apparent; the firmament, which has no other sensible motion, might be perfectly at rest; while the sun, the moon, and the five planets, might have no other movement beside that eastward revolution which is peculiar to themselves. By supposing the earth to revolve with the planets round the sun in an orbit, which comprehended within it the orbits of venus and mercury, but was comprehended within those of mars, jupiter, and saturn, he could, without the embarrassment of epicycles, connect together the apparent annual revolutions of the sun, and the direct, retrograde, and stationary appearances of the planets; that while the earth really revolved round the sun on one side of the heavens, the sun would appear to revolve round the earth on the other; that while she really advanced in her annual course, he would appear to advance eastward in that movement which is peculiar to himself; and, by supposing the axis of the earth to be always parallel to itself, not to be quite perpendicular, but somewhat inclined to the plane of her orbit, and consequently to present to the sun the one pole when on the one side of him, and the other when on the other, he would account for the obliquity of the ecliptic, the sun's seemingly alter-

nate progression from north to south and from south to north, the consequent change of the seasons, and different lengths of days and nights in the different seasons." This new account of things manifestly rendered the appearances of the heavens more coherent than had been done by any of the additions to the Ptolemaic system; and this it did by a more simple, intelligible, and beautiful machinery. It represented the sun, the great enlightener of the universe, whose body was allowed to be alone larger than all the planets taken together, as established in the centre, shedding light and heat on all the worlds that circulated around him in one uniform direction, but in longer or shorter periods, according to their different distances. The planets too, which formerly seemed to be objects unlike to every thing in nature, when supposed to revolve along with the earth about the sun, were naturally apprehended to be objects of the same kind with the earth, habitable, opake, and enlightened only by the solar rays: and thus this hypothesis, by classing them in the same species of things with an object which is the most familiar to us, took off much of that wonder and uncertainty which the strangeness of their appearance had excited. Yet, after all, this system at its first entrance into the world was almost universally disapproved of; and the learned in the sciences treated it with contempt: objection after objection was brought against it; and it must be confessed that on account of the ignorance respecting the real laws of motion which then reigned, the answers to these objections, though they displayed considerable skill, were by no means satisfactory.

206. This led *Tycho Brahe*, who had spent his life and his fortune upon the science of astronomy, to invent another hypothesis, in which the earth was supposed fixed in the centre of the universe, and all the stars and planets were imagined to revolve round

it in twenty-four hours; yet he allowed a menstrual motion to the moon, round the earth; and he supposed that the sun was the centre of the orbits of the planets, in which they were carried round him in their respective years, while the sun revolved about the earth in a solar year; and all these, as well as the sun, he still supposed to move round the earth in twenty-four hours. Such was the system of Tycho, evidently compounded out of those of Ptolemy and Copernicus: in some respects more complex than that of Ptolemy, as it supposed the different revolutions of the planets to be performed round two centres; the diurnal round the earth, the periodical round the sun. It was not to be expected that a system so perplexed and unnatural as this could long maintain any reputation in the world; but it fell in with deep-rooted prejudices, and therefore, as it has been remarked, while "those who considered the heavens only, favoured the system of Copernicus, which connected so happily all the appearances which presented themselves here," and which even met with confirmation from the astronomical observations of Tycho himself; still "those who looked upon the earth, adopted the account of Tycho, which leaving it at rest in the centre of the universe, did less violence to the usual habits of the imagination." However, this system being very embarrassing to the minds of ingenuous persons, and being in many respects in direct contradiction to the phenomena which began then to be observed with greater accuracy than at any preceding period, an attempt was made to amend it, by *Longomontanus* and others, who allowed the diurnal motion of the earth on its axis, but denied its annual motion about the sun: this hypothesis was called the *Semi-Tychonic*. But even this system could not, with all the ingenuity of its advocates, be made to correspond with the heavenly

phenomena, and of course it remained but a short period before it was completely abandoned.

207. About this period, the important invention of telescopes, together with the admirable sagacity and assiduity of *Galileo*, tended considerably to obviate the objections which had been urged against the Copernican system. This great, though unfortunate philosopher, removed an objection to the hypothesis of Copernicus, drawn from the nature of motion, by explaining the effects of the composition of motion, on principles now generally laid down in works on mechanics; and from which it naturally results that a ball dropt from the mast of a ship under sail would fall precisely at the foot of the mast; and, on a similar account, that the phenomena of falling bodies on the earth would be the same, whether it were at rest or actually in motion. It had been also objected to Copernicus, that his supposition of the moon moving round the earth, could not be admitted on the hypothesis of the earth revolving about the sun; unless it could be shewn that the other planets, or some of them, had moons revolving about them also: Galileo, by applying the telescope to astronomy, soon removed this objection; for he discovered that jupiter had satellites moving about him in a manner similar to the motion of the moon round the earth; and the subsequent discovery of the satellites of saturn, served completely to shew that the connection of the moon with the earth, was perfectly consistent with the analogy of nature. Another objection brought against the system of Copernicus was, that if venus and mercury revolved round the sun, in an orbit comprehended within the orbit of the earth, they would shew a variety of phases as the moon did. He answered, *that they undoubtedly did all this, and that some time or other their resemblance to the moon, in this respect, would be found out*: this very bold assertion was con-

firmed by Galileo. He discovered the phases of Venus, and sent an account of the discovery in a letter written from Florence, in 1611, to the Duke of Tuscany's embassador at Prague. From a translation of this letter the following passage is extracted: "We have hence the most certain, sensible "decision and demonstration of two grand questions, which to this day have been doubtful and "disputed among the greatest masters of reason in "the world. One is, that the planets in their own "nature are opake bodies, attributing to mercury "what we have seen in venus; and the other is, "that venus necessarily moves round the sun, as "also mercury and the other planets: a thing indeed well believed by Pythagoras, Copernicus, "Kepler, and myself; but never yet proved, as "now it is, by ocular inspection upon venus."

Galileo likewise discovered spots in the sun, and demonstrated by their motion (since he could easily shew that they adhered to the sun's surface) the revolution of that luminary round its axis; and this rendered it less improbable that the earth, a body so much smaller than the sun, should revolve round its axis in the same manner.

208. While Galileo in Italy was by his discoveries paving the way for the universal reception of the system of Copernicus, *Kepler*, a German philosopher of great genius, and greater industry, was employing himself to ascertain, correct, and improve it. A circle, as the degree of its curvature is every-where the same, is the simplest of all curves, and most easily to be conceived; it was therefore natural to conclude that a circular motion was the most perfect of all motions, and, since the universe was a perfect work, that the motions of the heavenly bodies were performed in circles: but Kepler saw reason to reject this opinion, which, though not exceedingly erroneous, was not quite consistent with the true state of

things. Having, pursuant to the invitation of the generous and noble Tycho Brahe, gone to reside with him at his observatory on a little island on the Baltic, the Danish astronomer communicated to him his observations upon mars, in the arranging and methodising of which his disciples were at that time employed. Kepler, on comparing them with one another, found that the orbit of that planet was not a perfect circle, that one of its diameters was somewhat longer than the other; and that it approached to an ellipsis, which had the sun in one of its foci. He found too that the motion of the planet was not equable; that it was swiftest when nearest the sun, and slowest when farthest from him; and that its velocity gradually increased or diminished according as it approached to or receded from him. Other observations of Tycho enabled Kepler to conclude, that the same things also, though in different degrees, were true of all the other planets. They exhibited the same things too of the sun, if supposed to move round the earth, and consequently of the earth if supposed to revolve round the sun.

209. This indefatigable astronomer, after many trials, at length found out a rule, which obtained in the real motions of the planets. Acording to him, if a straight line was drawn from the centre of each planet to the sun, and carried along by the periodical motion of the planet, it would describe equal trilineal areas in equal times, though the planet did not pass over equal spaces; the same rule he found to take place nearly with regard to the moon. Kepler, besides this, introduced another new analogy into the system, and first discovered that there was one uniform relation observed betwixt the distances of the planets from the sun, and the times employed in their periodical motions. He found that their periodical times were greater than in proportion to their distances, and less than in proportion to the square

of their distances. After committing several mistakes in determining the real analogy, he discovered at last, on the 15th of May, 1618, for he is so exact as to mention the precise day when he found, that "the squares of the periodical times are always in "the same proportion as the cubes of the mean dis"tances from the sun."

210. The observations of *Cassini* fully established the truth of these famous analogies of Kepler; for he discovered that the satellites of jupiter and saturn revolved round their primary planets in conformity to those analogies. And when it was determined that the motions of the four satellites of jupiter, and the five of saturn about their respective primaries, as well as those of all the primary planets (among which the earth is now included) about the sun, agreed with these analogies, the doctrine of Kepler was greatly confirmed, and his improvement of the Copernican system soon triumphed over every opposition.

211. Still, however, there remained to be discovered some extensive and general principle *, from which the analogies which obtained in the motions of the heavenly bodies should naturally flow; some invisible chain which should bind together those various bodies, and yet not oblige them to come into

* "The mind of man doth wonderfully endeavour, and extremely "covet this, that it may not be pensile; but that it may light "upon something fixt and immoveable, on which as on a firma"ment it may support itself in its swift motions and disquisitions. "Surely, as *Aristotle* endeavoureth to prove, that in all motion "of bodies there is some point quiescent, and very elegantly ex"poundeth the ancient fable of *Atlas*, that stood fixt and bare up "the heavens from falling, to be meant of the poles of the world, "whereupon the conversion is accomplished. In like manner "men do earnestly seek to have some *Atlas* or axletree of their "cogitations within themselves, which may, in some measure, "moderate the fluctuations and wheelings of the understanding; "fearing, it may be, the falling of their heaven."—*Bacon's Advancement of Learning*, lib. 5. G. Wats's edition, 1674.

immediate contact. This was eagerly sought after. Kepler attempted, as was usual with him, to discover it by some lucky guess: *Des Cartes* endeavoured to ascertain wherein this invisible principle consisted, and formed an hypothesis which will ever remain a monument of his admirable genius and lofty conception: *Spinosa* added largely, from the store of a rich imagination, to what he learned from *Des Cartes:* and the learned *Leibnitz*, after maturing his speculations, made the result of them known to the world. But these and others, though of great utility in striking out some particular and isolated principles, could only display, as general laws, such as merely had their origin in their own lively fancies. SIR ISAAC NEWTON was the first who successfully attempted to give a physical account of the motions of the planets, which should accommodate itself to all the constant irregularities ever observed by astronomers to take place in those motions. He adopted the maxim of his great precursor LORD BACON, "*that all theory was to be laid aside which was not* "*founded on experiment;*" and by a train of well-directed observations, aided by an excellent analysis, assailed Nature, as it were, with unwearied importunity, until he extorted from her the secret which regulated all her operations.

212. The superior genius and sagacity of this most eminent philosopher made the most happy, and we may now say, without fear of contradiction, the greatest improvement that was ever made in philosophy, when he discovered that he could connect together the motions of the planets by so familiar a principle as that of the gravitation of matter. "He demonstrated that, if the planets were supposed to gravitate towards the sun and to one another, and at the same time to have had a projecting force originally impressed upon them, the primary ones might all describe ellipses, in one of the foci of which that great luminary was

placed; and the secondary ones might describe figures of the same kind round their respective primaries, without being disturbed by the continual motion of the centres of their revolutions. That if the force which retained each of them in their orbits was like that of gravity, and directed towards the sun, they would each of them describe equal areas in equal times. That if this attractive power of the sun, like all other qualities which are diffused in rays from a centre, diminished in the same proportion as the squares of the distances increased, their motions would be swiftest when nearest the sun, and slowest when farthest off from him, in the same proportion in which by observation they are discovered to be; and that, upon the same supposition of this gradual diminution of their respective gravities, their periodic times would bear the same proportion to their distances which Kepler and Cassini had established betwixt them*."

§ 13. Nor was this all; for he not only endeavoured to shew that gravity *might* be the connecting principle of the planetary movements, but he proved that it really *was* so. Galileo had shewn that when a stone is projected, it is deflected from the rectilinear path into a parabola, in consequence of the gravity of the stone towards the earth. To shew that the moon is retained in its orbit by the force of gravity, *Newton* shewed that it was continually bent from the tangent of its orbit, in the same manner as Galileo shewed the deflection of the path of the stone, projected near the surface of the earth: that both these motions are directed to the same point, and agree in quantity: that, if the moon were to approach to the surface of the earth, the force by which it is retained in its orbit, would make it descend through the same space towards the centre of the earth, as an heavy body would fall through by its gravity in the same time; and that if a stone

* Dr. Adam Smith's History of Astronomy.

could be carried to the distance of the moon, and there projected with a sufficient velocity, it would revolve round the earth like a moon, in consequence of the same cause which bends its motion into a curve when projected near the earth's surface. Similar methods of reasoning were applied to all the planets; and as it appeared that the revolutions of the primary planets round the sun, and those of the satellites of jupiter and saturn round their primaries, are phenomena of the same kind with the revolution of the moon about the earth; likewise, as the centripetal powers of the primary are directed towards the centre of the sun, and those of the satellites towards the centre of their respective primaries; and lastly, as all these powers are reciprocally as the squares of the distances from the centres; it was safely concluded that the cause was the same in all, and therefore that the gravitation of matter is universally extended.

214. This system of *Newton* is the only one which accounts, in a satisfactory manner, for the various irregularities which astronomers have observed in the heavens. It assigns a reason why the centres of the revolutions of the planets are not precisely in the centre of the sun, but in the common *centre of gravity* of the sun and the planets: it is applied with peculiar success to the solution of the irregularities in the motion of the moon, and to the deviations from regularity in the motions and orbits of the various primaries and secondaries: that which had puzzled all preceding philosophers even to guess at the cause, namely, *the precession of the equinoxes*, has been explained on the principles of Newton, and calculations applied to the theory, the conclusions resulting from which correspond very accurately with the observations of astronomers: and other phenomena which have been discovered since the time of Newton, have met with a ready solution by a just appli-

cation of his principles. So that, although gravity be an occult cause, as was at first objected against it, yet, as in all the consequences deduced from it, the reasoning flowed from its effects without entering into the contemplation of causes, it must be acknowledged, that by making the phenomena of gravity the subject of physical consideration, Newton performed the most essential service to philosophy: and as his system has now acquired the most universal empire that was ever established in philosophy, we shall conceive ourselves fully justifiable in adopting his principles in every subsequent part of this work, where they will assist us in our enquiries.

CHAPTER VIII.

On the Truth of the Copernican System, as improved by Kepler and Newton.

ART. 215. IN deducing the laws by which the motions of the celestial bodies were regulated, as attempted to be shewn in the faint sketch given at the end of the last chapter, it is manifest that *Newton* must have directed his enquiries agreeably to the principles of the soundest reasoning. His rules of philosophising are, indeed, excellent; and, as they may often be of considerable utility in developing the causes of many natural appearances, they are here inserted.

216. RULE I. *More causes of natural things are not to be admitted, than are both true and sufficient to explain the phænomena.*

RULE II. *Therefore to the same natural effects we must always assign, as far as possible, the same causes.*

RULE III. *The qualities of bodies which admit neither retention nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.*

RULE IV. *In Experimental Philosophy, propositions collected from the phænomena by induction are to be deemed (notwithstanding contrary hypotheses) either exactly or very nearly true, till other phænomena*

occur, by which they may be rendered either more accurate, or liable to exceptions.

217. The first of these rules is founded upon the principle, that in Nature nothing is done in vain, but all is simple, and there are no traces of superfluous causes of things. The second naturally results from it; for no person who allowed that simplicity reigned in the operations of Nature, would ascribe respiration in man and in animals to different causes; or would feel a difficulty in admitting that the cause of the descent of stones was the same in America as in Europe. The third is consistent with the principle of analogy, agreeably to which the far greater part of human actions are regulated: by this rule, because extension, divisibility, hardness, mobility, gravity, the vis inertiæ, &c. are found in all bodies which fall under our cognizance, we conclude that they are original and universal properties of all bodies whatever. The fourth rule must, of necessity, be admitted, otherwise arguments by induction might at any time be destroyed by mere hypotheses, and reason would be subdued by fancy.

218. If the truth of the general laws of motion and gravity (as laid down by the authors on mechanics) be granted, and their truth has repeatedly been demonstrated both by theory and by actual experiments; we may, by a natural combination of these laws of motion, with the rules just given, be nearly as fully persuaded of the truth of the *Copernican System*, with the improvements of *Kepler* and *Newton*, as we should be by the most direct demonstration. Perhaps the only objection which can now be urged with any force, against this system, is drawn from the supposed impossibility of the earth's motion; for if once it be admitted that the earth is not fixed and immoveable, the mind will readily assent to every other part of this system. Now, it is manifest from constant observation, that either the earth

is at rest, and all the heavenly bodies perform their revolutions daily in circles parallel to each other, their several centres falling in the axis of the earth or heavens; or, that these celestial bodies, or the greater part of them, are at rest, and the earth revolves daily about one of its diameters as an axis. But that the first of these is highly improbable, may be thus shewn: Both the theory and practice of mechanics teach us, that forces are directed to some central body on which they physically depend; that the force of gravity decreases inversely as the square of the distances from its centre; and that when various bodies in a system revolve, their motions are performed about their common centre of gravity: the heavenly bodies are at widely different distances from the earth (varying from about forty millions of miles, to an almost infinitely greater distance, as will be hereafter shewn); and all these bodies, on this supposition, must move round the earth, or its axis produced, in twenty-four hours; and, besides this, the fixed stars, as appears by the precession of the equinoxes, have another motion round the axis of the ecliptic in about 25920 years: so that, if the earth be fixed, an innumerable multitude of bodies (for such, telescopic observations shew the stars to be) are daily moved with wonderful rapidity by forces directed to innumerable imaginary points on which they cannot possibly depend; and those forces must increase exactly in proportion to the distance from the imaginary centres, or they cannot all revolve in the same time; this is directly contradictory to the established laws of motion and gravity, and is, therefore, inadmissible by Rules I and II. Again, to come to a particular case, either the earth turns round its axis, or the sun revolves round the earth in twenty-four hours. Now, when one body revolves about another as its centre, it is necessary that the central body be always in the plane in which the revolving body moves; otherwise

it cannot be said to revolve about that body, but some other point: therefore, if the sun moves about the earth in a day, the earth must be in the plane in which the sun's motion is described, during the whole day. But at all times of the year, except when the sun is in the equinoxes, the sun describes circles which have not the earth for their centre; and therefore, according to the rules here laid down, can no more be said to revolve about the earth, than a body moving round the base of a cone can be said to revolve about the vertex. From these remarks it appears, that if we are at all able to determine what is *impossible*, respecting the different bodies in the universe, we must conclude against the possibility of the earth being at rest. On the contrary, if the rotation of the earth be admitted, we are furnished with an easy solution of various facts, which cannot otherwise be accounted for: thus, the figure of the earth is not spherical, but spheroidical (Art. 10.); but experiments prove that all the parts of the earth have a gravitation towards each other, and as a considerable portion of its surface is spherical, it would, if at rest, in consequence of the equal gravitation of its parts, be spherical. What then causes it to be spheroidical? We know that if an iron hoop be made to revolve swiftly round a fixed axis, the diameter corresponding with the axis about which it revolves will be shortened, and the diameter perpendicular to this will be lengthened, the hoop varying from a circular to an elliptical form: this we, without hesitation, attribute to the rotation of the hoop. In like manner, reasoning according to Newton's first and second rules, we may infer, that in consequence of the rotation of the earth, the parts most distant from its fixed axis have a greater tendency to fly off than those which are nearer, and therefore the diameter perpendicular to that axis must be the greatest. And this is proved (we may

venture to say) to demonstration, by the consideration that the diameter, about which the earth must revolve to correspond with the phænomena of the heavenly bodies, *is exactly that* which, from actual mensuration, is found to be the shortest.

219. The diurnal rotation of the earth being assented to, its annual motion is readily admitted: for since the planetary bodies are found to revolve on their axes, and the equatoreal diameters of some of them are known to be greater than the polar, they agree with the earth in these respects; and since they evidently have periodical revolutions from west to east, we are almost constrained to imagine the earth has such a revolution also, in order to render the analogy between it and the planets general and complete. But all the planetary revolutions, it is most natural to conclude, are performed about some common centre; and every phænomenon being easily and satisfactorily explained on the supposition that the sun is in, or near, that centre, we assume the sun as the centre of the *planetary*, or, as it is often called, the *solar system.* To this it may be objected, that if the motions of a system of revolving bodies are performed round their common centre of gravity, as the sun is one of the bodies in the system under consideration, he, as well as they, will move round the common centre of gravity, and will therefore appear to an inhabitant of the earth to have direct and retrograde motions, and to be sometimes stationary, as the planets are; whereas nothing of this kind is observed. We answer, without denying the principle on which this objection is founded, that the observations of astronomers have now determined with tolerable accuracy the dimensions of the sun (Art. 387.); and from a proper comparison of the dimensions and distances of the sun and planets, it results, that the common centre of gravity of the whole is al-

most always within the body of the sun, and can never be at the distance of its semi-diameter from it; so that, instead of the sun's motion round the common centre of gravity being obvious to common observers, it must require a considerable degree of care and patience to discover it at all, only as it is deduced from physical principles. On the whole, we think, the candid enquirer after truth will see much in the system here explained to recommend it to his notice: we therefore proceed to give the theory of apparent motions, and then to determine the distances, dimensions, &c. of the planets; in the course of which other evidences of the truth of this hypothesis will frequently present themselves.

CHAPTER IX.

On the Theory of apparent Motions, with its Application to the Phænomena of the Planets' Motions.

Art. 220. THE earth partaking of a twofold motion, one about its axis in the compass of a natural day, the other in an elliptical orbit about the sun, in a year, an observer of the heavens will be liable to optic illusions arising from the idea of the earth's being immoveable. We now propose to consider the circumstances of those illusions which are caused by the *annual* motion of the earth, and in order thereto, give the following theory of apparent and relative motions.

221. That which an observer, who is in motion, and fancies himself at rest in some fixed point, attributes to an object really at rest, is called the *apparent motion* of such object; and that motion which is attributed to an object in motion, by an observer also in motion who thinks himself at rest, is called *relative motion*.

222. By the *eye's true place* is meant, that point of the universe where the eye of the observer really is at a given time; and that point where the observer conceives himself to be at rest, we call *the eye's imaginary place*. Whether we suppose the real motions to be circular or elliptical, we call the path described by the observer *the eye's orbit*; and the path

which the object appears to describe in the heavens, *the optic orbit.*

223. The visual rays drawn from the eye's imaginary place, to every point of the optic orbit, will form, on a plane beyond it, a figure which may be called *the projection of the optic orbit:* or, to strengthen the idea, *the projection of the optic orbit* is that figure which would be formed on a plane by the boundary of the shadow of the orbit made by a light placed at the eye's imaginary place. If the visual rays intercepted by the plane are perpendicular to it, then, as is well known, the projection is called *orthographic.*

224. A plane passing through the object and the eye's imaginary place perpendicular to the plane of the eye's orbit, is called the *plane of comparison.* Consequently, if the object be fixed, the plane of comparison is also fixed; if it be moveable, that plane is likewise moveable, and with an angular velocity equal to that of the object.

225. If we have given in position the eye's imaginary place S (fig. 1, Pl. III.), and any number of points A, B, C, in the path of a moving body in any plane; with the points D, E, F, the eye's true places at those instants; and the optic path of the moving body be required: then, the lines A D, B E, C F, being drawn, from S draw S *a*, parallel and equal to AD, S *b* parallel and equal to B E, and S *c* parallel and equal to C F; *a*, *b*, *c*, will then be the corresponding points of the optic path. For, the right S *a* being equal and parallel to A D, the point *a* is situate in the same manner to S, both with respect to position and distance, as A is to D. The observer, therefore, conceiving his eye placed in S, will imagine he sees the object in *a*. For the same reason, when the observer is at E and F, supposing himself at S, the points B and C appear to be at *b*

and *c*. Hence may be deduced the following corollaries:

226. Corol. I. The true and imaginary places of the eye, and the true place of the object, with its place in the optic orbit, always form a parallelogram. The object's true place, and the eye's imaginary place, are at the extremities of one diagonal; the object's place in the optic orbit, and the eye's true place, are at the extremities of the other diagonal.

227. Corol. II. Consequently, the situation of the optic place of the object is always opposite to that of the eye's true place.

228. Corol. III. When the object is immoveable in O (fig. 2, Pl. III.), the optic orbit *a b c* is a line similar and equal to the eye's real orbit D E F, and is in a parallel plane. For, as the parallelograms D *a*, E *b*, F *c*, have the diagonal S O common, which is also the common intersection of their planes; and, as their bases S D, S E, S F, are situated in one plane, namely, the plane of the eye's orbit; their parallels and equals O *a*, O *b*, O *c*, must also be in one plane parallel to that of the eye's orbit; making the angles *a* O *b*, *b* O *c* equal respectively to the angles DSE, ESF. Consequently, the points *a*, *b*, *c*, must be in a line equal to the line D E F, and in a parallel plane, but in an opposite situation. If the eye's path be supposed a circle, with the imaginary place at the centre; and the object is immoveable either within or without that path; it is evident from this corollary that the optic path will also be a circle: this, by way of distinction, we call the *epicycle* of the object.

229. Corol. IV. When the object is immoveable, and in the plane of the eye, its optic orbit will also be in that plane.

230. Corol. V. When the object is immoveable, and situated in the eye's imaginary place, it appears to be at the extremity of a radius, equal, and in the

same direction, with that drawn from the eye's true place to the centre. Hence, if the eye describe a circle or an ellipsis, the centre of which is the eye's imaginary place, as well as the real place of the object, the object will appear at the extremity of a diameter passing through the eye, and consequently it will seem to describe the same orbit with the eye.

231. COROL. VI. When the object is immoveable, its apparent velocity in its optic orbit is equal to the eye's real velocity in its orbit.

232. LEMMA I. *Let* Q I (fig. 3, Pl. III.) *be the plane of a great circle of the sphere;* F G *a plane touching the sphere in* H; H L *the plane of a small circle parallel to* Q I, *and passing through* H: *then the inclination of the plane* H L *to the plane* F G, *is measured by the complement of the arc* Q H; *and the radius of* H L *is to the radius of* Q I, *as the co-sine of* Q H *is to radius.* For, drawing from the centre I, the radius I H, the angle M H L = inclination of the plane F G, with the plane of the parallel H L, is the complement of the angle L H I, or it is the complement of the angle H I Q, measured by the arc Q H: and H L is manifestly the co-sine of Q H, Q I being radius.

233. LEMMA II. *If from all the points* A, M, D, B, C (fig. 4, Pl. III.), *of the circumference of a circle situated in a plane* N H K S, *inclined to another plane* N G I H, *the lines* A a, M m, D d, B b, C c, *&c. be drawn perpendicular to the latter plane; they will terminate in the circumference of an ellipsis* a d b c, *whose greater axis* a b *will be equal and parallel to that diameter of the circle which is parallel to the plane* H G, *or to its intersection* H N; *and the less axis* c d *will answer to the diameter* C D, *perpendicular to the plane* H G, *or its intersection* H N, *and will be to the greater axis* a b, *as the co-sine of the inclination of the*

planes H G, H K, *is to radius.* For, since A B, the diameter of the circle, is parallel to the plane H G, and the perpendiculars A *a*, B *b*, are parallel to each other, the figure A *a b* B is a right-angled parallelogram, and consequently A B = *a b*. The diameter C D, perpendicular to H N and to A B, bisects these two lines; also the plane *c* V C containing the perpendiculars C *c*, D *d*, is therefore perpendicular to the plane H G, and equally distant from the parallel planes *a* H A, *b* N B. Consequently, *c d*, perpendicularly bisects *a b*, the angle *c* V C is equal to the inclination of the planes H G, H K, and, because of the similar right-angled triangles *c* V C, *f* V F, *d* V D, the parts *c f*, *f d*, of the right line *c d*, are equal to one another, since E F is evidently equal to F D. The same triangles shew that *c d* : C D or *a b* : : *c* V : V C : : co-sine *c* V C : radius, because C *c* V is a right angle. Hence, *c d*, *a b*, are to one another, as the co-sine of the inclination of the planes to radius. We have now to shew that the curve *a c b d* is an ellipsis, of which *a b*, *c d*, already shewn to bisect each other perpendicularly, are the axes; in order to which, draw any ordinate M P to the circle, and draw M *m*, P *p*, perpendicular to the plane H G. The point P being in the diameter A B, the perpendicular P *p* is parallel to A *a*, B *b*, and in the plane of the rectangle A *a* B *b*; it consequently falls upon the line *a b*, making *a p*, *p b*, respectively equal to A P, P B. But P M being perpendicular to A B and to H N, and parallel to D C, the plane P X *p* containing the perpendiculars P *p*, M *m*, is parallel to the plane C V *c*; therefore *p m* is perpendicular to *a b*, and is an ordinate to the curve *a c b d*; alſo the triangles *p* X P, *m* X M, *c* V C, *d* V D, are similar; hence *p m* : PM : : *c d* : C D or *a b*; or by the nature of proportion $pm^2 : PM^2 :: cd^2 : ab^2$. Now $PM^2 = AP \times PB = ap \times pb$. Substitute this instead of PM^2 in the foregoing analogy, and there arises $pm^2 : ap \times pb :: cd^2 : ab^2$: this is a well-

known proportion between the abscissæ and ordinates of an ellipsis; and as the same might be shewn of any other ordinate in *a c b d*, that curve is an ellipsis. The truth of this lemma is, therefore, manifest.

234. From this lemma we might deduce as corollaries, the method of dividing any arc of the ellipsis into degrees corresponding with the degrees in an arc of the primitive circle; a method of describing an ellipsis on two given axes, which should at the same time be divided at pleasure; a way of marking the degrees on an ellipsis infinitely narrow, which is always the case when the eye is in the place of the projecting circles; with other things generally shewn in the treatises on Orthographic Projection *.

235. THEOREM I. *If the eye's true orbit is a circle, and the object, placed at an immense distance from the eye, be immoveable; the part* HNGI *of a great circle in the heavens, occupied by the projection of the optic orbit* ACBD (fig. 4, Pl. III.) *is so small, that it may be considered as a plane; this projection is an ellipsis* acbd, *whose greater axis* ab *is in a parallel position to the plane* QR *of the eye's orbit, and equal to the celestial arc* ab *intercepted between the radii* Sa, Sb, *drawn from* S, *the imaginary place of the eye, to the extremities of the diameter* ab, *which is in a parallel position to the plane* HNGI; *its less axis* cd *is in the plane of comparison, or in a plane perpendicular to the plane of the eye's orbit: also, the less axis is to the greater, as the sine of the arc* QH, *measuring the distance of the plane* QR *of the eye's orbit from the plane* HK *of the optic orbit, is to radius.*

DEMON. The circle ACBD being supposed at

* It gives the Authour pleasure to have an opportunity of recommending to the notice of the reader, a piece on the *Projection of the Sphere*, so original, perspicuous, truly geometrical, and in every respect so worthy of attention, as that given in *Bishop Horsley's Elementary Treatises on the fundamental Principles of Practical Mathematics.*

almost an infinite distance from the eye S, and the radii drawn from the centre to the surface of a sphere being perpendicular to it; then the right lines A *a*, B *b*, C *c*, D *d*, &c. parallel to each other, are considered as perpendicular to, and terminating at, the plane H N G I, inclined to the plane H K. Therefore (Art. 233.) the curve *a c b d* is an ellipsis, the greater axis *a b* of which is equal to the diameter A B of the circle A C B D; its less axis *c d* is perpendicular to the plane of the eye's orbit, and is to the greater axis *a b*, as the sine of the arc Q H to radius.

236. Corol. I. The projection *a c b* of that part of the circle A C B nearest the eye, is the part of the ellipsis raised most above the plane of the eye, or the part most distant from that plane: on the contrary, the projection *b d a* of that part of the circle B D A opposite to the eye, or most distant from it, is the interiour part of the ellipsis, that is, the part nearest to the plane of the eye's orbit.

237. Corol. II. The more perpendicular the plane of the optic orbit is to the eye's true orbit, the more its elliptic projection approaches to a circle; and *vice versa*: so that, if the optic orbit answered to the pole Z of a great circle in whose plane the eye's orbit is placed, its projection would then be a circle, because both axes of the ellipsis would be equal: but if the optic orbit were in the same plane with the eye's orbit (the less axis being then indefinitely small), the elliptic projection would become a right line equal to the greater axis of the ellipsis, or to the diameter of the optic orbit.

238. Corol. III. Hence, when a celestial object, F, is immoveable in the heavens, and at a very great distance from the eye of the observer, who conceives himself at rest in S, the centre of the circle he really describes—1st, The object is never seen at the true place, *f*, where the visual ray terminates, but appears to describe an ellipsis, *a c b d*, about the true place, *f*,

at each revolution of the eye in its orbit; and consequently appears going sometimes in one direction, *a c b*, in describing one semi-ellipsis, when it is said to be *direct*; and then in a contrary direction, *b d a*, when it describes the other semi-ellipsis, which apparent motion is called *retrograde*: 2dly, Its velocities must also be very unequal: for when the object in its apparent motion is near one end, *c* or *d*, of the less axis, the space it runs through being in full view, it then seems to move very fast; but, as it gradually approaches the ends *a* or *b* of the ellipsis, it then seems to slacken its pace, because the arc that it describes is more oblique to the visual rays; and when the object is at either extremity of the greater axis of the ellipsis, it seems for some time *stationary* or immoveable; after that, changing its direction, it revolves under different circumstances, as just described.

239. Lemma III. If on the circumference and in the plane of a circle, A B C (fig. 5, Pl. III.), another circle, AMD P, be supposed, that at first has no other motion than to turn on its centre O, in the direction of the letters A, M, D, P; it is obvious that although any point, A, of its circumference, in turning, has all kinds of directions, yet with respect to the centre S it has properly but two opposite directions, the one *direct*, in describing the *superiour semicircle* M D P, the other *retrograde*, when it describes the *inferiour semicircle* P A M.

But, if the centre O of the circle A M D P be supposed to move in the direction O I N, so that it roll as a wheel on the circumference A B C; then it is evident,

240. 1st, That the track of this centre is a concentric circle O I N greater than the circle A B C.

2dly, That the point A returns to the circumference of the circle A B C, when it has described, besides a whole rotation on its centre O, an arc BL of its circle similar to the arc AB, comprehended between

the point A departed from, and the point B returned to: for, the point A being at its departure relative to a fixed point infinitely distant in the direction of the radius O H, its whole rotation on its centre is finished when it returns to the radius directed to that fixed point: when, therefore, the point A is arrived at B on the circle A B C, if through the centre I, I L *h* be drawn parallel to A H, these two right lines will coincide at that infinitely distant fixed point, and the point L is the relative situation of the point A; and the centre being in B, it has described its whole circle + the arc B L, containing a number of degrees equal to O I or A B, because of the parallels I *h*, A H; though respectively to S the point A seems to have made but one rotation about its centre.

3dly, Marking the track A *e* Q *g* B K C described by the point A at each return to the circle A B C; by the first return is described a curve A *e* Q *g* B, called an *Epicycloid;* the second forms another epicycloid B K C, and so on. Now there may occur three cases.

241. Case I. If the circle A B C remain immoveable, while A M D P (Fig. 5, Pl. III.) runs on its circumference, then every epicycloid, as A *e* Q *g* B, is called *simple* or *ordinary*, and the following properties respecting it are evident:

1st, Though the velocities of the circle be ever so unequal, the arc AB (called the *base* of the epicycloid) is equal to the circumference AMDP.

2dly, If the arc OI, similar to the arc A B, be divided into four equal parts at the points E, F, G; the centre O will have arrived at the point E, when the point A has described a fourth part of its track in returning to the circle A B C: the same happens at the points F, G, I, where the centre of the circle A M D P is found at the end of each fourth part of its return to the circumference A B C. Therefore,

if through the point F be drawn a radius S F, prolonged till Q F = F T, it will give the point Q for the vertex of the epicycloid; and the part Q T will be its axis, being equal to the diameter of the circle A M D P. And if from the point E as a centre, with a radius equal to O A, an arc of a circle be described towards A, it will intersect the epicycloid in *e*, the situation of the point A when the point O was at E. In the same manner the point A will also be found to have been in *g*, when the centre was in G; whence it follows, that the arc A *e* was described while the point A was retrograde in describing a part A M of its inferiour semicircle; and the arc *e* Q *g* was described while the point described its upper semicircle M D P with a direct motion: lastly, the arc *g* B was described while the point A was retrograde in the other part P A of its inferiour semicircle.

3dly, The epicycloid is bisected in the vertex Q; and the legs Q *e* A, Q *g* B, of the curve are equal, and similarly posited with respect to the axis TQ.

4thly, The arcs A *e*, *g* B, bend towards the axis T Q, because the retrograde motion of the point A carries them in that direction: but they do not return into themselves (as in fig. 8, Pl. III.); because the path O I of the centre A being, in every revolution, greater than the arc A B, which is equal to the circumference A M D P, the motion of that centre in the direction O I N draws the point A towards K, more than its retrograde motion about the centre O carries it in an opposite direction.

5thly, The describing point A, in passing from the leg Q *g* B into the leg B K of the next epicycloid, makes an angle at B: for the point describing the last leg of the epicycloid A Q B, the instant it comes to B, begins to rise again into the first leg of the other epicycloid B K C, and describes no space between those legs.

6thly, The point A always appears direct when viewed from S.; but the velocity seems to be accelerated all along the leg A *e* Q, to be greatest at the vertex Q, and then to be retarded along the leg Q *g* B, so as to become nothing in the point B: after this it undergoes the same changes in the second and subsequent epicycloids, as in the first.

242. Case II. If, while the circle A M D P rolls on A B C (fig. 7, Pl. III.) the latter should also turn on its centre S, in the same direction A B C; or, in general, if the centre of the circle A M D P advances in the circle O F I, with a greater velocity than the point A has in revolving in the circle A M D P; then the epicycloid described during the revolution of the point A, would be *lengthened*, and evidently have the following properties:

1st, Since all the points of the simple epicycloid would advance towards C, although the circle ABC were at rest; therefore, if A B C advance also in the same direction, the velocity of the describing point must be greater in that direction, and consequently make the spaces longer in a given time; so that the base A B of the epicycloid must be equal to the sum of the circumference A M D P, and that arc which the circle A B C has described during the interval occupied by the point A in leaving A B C, and returning to it again.

2dly, If the motions of the two circles are uniform, the points G, F, E, dividing O I into four equal parts, will determine, by the same method as in *Case* I. the epicycloidal arcs A *e*, *e* Q, Q *g*, *g* B, corresponding with the motion of the point A in the inferiour and superiour parts of its circle; the axis T Q will here also be equal to the diameter A D, and the legs Q *e* A, Q *g* B, will be equal, and similarly posited with respect to the axis.

3dly, The greater the velocity of the circle A B C, the less the arcs A *e*, *g* B, bend towards T Q.

4thly, The describing point does not form an angle in passing from the leg Q *g* B into the leg BK, but makes a curve in B, which is more obtuse as the velocity of the circle A B C is greater; for this velocity makes the point A descend more obliquely along the leg Q *g* B; and when arrived at B, causes it to describe a small space on A C B before it ascends into the leg B K, so that the passage is through a curve *g* B K.

5thly, The point A is always direct when viewed from the point S; and its velocity is accelerated from A to Q, from B to K, &c. but is retarded from Q to B, where it appears to be only equal to the velocity of the circle A B C.

243. Case III. If during the interval between A's leaving the circle A B C and returning to it again, that circle should also turn on its centre, but in an opposite direction; or, in general, if the centre of the circle A M D P advances in the circle O F I, with less velocity than that with which the point A moves in the circle A M D P, then the epicycloid will be *shortened.* This case might be subdivided into several others, but as it is not desirable to render this theory very complex, we shall only consider that in which the retrograde velocity of the circle ABC is supposed less than that of the point A in its circle AMDP. Now on this hypothesis it may be remarked:

1st, If the circle ABC (fig. 8, Pl. III.) had remained immoveable, none of the points of the epicycloid would have become retrograde; but, as the motion of A B C is retrograde, it must augment the velocity of the point A in the inferior semicircle where it is retrograde, and diminish it in the superior semicircle where it is direct. Hence it follows, that the extent of the epicycloid will be contracted in the direction A B C, its base becoming equal to the difference between the circumference of the circle

AMDP, and the arc which the circle A B C has described in the intervening time between A's leaving and returning to that circle.

2dly, The motions of the two circles being uniform, the arcs A *e*, *e* Q, Q *g*, *g* B, answering to the upper and lower parts of the circle AMDP, will be found as in the former cases; here also the axis T Q = A D, and the legs Q *e* A, Q *g* B, are equal, and in similar positions relatively to T Q.

3dly, The arcs A *e*, *g* B, corresponding to the retrograde motion of the point A, are more curved towards the axis T Q, in proportion as the velocity of the circle A B C is greater; and at the point B the arc must turn inwards; because then the point A moves contrary to the tendency towards C by the sum of both motions.

4thly, The passage of the point A through the point B is made by a curve *g* B *m* returning towards itself: for when A has described the leg Q *g* B and arrives at B, the retrograde velocity of the circle A B C causes it to describe a small space before A ascends by the leg B *m* K.

5thly, The point A viewed from S, appears sometimes direct, sometimes retrograde, and sometimes immoveable or *stationary*. For the tangents S *n*, S *m*, S *t*, S *y*, &c. being drawn from the point S, shew that A appears direct in describing the arc *n* Q *t*, all the parts of that arc being passed over by an angular motion of S *n* produced from the position S *n* to S *t*: when the generating point A is in the tangent S *t*, it must appear stationary, since it tends towards neither C nor A during the time of describing the arc of the curve coinciding with the tangent; afterwards while the arc *t* B *m* is described, the generating point seems to have a retrograde motion between the tangents S *t*, S *m*: in the point *m* it again appears stationary, for the same reason as at the point *t*: it seems direct in the arc *m* K *y*, stationary at

y, retrograde in *y* C, and so on. Hence, the point A appears to be twice retrograde, twice stationary, and once direct.

6thly, The velocity of the point A must appear equal to nothing in the stationary points; then it appears accelerated until A arrives at the vertex of the curve, where the velocity seems greatest; afterwards it is retarded from the vertex to the next stationary point: the retrograde velocity increases from *t* to B, then diminishes from B to *m*, the following stationary point.

7thly, The greater the velocity of the circle A B C, the greater is the arc of retrogradation, *t* B *m*.—From all this it appears that, in these kinds of motions, the nature of the epicycloid generally depends upon the relation of its base A B to the circumference of the circle in which is the generating point.

244. Scholium. If in the second and third cases, the motions of the circles had not been uniform, or had not accelerated and retarded in a constant ratio to each other, the epicycloid would have been less regular, and the axis T Q not in the middle between the legs Q *e* A, Q *g* B; nor would the legs be equal, or in like positions; for one would be longer than the other, according as the combination of the velocities urged more on one side than on the other. But still the epicycloid would retain nearly the same general figure; that is, it would always have a curvature towards B, disposed as pointed out in the respective cases.

245. Theorem II. *If an object and the eye turn in the same direction with uniform angular velocities* V, u, *in two concentric circles whose radii are* R, r, *the eye's imaginary place being at the centre; the optic orbit of the object is a curve compounded of a number of epicycloids, equal to the number of times the eye is found in the plane of comparison* (Art. 224.) *on the same side; and if the eye run*

through the inner circle, these epicycloids are simple, lengthened, or shortened, according as V R — V r *is equal, greater, or less than* u r — V r; *but if the eye run through the outer circle, the nature of these epicycloids will be ascertained by the relation of* u r′ — u R *to* V R — u R.

Demon. When the object is immoveable, and the eye revolves in a circle, the observer, who thinks himself immoveable at the centre of his orbit, never sees the object in its true place, but it appears to him as describing a circle about its true place, equal to the circle described by the eye, and with equal velocity (Art. 230.) Therefore if the object and the eye both move at the same time in concentric circles, the motion of the object will appear, with respect to the eye's imaginary place, as compounded of a motion revolving in an epicycle (Art. 228.) about the object's true place, and of a real motion of the centre of that epicycle in the object's orbit. The object is, therefore, relatively to the eye's imaginary place, in a similar case to that of the point A (figs. 5, 7, 8, Pl. III.) with respect to the point S. The optic path of the object must then be a series of epicycloids equal to the number of times the eye returns to the same side in the plane of comparison (for by these returns only the object appears to have made a revolution in its epicycle, with regard to the point S); the nature of these epicycloids depends on the relation between the circumference of the epicycle, or of the eye's orbit, and the magnitude of the arc A B, similar to the arc O I, which the object describes during the interval of the eye's quitting and returning to the plane of comparison.

246. I. To find the expressions for this relation, it must be observed that, as the eye and the object move in the same direction, the eye advances towards the plane of comparison only by the excess of the eye's velocity over that of the object. Therefore, as

the excess, $u-V$, of the eye's angular velocity above that of the object for any time, is to V, the object's velocity during the same time; so is 360°, the sum of the excesses of the angular velocities gained by the eye upon the object in returning to the plane of comparison, to the sum of the angular velocities during the interval of that return: this produces $\frac{360° \times V}{u-V}$ for the expression of the angle OSI or ASB (figs. 5, 7, 8, Pl. III.) equal to that sum. Now the radius of the arc AB is $SO - AO = R - r$; and as the arc varies in proportion to the product of the angle into the radius, therefore the arc $AB = \frac{360° \times V}{u-V} \times \overline{R-r}$. The epicycle APDM is by the same reason $360° \times r$. Consequently the arc AB, is to the epicycle APDM, as $\frac{360° \times V}{u-V} \times \overline{R-r}$, to $360° \times r$; or, as $\frac{V}{u-V} \times \overline{R-r}$ to r; or, as $VR - Vr$ to $ur - Vr$; or lastly, as $\frac{R-r}{r}$ to $\frac{u-V}{V}$.

247. II. When the orbit MDP of the object (fig. 6, Plate III.) is included in that of the eye; as when the eye really describes the arc *ab* in the time it returns to the plane of comparison on the same side; then the centre S of the object's orbit will seem (Art. 231.) to describe the equal arc OI; and since the object makes at the same time a revolution in its circle MDP, therefore to an eye situate in the imaginary place S, the object appears to move in a circle equal to MDP, while the centre of that circle describes the arc OI. Hence the optic orbit of the object must, in this case also, be a curve AQBKC composed of epicycloids, whose nature depends upon the relation between the arc AB, and the circumference of the circle MDP. Now the arc AB is the sum of the angular velocities of the eye; the

radius of that arc is $r - R$; and MDP is manifestly represented by $360° \times R$; the arc AB $= \frac{360° \times u}{V - u} \times \overline{r - R}$. Consequently, the nature of these epicycloids depends upon the relation of $ur - uR$ to $VR - uR$; or of $\frac{r - R}{R}$ to $\frac{V - u}{u}$.

248. Corol. I. Whether the velocities of the eye and the object be uniform or not, the object's apparent place is always at the vertex of the epicycloid, if the eye's true place is in the plane of comparison, and its imaginary place between the object and the eye; likewise, the object's apparent place is always at the bottom of the epicycloid, when the eye's true place being in the plane of comparison, is between the object and the eye's imaginary place, or the object between the eye's true and imaginary place. For the vertex of the epicycloid is the farthest point from the centre S, and its base the nearest to it. Now it is obvious that when the eye is in the plane of comparison and beyond the eye's imaginary place relatively to the object, the eye is then at its greatest distance from the object: but when the eye is on the nearer side, or the object is between the eye and its imaginary place S, then the eye is nearest to the object. Therefore, in the first case the object's apparent place is at its greatest distance from the eye's imaginary place; and in the two other cases it is nearest.

249. Corol. 2. The projections of epicycloids in the concavity of the heavens, must be a kind of elliptic epicycloids, whose axes TQ are to one another as the sines of the celestial arcs measuring the distance from the plane of the eye to that of the object's epicycle (Art. 235.), and whose vertices Q, K, must appear as the points nearest to the eye's plane; also the extremities A, B, C, must appear as the points most distant from the eye's orbit. (Art. 236.)

250. In applying the preceding theory to the phenomena arising from the earth's annual motion, it must be observed, 1. That what has been called *the plane of the eye's orbit*, is the plane of the ecliptic. 2. That the *eye's orbit* is the ellipsis described annually by the earth about the sun, which ellipsis differs but little from a circle (Art. 311.). 3. That as the sun apparently describes this ellipsis about the earth, and the earth appears to be in the centre of the celestial motions, though it be quite the contrary, what has been called the *eye's imaginary place*, is the true place of the sun's centre. 4. That the *plane of comparison* is the plane of a great circle perpendicular to the ecliptic, passing through both the sun and the heavenly body observed. Therefore (Art. 46.) when the eye is placed in the plane of a circle of latitude, if it sees the sun and a star or planet on the same side, the star or planet appears to have the same longitude with the sun, and is said to be in *conjunction with the sun*, and this may happen in two cases when the earth's orbit includes that of the star or planet; but in only one case when the star or planet's orbit includes that of the earth. For when the earth's orbit includes that of a planet, it may be seen on the same circle of latitude with the sun, but beyond him, which is called the *superiour conjunction;* or it may be seen between the earth and sun, which is called the *inferiour conjunction.* But when the eye is in the plane of comparison between the star and the sun, and they appear to be distant from each other 180 degrees of longitude, the star is then said to be in *opposition to the sun.* The general term *Syzygies* refers both to conjunction and opposition. When the eye is so situated that the arc of the ecliptic comprehended between the sun and the plane of a star's circle of latitude, is 90°, such star is then said to be *in quadrature with the sun.* Leaving what we have to say respecting the apparent mo-

tion of the fixed stars, to be given under the head of *Aberration* in a subsequent chapter (Chap. XXII.), we will in a few words apply this theory to the relative motions of the planets.

251. The planets having a proper motion in orbits nearly circular, whose planes (as will soon be shewn) are inclined in very small angles to that of the ecliptic, they must appear to describe in the heavens very flat elliptic epicycloids. But the earth's orbit being included (Art. 205. 360.) within the orbits of ♂, ♃, ♄, and ♅, while it includes those of ♀ and ☿; it becomes necessary to distinguish these planets into two kinds, relatively to the earth. The former four, ♂, ♃, ♄, and ♅, are called *superiour* or *exteriour planets;* the latter, namely, ♀ and ☿, *inferiour* or *interiour planets:* of these the terms *superiour* and *inferiour* are in most general use.

252. To know what kind of epicycloid each of these planets describes, take for the *superiour* the expression of the proportion of $RV - rV$ to $ru - rV$; and for the *inferiour* that of $ru - Ru$ to $RV - Ru$, in which V must express the planet's mean angular velocity for any given length of time, R the radius of its orbit; u the earth's angular velocity for the same time, r the radius of its orbit; or R and r may be any two numbers bearing the same ratio to each other as the semidiameter of the planet's orbit to that of the earth's orbit. The mean angular velocities for a day may easily be found, when the times of the periodical revolutions are known, for they will be expressed by the quotients of 360° by the days in which the revolutions are performed. The student may, therefore, after the mean distances and periodical times of the planets are determined, find the relation of $RV - rV$ to $ru - rV$, or of $ru - Ru$ to $RV - Ru$, and he will perceive that in every case the antecedent will be much less than the consequent. But without

entering into the minutiæ of these calculations, since they would not only be of no utility, but could not possibly be inserted here without assuming too much that does not yet appear, we shall proceed to deduce from this theory a few inferences.

253. I. Since, with respect to all the planets, R V — *r* V is less than *r u* — *r* V, or *r u* — R *u* less than R V — R *u*, it follows (Art. 245.) that *the relative motions of all of them are in shortened epicycloids.*

254. II. *All the superiour planets are direct at the time of their conjunction with the sun* (Art. 243.), *retrograde at that of their opposition, and stationary some time before and after their opposition.* And as the inferiour planets cannot be in opposition to the sun (because the earth cannot pass between them and the sun), but must have two conjunctions (Art. 250.), it follows that *the inferiour planets are direct in their superiour conjunctions, retrograde in their inferiour conjunctions, and stationary some time before and after.*

255. III. *The apparent velocities of the planets, whether direct or retrograde, are accelerated from one of the stationary points to the midway between that and the following stationary point, from thence they are retarded until the next station* (Art. 243.); *their greatest direct velocity is in their conjunctions, and their greatest retrograde velocity is in the opposition of the superiour planets, and in the lower conjunction of the inferiour planets.*

256. IV. Since (Art. 254.) the inferiour planets are always direct in their superiour conjunctions, and retrograde in the inferiour, *they appear least when their motion is direct, and greatest when the motion is retrograde.*

257. V. *The shorter the periodic time of an inferiour planet, the more frequent are its stations*

and retrogradations (Art. 245.), *the shorter time they continue, and the less they are in quantity.*

258. VI. *The longer the periodic time of a superiour planet, the more frequent are its stations and retrogradations; but they are less, yet continue a longer time.*

259. VII. *The inferiour planets never go farther from the sun than such an angle as measures the apparent semidiameters of their orbits as seen from the earth.* This agrees with what was observed (Art. 199. 200.) respecting *venus* and *mercury*; the former of which never comes to the meridian more than $3^h\ 10^m$, or 47° sooner or later than the sun, and the latter never more than $1^h\ 50^m$, or $27\frac{1}{2}^\circ$, earlier or later. Hence, when the semidiameter of the earth's orbit becomes known, we may by means of this angle under which the semidiameter of an inferiour planet's orbit appears (called its *greatest elongation*) approximate to the real semidiameter of its orbit. But this is merely mentioned here.

260. VIII. *When the planets are in their syzygies, their longitude seen from the earth is the same as their longitude seen from the sun; except in the lower conjunction of an inferiour planet, when its longitude seen from the earth differs* 180° *from its longitude as seen from the sun.* For then the sun, earth, and planet, are all in the plane of the same circle of latitude (Art. 228.). But the planets moving in orbits whose planes are somewhat inclined to another, and consequently always have some latitude, except when they are in the intersection of the plane of their orbit with that of the ecliptic; in the syzygies *the parallax of the annual orbit* (that is, the difference between a star's true heliocentric place and its apparent geocentric place) *is very sensible in latitude; and out of the syzygies, the parallax is sensible both in longitude and latitude.*

261. In the preceding articles the motions of the

planets are supposed uniform, and their orbits to be circles, with the sun and the eye's imaginary place at the centre. From the near correspondence, therefore, of the last eight articles, with what is actually observed to take place with respect to the planets (Art. 199. &c.), we may safely conclude—In the first place, *that the orbits of the planets are nearly circular*;—and in the next, that *the orbits of mercury and venus are nearer the sun than the orbit of the earth is, that is, that they are inferiour planets; and that all the rest are superiour planets.* A careful comparison of the stations, retrogradations, and direct motions, of the planets, with the appearances which ought to be observed as resulting from the foregoing theory, furnishes us, therefore, with an additional argument in favour of the *Copernican system*: and on this account the theory has been given here; otherwise it would have been deferred until the real distances and motions of the planets were established from observation *.

* Those who are desirous of seeing the tracks which the planets describe for a considerable time, relatively to the earth, are referred to *Ferguson's Astronomy*, *Long's Astronomy*, and a paper of *Cassini's* in *Mémoires d'Acad. Royale, ann.* 1709.

CHAPTER X.

On the Law by which the Planets are retained in their Orbits.

ART. 262. PREVIOUS to determining the elements necessary for ascertaining the distances, revolutions, magnitudes, and rotations, of the planets, it may be proper to employ a few pages in shewing the conformity of the opinion that the orbits of the planets are elliptical, with the present received principles of mechanical philosophy; or in proving that the motions in such orbits correspond with the known effects of the law of gravity. In order to this, we shall just recapitulate a few of such principles and theorems (demonstrated or explained in every scientific treatise on mechanics) as are more peculiarly applicable to the object of our present enquiry.

263. Thus, we admit the *laws of motion,* and the doctrine of the *composition* and *resolution* of forces, as now taught: we call that a *uniform motion* which a body receives by an instantaneous action of any power; or rather that by which a body describes equal spaces in equal times: and that, a *motion uniformly accelerated,* which is produced in a body by a continual repetition of equal actions, uninterruptedly and equally augmenting its velocity at each instant.

264. We take it as proved, that in a motion *uniformly accelerated, the velocities are as the times.* Also,

265. That, the space s, run through in a certain finite time t, by a motion uniformly accelerated, is

equal to *half* the space run through in the same time by a uniform motion with a velocity u, equal to that acquired by the acceleration at the end of the time t; that is, in a motion uniformly accelerated, $s = \frac{tu}{2}$, whereas in a uniform motion $s = tu$.

266. That, the spaces described by a motion uniformly accelerated, are as *the squares of the times*, reckoned from the beginning, or as *the squares of the velocities* acquired at the end of the times.

267. That, a body urged by two forces at once, *continues in the plane of the directions of those forces.*

268. That, a body urged at the same time by two powers inclined to one another, describes *a right line if the powers are of the same nature*; that is, both uniform, or both variable, according to one and the same law. But if these powers are of *a different nature, they cause the body to describe a curve, or trajectory*, whose species depends upon the relation the two powers have to each other at each instant.

269. That, when a body is urged by two forces, *one uniform*, and consequently tending to make it move uniformly in the direction it has at the end of each instant, and the other either *constant or variable, but always directed to one and the same fixed point*, towards which the body is continually driven; this body describes *a curve always concave towards the fixed point.*

270. That, a trajectory described by virtue of *two forces, one uniform, the other central*, will have no point of contrary flexure; but, *cæteris paribus*, its curvature *is greater* in proportion *as the central force exceeds the uniform*; and reciprocally.

271. That, if two right lines are drawn to the extremities of any arc of a trajectory, from a central point, *the area of the included sector is always proportional to the time of running through the arc.*

This is analogous to Kepler's law (Art. 209.), *that a line drawn from the centre of a planet to the sun, would, when carried along by the periodical motion of the planet, describe equal areas in equal times.*

272. That, the velocity of a body in any point of its trajectory *is reciprocally as a line drawn from the central point, perpendicular to a tangent to the trajectory* at the point where the body is.

273. That, the true angular velocity of a body, describing in a very small time any small arc of its trajectory, is always *reciprocally as the square of the distance from the point in which the central force resides, to the point where the body is found at the middle of the time.*

274. These premises being considered as proved, we proceed to the motions of the planets, which, as they are not uniform, must be affected by a variable central force: the nature of the law causing this variation we shall now attempt to determine.

275. Theorem I. *Any variable central force, is a constant accelerative force during a very short time.* This will be demonstrated by shewing that any variable central force causes the body it acts upon to describe spaces towards the centre, which are to one another as the squares of the number of instants contained in a very short space of time (Art. 266.). Let P, Q, p (fig. 9, Pl. III.), be three points of an indefinitely small arc of any trajectory, A P D. This arc may be considered as uniformly described in the exceedingly small time t, and therefore the spaces P Q, P p, are to each other as the parts of that time t, estimated from the instant the body was in P. Let P K be a tangent to that arc, and consequently the path that body would describe, if, when it was in P, the central force should cease to act, and the uniform force only remain. From the centre of force S, draw through the three points P, Q, p, the rays S P, S R, S F, prolonged till they meet the tangent.

Then it is manifest, 1st, that the small excesses Q R, p F, represent the effects of the central force at the points Q, p, because they are the spaces through which the central force has drawn the body from its rectilinear path P K, towards the centre S. 2dly. The rays S P, S R, S F, being infinitely near, and the point S at an immense distance, they may be taken as right lines parallel to each other; the same may be said of the excesses Q R, p F. Through the points P, Q, p, describe the circumference of a circle P B N, with which the arc P Q p of the trajectory A P D will coincide: the diameter P N, drawn from the point P, will be perpendicular to P K, a tangent common to both the circle and the trajectory. Then through the points Q p, draw Q E, p H, perpendicular to the diameter P N; and join Q N, p N. Now the triangles P Q N, P p N, P Q E, P p H, are both rectilinear and right-angled, since P Q, P p, may safely be taken as right lines. Therefore, E P, P Q, P N, are continually proportional, as are also P H, P p, P N; consequently $PQ^2 = EP \times PN$, and $Pp^2 = PH \times PN$. Hence $PQ^2 : Pp^2 :: EP \times PN : PH \times PN :: EP$ or $QI : PH$ or pi. But because of the parallels Q I, $p\,i$, and Q R, F p, the triangles Q R I, p F i, are similar; therefore $QI : pi :: QR : pF :: PQ^2 : Pp^2$. If then a body describe any trajectory A P D, with a uniform force, and a central force combined, the effects of that central force are in any very small arcs, as the squares of the times required to run over those arcs.

276. COROL. I. We may therefore apply the formulæ commonly applied to uniformly accelerated forces, to central forces: thus, let f be the central force, s the space run through in a very short time t, then $f = \frac{s}{t}$, $s = f\,t\,t$, $t = \sqrt{\frac{s}{f}}$; and from a comparison of these formulæ, others might be deduced. But in the present case, p F expresses the

space whereby the central force has drawn the body towards the centre S, while the body is describing the arc P p; therefore the first formula may be thus expressed, $f = \frac{Fp}{t^2}$.*

277. Corol. II. If the times t are equal, $f = p F$, that is, *in indefinitely small equal times the central forces are to each other as the right lines* p F, *drawn from one of the extremities* p *of each arc run over, parallel to the ray* S P, *passing through the other extremity* P, *and terminated by the tangent to the trajectory in the same point* P.

278. Corol. III. Then, since the areas comprehended between the rays (Art. 271.) represent those times, the indefinitely small time t, wherein the body runs through the arc P Q p, may be expressed by the area of the triangle S P p; hence $t = \frac{1}{2}$ S P $\times$ p M, and $t^2 = \frac{1}{4}$ S P^2 $\times$ p M^2. The formula $f = \frac{Fp}{t^2}$, therefore, becomes $f = \frac{Fp}{\frac{1}{4} SP^2 \times pM^2}$; or, since the constant coefficient $\frac{1}{4}$ may be neglected without changing the proportion, $f = \frac{Fp}{SP^2 \times pM^2}$.

279. Lemma I. If from the extremity P (fig. 12, Pl. III.) of an ellipsis, P D be drawn perpendicular to the conjugate diameter N n, then P D : C H : : C B : C N. Or, the rectangle B C H L, under the two semi-axes C B, C H, is equal to the parallelogram C N R P, formed by any two conjugate semidiameters, C N, C P; for then C H $\times$ C B = C N $\times$ P D, and consequently P D : C H : : C B : C N. It is shewn by the writers on conics (See *Hutton's Conics*, p. 35; *Vince's Conics*, p. 25; *Hamilton's Conics*, &c.), that all the parallelograms circumscribed

* In this and some of the following articles, the sign = does not always denote *equality*, but often *proportionality*, for which the sign $\propto$ is sometimes used.

about an ellipsis are equal to one another, and each equal to the rectangle of the two axes. But, if the parallelograms of which P C R N, B C H L, are the fourth parts, are equal, those fourth parts are equal also; whence the truth of the lemma is manifest. Or the equality of the parallelograms B C H L, P C N R, is directly proved, both with respect to the ellipsis and hyperbola, at p. 74, *Newton's Conics*, and the same analogy deduced as given in the enunciation of the lemma.

280. Lemma II. In an ellipsis where A H is the greater axis, and B L the less (fig. 10, Pl. III.), if from the extremity, P, of any diameter, P K, be drawn a right line P S to either focus, as to S, that line will be intersected in E, by the diameter N *n* conjugate to P K, so that P E = C A or C H. From the other focus Q, draw Q P, and Q G parallel to N *n*, or to the tangent T V; then the triangle P Q G is isosceles; for the angles P Q G, P G Q, are equal to the angles V P Q, T P G, by reason of the parallels T V, G Q; and the latter two angles are equal by conics: (see *Hutton*, p. 25; *R. Simson*, p. 92): hence P Q = P G. Also, since the triangles Q G S, C E S, are similar, and C S = C Q, S and Q being the foci, therefore E S = G E. Hence P G + G E = P Q + E S = P E; and P E + E S + P Q, being equal to A H (*Hutton*, p. 18), also equal to 2 P E; consequently P E, or P O, = $\frac{1}{2}$ A H = A C. This also is deduced generally, both as it regards the ellipsis and the hyperbola, in a *Corol.* to *Prop.* 47. *Newton's Conics.*

281. Theorem II. *The central force urging a planet*, P, *and causing it to describe an ellipsis, in whose focus*, S, *that central force is placed, varies in a reciprocal ratio of the square of the ray* S P; *that is, inversely as the square of the planet's distance from that focus.*

Demon. Through P, the planet's place, draw the

tangent P T (fig. 10, Pl. III.). Suppose P p an indefinitely small arc described by the planet; draw the lines P S, p S, and p F, which, for a reason assigned before (Art. 275.), we imagine parallel to S P meeting the tangent; then the line p F expresses the planet's central force (Art. 277.) while running through the arc p P, and $p \, F = \frac{1}{SP^2}$. Put R to denote the latus-rectum or parameter of the greater axis A H; then, by conics, $R = \frac{4\,CB^2}{2\,AC} = \frac{2\,CB^2}{AC}$. Therefore $R \times AC = 2\,CB^2$. From p draw pI, an ordinate to the diameter P K (or parallel to the tangent T V), meeting S P in i. The triangles P I i, P C E, being similar, and P i equal and parallel to F p, we have P i or F p : I P : : P E or A C (Art. 280.) : P C. Consequently

(a) $Fp \times R : IP \times R :: AC : PC$; and, for the same reason,

(b) $IP \times R : IP \times KI :: R : KI$; also, by the property of the ellipsis,

(c) $IP \times KI : Ip^2 :: PC^2 : CN^2$. From the point p draw pM perpendicular to S P; then the right-angled triangles $p\,i$ M, P E D, are similar, because the parallels $p\,i$, E D, make the angles $p\,i$ M, P E D, equal; therefore $i\,p$ (or because of the proximity of the points I, i), $Ip : pM :: PE : PD$. But (Art. 279.) CA or $PE : PD :: CN : CB$; consequently $Ip : pM :: CN : CB$; and therefore,

(d) $Ip^2 : pM^2 :: CN^2 : CB^2$.

Multiply together the corresponding terms of the four proportions, marked a, b, c, d, the products give $Fp \times R^2 \times IP \times IP \times KI \times Ip^2 : IP \times R \times IP \times KI \times Ip^2 \times pM^2 :: CA \times R \times PC^2 \times CN^2 : PC \times KI \times CN^2 \times CB^2$. Now the first ratio divided by $IP \times R \times IP \times KI \times Ip^2$, and the second by $PC \times CN^2$, gives $Fp \times R : pM^2 :: CA \times R \times PC : KI \times CB^2$. Or, since

$AC \times R = 2CB^2$, $Fp \times R : pM^2 :: 2CB^2 \times PC : KI \times CB^2$; and, dividing the second ratio by CB^2, we get $Fp \times R : pM^2 :: 2PC : KI$. The distance PI being indefinitely small, $KI = KP = 2PC$; therefore $FP \times R = pM^2$; for if the second ratio in the proportion be a ratio of equality, the first will be so too. This last equation multiplied by $\frac{1}{SP^2 \times pM^2}$, gives $\frac{Fp \times R}{SP^2 \times pM^2} = \frac{1}{SP^2}$; or, because of the constant quantity R, $\frac{Fp}{SP^2 \times pM^2} = \frac{1}{SP^2}$. But (Art. 278.) $\frac{Fp}{SP^2 \times pM^2}$ is a general expression of the central force in any trajectory, therefore in the ellipsis the central force is always as $\frac{1}{SP^2}$.

282. Corol. The force which retains the planets in elliptical orbits, produces similar effects with the power of gravity near the earth's surface, and therefore (Art. 216.) is of the same nature.

283. Theorem III. *If several planets move each in a separate ellipsis, by virtue of a central force that is always reciprocally as the square of each planet's distance from a focus common to all those ellipses, and wherein that central force is placed; then,*

I. *The areas of the sectors described in the same time, are to one another as the square roots of the parameters of the greater axes of the ellipses.*

II. *The velocity of each body in its ellipsis is as the square root of the parameter of the greater axis, divided by a line drawn from the focus perpendicular to a tangent passing through the place of the body.*

III. *The area of each ellipsis is in a ratio compounded of the square-root of the parameter of the greater axis, and of the time of the planet's revolution.*

Demon. I. In every ellipsis $Fp \times R = pM^2$ (Art. 281.): but by the hypothesis $Fp = \frac{1}{SP^2}$;

therefore, $R = pM^2 \times SP^2$, and $\sqrt{R} = pM \times SP$. But $pM \times SP$ is as the area of the triangle or sector SPp; consequently the area of any sector is as $\sqrt{R}$.

II. The velocity in an indefinitely small time is as the arc described, pP: now the similar right-angled triangles SPT, pMP, give $ST : SP :: pM : pP$; therefore $Pp = \frac{SP \times pM}{ST}$. But (pr. *Dem.* I.) $\sqrt{R} = pM \times SP$; consequently pP, or the velocity, is as $\frac{\sqrt{R}}{ST}$.

III. The greater the area of the ellipsis, and the less the portion of it described by the planet in a given time, the longer also is the time t of the periodic revolution. Therefore the time of the revolution is as the area (a) of the ellipsis, divided by the area (s) of a sector described in a given time, or $t = \frac{a}{s}$. But $s = \sqrt{R}$, as just shewn; therefore $t = \frac{a}{\sqrt{R}}$, and $a = t\sqrt{R}$.

284. From the preceding theorems we readily deduce the second of Kepler's famous analogies (Art. 209.), *that the square of the time of each planet's revolution is proportional to the cube of the principal axis* (d) *of its ellipsis.* For if c be the less axis, and R the parameter of the greater axis, then (by conics) $dR = c^2$, and $d^3 R = c^2 d^2$, by multiplying each side of the equation into d^2. But the whole area (a) of the ellipsis is as the product of the axes, or $a = cd = t\sqrt{R}$ (Art. 283. III.); consequently $c^2 d^2 = t^2 R$: and as $c^2 d^2 = d^3 R$, therefore $d^3 R = t^2 R$, and d^3 is proportional to t^2.

285. Corol. I. If the ellipsis change into a circle, *the time of a revolution will be as the square root of the cube of the diameter, or of the radius.*

286. Corol. II. Hence it follows, that, *the times of the revolutions of the planets being known, the*

relations of the principal axes of the ellipses they describe may be found. Consequently, if the relative dimensions of each planetary orbit be determined by observation, and the chief axis of any one of them be ascertained, the respective major axes of the others may be found by the above analogy: and indeed they are generally determined in this manner; though other methods are in some cases had recourse to, by which the accuracy of this simple method is clearly proved.

287. When the mutual actions of the sun and primary planets upon each other are taken into consideration, the process is thereby rendered more tedious; but it would then appear that the orbit of each planet is nearly *an ellipsis*, whose focus is in the common centre of gravity of the sun and all the planets inferiour to it; and this centre of gravity (Art. 219.) is always either in the body of the sun, or very near it.

288. The celebrated M. *John Dominic Cassini*, in *a Treatise on the Origin and Progress of Astronomy*, proposed a curve for the orbit of a planet, somewhat different from the common ellipsis: in the common ellipsis, the *sum* of two lines drawn from the foci to any point of the curve is equal to the *transverse diameter*; but in the Cassinean ellipsis, the *product* of two lines from the foci to the curve is equal to a *constant* or *given number*. In this figure, if the less axis exceeds the distance of the foci, the curve is every-where concave towards the centre: if, while the principal axis remains the same, the distance of the foci is lessened, the minor axis will be increased; and when the foci meet in the centre, the ellipsis will become a circle: but if, on the contrary, the distance of the foci be increased, the less axis will be lessened, and the curve will at length have a point of contrary flexure, and will, at the ends of the minor axis, be convex towards the

centre; and when the distance of the foci is so far increased, as to be in the same proportion to the major axis as the side of a square to the diagonal, that is, as 1 to $\sqrt{2}$, the less axis will become nothing, and the curve extend to the centre on each side: if the distance of the foci be greater in proportion to the major axis, than in the above ratio, the figure turns into two conjugate ones *at a distance from each other;* and as the distance of the foci is farther increased, the two conjugate figures will at last become merely *points.* It must be evident, after thus tracing a few of the properties of this curve, that it cannot possibly be the orbit of a planet: for it is certain, that in all those cases where it passes into two conjugate figures, it deviates from what is essential to the nature of an orbit, namely, *continuity;* and in all those cases where it is, at the end of the minor axis, convex towards the centre, the planet would need a centrifugal force to describe such parts of its orbit; that is, it would require at *equal distances* from the sun sometimes a *centrifugal,* and sometimes a *centripetal* force to retain it in its orbit, which is totally incompatible with all the laws of nature. It may fairly be concluded, that when all the species of a figure beyond a certain limit are unfit for discharging any office of nature, the remaining species on the other side of the limit should be rejected also: and when, in addition to this, it is considered that the celestial observations are not consistent with this curve, it can by no means be admitted into astronomy*.

* A very beautiful elementary treatise of Physical Astronomy may be seen in the learned Dr. *Stewart's Tracts, Physical and Mathematical.* The doctrine of centripetal forces is there laid down in a series of twenty-eight propositions, demonstrated (if the quadrature of curves be admitted) with the utmost rigour, and requiring no previous knowledge of any part of the mathematics, besides the elements of plane geometry, and of conic sections. But the most comprehensive, elaborate, and profound work, on Physical Astronomy, *in all its branches,* is that published by M. *La Place,* under the title of *Traité de Mecánique Céleste.*

CHAPTER XI.

On the Magnitude and Situation of the Earth's Orbit.

ART. 289. IT is related of the ancient philosopher *Archimedes*, that when *Hiero* was admiring the skill and judgment displayed in his machines, he replied to the monarch, "*These are nothing; but* "*give me another place to stand upon, and I will* "*move the earth*:" with much more truth it may be said by a modern astronomer, '*Give me a proper* '*basis for my geometrical operations, and I will* '*scale the heavens.*' This basis Nature has furnished us with, in the elliptical orbit of the earth; the dimensions of which we, therefore, now proceed to determine, as a necessary preliminary in ascertaining the magnitudes and distances of the planets, and the dimensions of their orbits. But we must first premise a few definitions of terms used in this part of astronomy.

290. The path in which the sun appears to move, is in the same plane as that in which the earth really moves; therefore (Art. 40.) the earth's orbit is in the plane of *the ecliptic*: the points in which the orbits of the planets cut the orbit of the earth, or the ecliptic, are called the *nodes*; as are also the points where the orbits of the secondary planets, or satellites, intersect their respective primaries. That node is called *ascending* where the planet passes from the south to

the north side of the ecliptic, and is marked thus ☊: the other node is called *descending*, and is denoted by this mark ☋. An imaginary line drawn from one node to the other is called *the line of the nodes.*

291. That angle which the plane of a planet's orbit makes with the plane of the earth's orbit, is called the *inclination* of that planet's orbit.

292. If a perpendicular be let fall from a planet to the ecliptic, the angle at the sun between two lines, one drawn from it to that point where the perpendicular falls, and another to the earth, is called the *angle of commutation.*

293. The *curtate distance* of a planet from the sun or the earth, is the distance from the sun or the earth to the planet, reduced to the ecliptic; or the line between the sun or the earth, and that point where the perpendicular let fall from the planet meets the ecliptic.

294. The *mean distance* of a planet is the line drawn from that focus of its elliptical orbit in which the sun is placed, to either end of the conjugate axis of its orbit: it is manifest from the nature of the ellipsis that this line is equal to half the transverse axis. Some authors call a mean proportional between the two axes, the *mean distance.*

295. The distance from the centre of the orbit to either focus is called its *excentricity.*

296. The *apsides*, or *apses*, are the two points in the orbit of a planet, where it is at its greatest and least distances from the sun. The point at the greatest distance is called the *higher apsis*, that at the least distance, the *lower apsis.* The higher apsis is also called the *aphelion*; the lower, the *perihelion.* The diameter which joins these two points is called the *line of the apsides*, and is supposed to pass through the centre of the sun. But it is not true that the apsides are always in the same straight line passing through the sun; for they are sometimes out of a

right line, making an angle greater or less than 180°; and the difference from 180° is called the *motion of the line of the apsides.* When the angle is less than 180°, the motion is said to be *in antecedentia*, or contrary to the order of the signs; when it is greater than 180°, the motion is *in consequentia*, or according to the order of the signs.

297. The sun and moon are in *perigee*, when at their nearest distance from the earth; and in *apogee*, when their distance from the earth is greatest.

298. The *argument of latitude* is the angle at the sun between the planet and its ascending node, estimated in the planet's orbit.

299. The *true anomaly*, or *equated anomaly*, as it is sometimes called, is the angle at the sun which is formed by the *radius vector*, or line drawn from the sun to the planet, and the line drawn from the sun to the aphelion of the planet: the *mean anomaly* is the angular distance of a planet from its aphelion (taken at the same time with the true anomaly), supposing it to move uniformly with its mean angular velocity. The difference between the true and mean anomaly is called the *equation of the centre*, or the *prosthapheresis.*

300. If a circle be supposed drawn on the line of the apsides as a diameter, and through the place of the planet a perpendicular to the line of the apsides be drawn till it meet the circumference of the circle; then the angle formed by two lines, one drawn from the centre of the planet's orbit to the aphelion, and the other to the point where the perpendicular through the planet's place intersects the circumference of the circle, is called the *excentric anomaly*, or the *anomaly of the centre.*

301. Prob. I. *To find the time in which the earth performs a revolution about the sun.*

A solution to this problem may be found in Chap. III. where we determined the length of the sidereal

year to be $365^d\ 6^h\ 9^m\ 12^s$ (Art. 56.). For the sun passes through the twelve signs of the ecliptic in the space of a solar or tropical year (Art. 55.): but the equinox or point of the ecliptic from which the sun appeared to depart, advances as it were to meet the sun, over an arc equal to the annual precession of the equinoxes; therefore the time required for the earth (or sun, apparently) to pass over this arc, must be added to the solar year to give the earth's complete periodic time, which is consequently equal to the sidereal year.

302. Prob. II. *To determine the time of either solstice.*

In fig. 11, Pl. III. imagine T R to represent the tropic, E C the ecliptic, touching it in the solstitial point S; then the lines dl, dl, &c. perpendicular to T R, will represent the changes of declinations when the sun appears in the points l, l. And since the part of the curve E S C is very small, the right lines dl, dl, are as the squares of the arcs S l, S l; or, since S l, S l, are exceedingly nearly equal to S d, S d, dl, dl, will be as the squares of S d, S d. But this is the property of the parabola; therefore a small portion of the ecliptic near the solstice, when considered relatively to the tropic, may be looked upon as part of a parabola. We may safely suppose this to be the case for four or five days before, and as many after, the solstice: then the abscissæ S P, S P, are what the declination of the sun is less than the greatest declination; and consequently the differences P P are as the differences of the declinations; that is, as the intervals of the altitudes of the sun above the horizon (or of their zenith distances), in the meridian or any other vertical circle: and the ordinates P l, P l, are respectively as the times wherein the sun acquires those declinations; and the differences of the one, are as the differences of the other.

These things being admitted, let a gnomon, A B (fig. 13, Pl. III.), be erected, whose shadow caused by the sun, when near either solstice, marks out the point H on a plane C D, to which the line B H, drawn from the end of the shadow to the vertex of the gnomon, is nearly perpendicular. Three or four days after observe the point F, where the extremity of the shadow of the gnomon falls when the sun is on the same vertical circle. Repeat this again after two or three days more, and let the extremity of the shadow mark out the point G. The lines H F, G F, are proportional to the distances of the sun in the same vertical circle from the horizon (that is, by what has been above said, to the change of declination); for, because of the minuteness of the angles F B G, H B G, and the distance from the top of the gnomon to the plane, the line F G H will not differ sensibly from the arc of a circle described from the centre B, with either the radius B G, B F, or B H, as these radii will be very nearly equal. And if, instead of the plane C D, the shadows should be taken upon the plane C E, inclined to C D by an angle D C E not very large, marking the extremities in the points *f*, *g*, *h*; then the right lines *h f*, *g f*, will have nearly the same ratio as H F, G F; for the lines *f* F, *g* G, *h* H, meeting at the point B, at a considerable distance with respect to *f g*, or *g h*, may be considered as very nearly parallel. The problem, therefore, for finding the solstice from the points F, G, H (or *f*, *g*, *h*), given, together with the moments of time in which the sun casts the shadow of the point B to them, is now reduced to this: *given, in the parabola* K V M, *the parts of the axis* F G, G H (fig. 14, Pl. I.), *and the distances* A T, T E, *of three lines* A B, T Y, E S, *each parallel to the axis* C D; *to find the distance of any one of them, for instance* T Y, *from that axis.* Here we suppose F G, and G H, in the parabola, to be respectively equal to F G and G H, as marked by

the extremities of the shadow; and A T and T E to represent the respective intervals between the observations of the points H and F, and between those of F and G. Then K H, L F, and G M (ordinates of the parabola), will represent the times between the moments the shadow was observed at the points H, F, G, and the exact instant when the sun was in the solstice. Let A T be denoted by a, T E by b, F H by c, F G by d, T C or L F by x, and the right parameter of the parabola by r; then A C or K H $= a+x$, and C E or G M $= b-x$. From the nature of the parabola $x^2 = \text{VF} \times r$, consequently $\text{VF} = \frac{x^2}{r}$. In a similar manner we get $\text{VG} = \frac{b^2 - 2bx + x^2}{r}$, and $\text{VH} = \frac{a^2 + 2ax + x^2}{r}$; whence $c = \text{FH} = \text{VH} - \text{VF} = \frac{a^2 + 2ax}{r}$, and $d = \text{FG} = \text{VG} - \text{VF} = \frac{b^2 - 2bx}{r}$. By means of these two equations, after exterminating r, we have $\frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$; and from this we readily find $x = \frac{b^2 c - a^2 d}{2ad + 2bc}$, the expression for the space of time between the observation of the shadow at F, and the instant of the solstice sought. Had the point G fallen above or beyond the point F with respect to H, then the observations would all have been made on the same side of the solstice, G M would have been on the same side of the axis as L F and K H, and the final equation would have been $x = \frac{b^2 c - a^2 d}{2ad - 2bc}$. If the observations be made at the end of equal intervals, then A T or a = T E or b, and the equations will be more simple; x will be found $= \frac{ac - ad}{2d + 2c}$, $a + x = \frac{3ac + ad}{2d + 2c}$, and $b - x = \frac{ac + 3ad}{2c + 2d}$.

303. This method of determining the solstices was first given, in the *Philosophical Transactions* for 1695, by Dr. *Halley*. When the gnomon is properly constructed, and the observations made carefully, the times of the solstices may be found thereby, with nearly as much accuracy as the times of the equinoxes can by the methods already given (Art. 54.); therefore, the length of the solar year may be ascertained nearly as well by means of the solstices, as by the equinoxes.

304. Prob. III. *To determine, by observation, the ratio of the axes of the ellipsis in which the earth moves, and the excentricity of the orbit.*

1*st Method.* It is an established law in optics, that *the tangents of the angles under which the diameter of an object appears, are reciprocally as the distances of the object from the eye;* or, since in small angles the angles are nearly as their tangents, the distances will be nearly in the inverse ratio of the apparent angles. Therefore, the earth is in the aphelion when the apparent diameter or semidiameter of the sun is the least, and in the perihelion, when the apparent diameter, or semidiameter, of that luminary, is the greatest. The apparent semidiameters of the sun being accurately taken, when the earth is in the aphelion and perihelion, we have then the ratio of the perihelion and aphelion distances; that is (fig. 10, Pl. III.), we know the ratio of S H to S A, S being the place of the sun, H and A the places of the earth at the respective times. From this, by composition and division, we get the ratio of S A + S H to S H, and of S Q to S H; and, consequently, that of A H to S Q. Or, half the difference between the antecedent and consequent of the ratio of S H to S A, will give S C or C Q, in terms bearing a relation to either the antecedent or consequent of that ratio.

Ex. The greatest apparent semidiameter of the

sun is 16′ 19″·3, the least 15′ 46″·9; from this the proportion of the major axis, to the excentricity of the earth's orbit, is required.

The natural tangents of the semidiameters are 47477 and 45908 respectively: these express the ratio of S A to S H. Half the sum of these numbers is 46692·5, and half the difference is 784·5; these express the ratio of A C or S B to S C; that is, A C : S C :: 46692·5 : 784·5 :: 100000 : 1681, the ratio of the mean distance to the excentricity.

305. 2d *Method.* Let the apparent motion of the sun be observed in the ecliptic when he is swiftest, that is, when in a given space of time he describes the greatest arc towards the east, for in that case the earth is in the perihelion H; also observe his apparent motion when slowest, at which time the earth is in the aphelion A. Then the apparent motion of the sun, or the real motion of the earth in perihelion, is to the motion in aphelion (Art. 273.) as $SA^2 : SH^2$; whence S A and S H become known, and the rest as in the former method.

Ex. The sun's greatest daily motion in the ecliptic is 1° 1′ 12″, the least motion 57′ 11″; what is the excentricity of the earth's orbit to the mean distance 100000?

The square roots of the respective daily motions, in seconds, are 60·597 and 58·5747, which are in the ratio of S A to S H; whence, by proceeding as in the example to the last article, we get 1691·3 for the excentricity to the mean distance 100000.

306. 3d *Method.* The above methods require such exceedingly accurate observations, that they cannot be relied upon in practice, although they are strictly true in theory; we, therefore, add another method, better fitted for astronomical purposes. Here we only require three places of the sun in the ecliptic, or the opposite points to these, which are the places of the sun at the same times, and the

intervals between the observations. Then let ADPB (fig. 15, Pl. III.) represent the earth's orbit, S the sun in one of the foci, F the other focus, AP the line of the apsides, B, C, D, the three observed places of the earth: then we have to solve this problem—*to describe such an ellipsis, having a given focus* S, *as that its whole area may be to its parts contained under the lines* SB, SC, SD, *in given ratios.* In order to this, from the centre F suppose the circle MRGY described, with a radius FM equal to AP. From F through B, C, D, draw FBR, FCZ, FDY, and draw SR, SZ, SY. Then the angles BSC and CSD are known, being measured by the arcs of the ecliptic. But in the present case, as the ellipse approaches nearly to a circle, the angles about the focus F will be nearly proportional to the times of describing the respective arcs, and the periodic time of the earth being nearly known (Art. 301.), the angles BFC, CFD, will thence be known. Now since $FB + BR$ or $FR = AP = FB + SB$, therefore $BR = SB$: hence the triangle SBR is isosceles, the angle $BSR = BRS$, and $FBS = 2BSR$. But $AFB = ASB + FBS = ASB + 2BSR$; whence $2BSR$ or $2FRS = AFB - ASB$. By similar reasoning, $2FZS = AFC - ASC$. These added together give $2FZS + 2FRS = BFC - BSC$; consequently, $FRS + FZS$ is known, since BFC and BSC are known. But $FRS = RFA - RSA$, and $FZS = ZFA - ZSA$; whence, by addition, $FRS + FZS = RFZ - RSZ$ a known angle; therefore, since RFZ or BFC is known, RSZ becomes known; and by tracing similar steps RSY becomes known, and consequently ZSY.

Draw the lines YSG, GR, RZ, and ZG, and assume SZ of any value (as 100000), in order to get the value of the other parts of the figure in relative terms. In the triangle ZSG there are given the

angles Z S G, Z G S, and side Z S, to find S G; for Z S G is manifestly equal to supp. of Y S Z, and Z G S = half Z F Y. Also, in the triangle S G R are given, the angle R S G = supp. R S Y, S G R = half Y F R, and S G, to determine S R. Again, in the triangle S Z R, are known S Z, S R, and the angle Z S R, to find Z R, and the angle S Z R. Then, in the triangle Z F R, are given Z R and angle Z F R to find Z F; which, by the construction, is equal to A P, the major axis of the orbit: we also, in the same triangle, find R Z F, which taken from R Z S leaves S Z F. Finally, in the triangle F Z S, we know F Z, Z S, and the angle F Z S, whence we find Z S F, which shews the position of the apsides, and S F, which is double the excentricity.

307. This solution, being on the supposition that the angles A F C, A F B, &c. are as the times, which is not strictly true, will stand in need of correction. As the excentricity of the orbit is small, the place of the earth may be corrected, by saying, in the first and fourth quadrants, or arc O A D, as the minor axis to the major, so is the tangent of A F C to the tangent of A F C corrected; and the same may be done for B. But in the 2d and 3d quadrants, or arc D P O, the correction must be made by reversing the antecedent and consequent of the ratio. In orbits of greater excentricity, other modes of correction must be adopted.

308. *4th Method.* Since the species of an orbit may be best determined by taking the medium of results from different methods, one more method is added, similar to the latter in some respects, but admitting of a more speedy calculation. Find three places of the sun in the ecliptic, and carefully measure the semidiameter of that luminary when in those places: from these observations we have the earth's places in the ecliptic, and the relative distances from the sun; whence the problem is reduced to

this: *given the length and position of three lines meeting in the focus of an ellipsis, to describe that ellipsis.* This may be easily performed geometrically; for if C (fig. 1, Pl. IV.) be the centre, and S the focus, of an ellipsis, and in the transverse axis produced there be taken C D, a third proportional to C S and C A, and from any point G in the curve, G E be drawn parallel to B D, and meeting D E perpendicular to it in E, and S G be drawn; then S G will always be to G E in the constant ratio of C S to C A, and D E will be the directrix of the curve, as is shewn by the writers on conics. Wherefore, if G, H, I, be the extremes of the given lines S G, S H, S I, drawn from the focus S, and H G, I H, be joined and produced to K and L, so that H K be to G K as S H to S G, and I L to H L as S I to S H; then, if K L be drawn, and perpendicular to it I N, H M, G E, S D; and, lastly, S D be divided in A in the given ratio of S G to G E; then A will be the end of the transverse, and the whole of the ellipsis is thence determined. For K H being to K G as S H to S G, by the construction, and K H to K G as M H to E G, by similar triangles; therefore, S H : S G : : M H : E G, or S H : M H : : S G : E G; and in the same manner, S I : I N : : S H : H M; wherefore, in general, S I : I N : : S H : H M : : S G : G E : : (per above) C S : C A; and the species of the ellipsis becomes known. From this construction, the calculation is readily deduced.

309. M. *de la Caille* constructs the problem, as stated in the last article, in much the same manner: his construction, with the mode of calculation resulting from it, are as below. Let S B, S C, S D (fig. 2, Pl. IV.), be the three lines given in magnitude and position; draw the indefinite right lines C B, C D, and make S B : S C : : E B : E C, and S C : C D : : C F : D F; these analogies give $CE = \frac{SC \times BC}{SC - SB}$, and

$CF = \frac{SC \times CD}{SC - SD}$. Then the right line passing through the points E, F, will be the directrix of the ellipsis; for BH, CI, DK, being drawn perpendicular to FE, the triangles ICE, HBE, are similar; therefore IC : HB : : EC : EB. Now, by construction, EC : EB : : SC : SB; hence IC : HB : : SC : SB, or IC : SC : : HB : SB; also IC : KD : : CF : DF : : SC : SD. Therefore the perpendiculars HB, IC, KD, are always in the same proportion as the lines SB, SC, SD; consequently EF is the directrix of the ellipsis passing through B, C, D. Through S draw ASG perpendicular to FE; make CI : CS : : GA : AS : : GP : SP; or take $SP = \frac{SC \times SG}{CI + SC}$, and $SA = \frac{SC \times SG}{CI - SC}$; and the vertices A, P, of the ellipsis will thence be determined.

310. *Calculation.* In each of the triangles SBC, SCD, two sides and the included angle are known, whence are deduced the sides BC, CD, and the angles BCS, SCD, BCF. Also CE and CF may be found by the equations relating to them in the last article; then in the triangle CEF, two sides and the included angle are known, whence the angle CEF is determined. In the right-angled triangle CIE, CE and the angle CEI being given, CI is found. Draw SI; then in the triangle SIC, the sides CI, CS, and the angle SCI (=ECI—BCS), being given, the angles CIS, CSI, the side SI, and the angle SIG, the complement of CIS, become known. In the right-angled triangle SIG, we know SI and the angle SIG, whence SG is easily deduced. Then SP, SA, are found by their respective equations (Art. 309.); their difference SO is twice the excentricity, or the distance between the foci O, S; and their sum gives the major axis PA. Laſtly, in the triangle BSO, we know BS, SO,

and BO = PA − SB; whence the angle BSO or BSA is determined, which gives the position of the line of the apsides PA relatively to SB.

311. Dr. *Halley* gave an ingenious method of performing this problem; but as it supposes the periodic time of the planet mars to be known, it cannot be given with propriety in this place. According to one or other of these methods, the excentricity of the earth's orbit has been ascertained by various astronomers: *Cassini* makes it 1690; Dr. *Halley*, 1691·9; M. *de la Lande*, 1681·395; to the mean distance 100000.

312. Prob. IV. *To find the greatest equation of the centre* (Art. 299.) *in the earth's orbit, by observations on the sun.*

Here it will be necessary to compare the motion of the earth in different parts of its orbit, with an equal and uniform motion of a body moving in a circle. Let, therefore, the ellipsis AEBF (fig 3, Pl. IV.) be the orbit of the earth, in the focus S of which is the sun. At the centre S, and with the distance $SE = \sqrt{AK \times OK}$, a mean proportional between the two semi-axes, describe the circle CEGF; then it is well known that the area of this circle is equal to that of the ellipsis. Suppose that, at the same time the earth departs from A the aphelion, a body begins to move with a uniform motion from G through the periphery CEGF, and performs a whole revolution in the same period that the earth describes the ellipsis; the motion of this body will represent the equal or mean motion of the earth, and it will describe round S areas or sectors of circles, which are proportional to the times, and equal to the elliptic areas described in the same times by the earth. Let now the equal motion, or the angle about S proportional to the time, be CSM, and take the area ASP equal to the sector CSM; then the place of

the earth will be P; M S C will be the *mean anomaly*, D S C the *true anomaly*, and M S D the *equation of the centre*. Since the sectors C S M and A S P are equal, and the part C S D is common to both, P A C D and S D M are equal; and since the areas of circular sectors are proportional to their arcs when the radii are equal, the equation of the centre is *a maximum* when the area A C D P is *a maximum*, that is, at the point E, where the ellipse and circle intersect each other. For when the earth descends further, to R for instance, the equation becomes proportionable to the difference of the areas A C E and *m* E R, or to the area G B R *m*, V being the situation of the body moving equably; for the sector C S V will be equal to the elliptic area A S R, and, taking away the common spaces, A C E — R E *m* = to the sector V S *m*, or the equation. At the points E and F, where the circle and ellipsis intersect, the *radius vector* of the earth, and the radius of the circle of equable motion, are equal, and of course those radii then describe equal areas in equal times; wherefore, when the real motion of the earth is equal to the mean motion, the equation of the centre is greatest; hence, we have this practical rule: observe the motion of the sun in the ecliptic for several days, until the true motion be equal to the mean motion, found by dividing the whole circumference 360°, in such ratio as the time for which the mean motion is required, bears to the time of a periodic revolution; then is the earth in the place of its *greatest equation*. Between five and six months after this, observe again the true place of the sun, or of the earth, when its true and mean motions are equal, for then also its equation is greatest; consequently, take the difference of the two places of the earth, or sun, for their real angular motion; and, after computing the mean motion for the same time, take half the difference of the mean and angular motions for the greatest

equation. In order to arrive at great accuracy, several observations should be made near the times when the equation is greatest, and, after comparing them two by two, the medium of the results may be relied upon as very near the truth.

Ex. In 1744, M. *de la Caille* observed the sun's place when the true motion corresponded with the mean;

	h.	m.	s.		s.	°	′	″
On September 30, at	18	44	45	.	6	8	22	31
1745. March 28, at	0	5	7	.	0	7	53	46
Difference of longitude				.	5	29	31	15
Mean motion corresponding to 178^d 5^h 20′ 22″				.	5	25	39	52
						3	51	23

Half this difference gives 1° 55′ 41½″ for the greatest equation. From the mean of several observations, M. de la Caille made it 1° 55′ 32″; but *Cassini* in his tables makes it 1° 55′ 51″. In *Mayer's* tables it is 1° 55′ 31·6″; and M. *de Lambre*, from the very accurate observations of Dr. *Maskelyne*, states it at 1° 55′ 30·9″ for the year 1780; and it is generally allowed, that both it and the excentricity are subject to a regular diminution.

313. PROB. V. *To find the position of the earth's apsides, and the time when it is in the aphelion.*

1*st Method.* The position of the earth's apsides may be found, having three places of the sun, and the apparent semidiameter of the luminary when in each place, by the method of calculation given in Art. 310.; and the time may be found, by determining the time when the sun is in that part of the ecliptic, opposite to the place of the earth's aphelion.

314. 2*d Method.* Observe two places of the sun at the distance of three or four signs of the ecliptic

from each other, where the apparent motion is the same, or the apparent magnitude the same: let the opposite places to these be C and B (fig. 15, Pl. III.), one on each side the aphelion. Bisect the arc B C by the line A P, which will be the line of the apsides; and the middle between the times of observation, is the time when the earth is in the aphelion; that is, if the magnitude of the sun at S, when observed at B, was decreasing; for, if it were increasing, the earth would then be approaching the perihelion.

315. *3d Method.* Having found the greatest equation, we may find the place of the aphelion according to the following directions given by M. *Cassini*: observe the earth at *r* near the aphelion A (fig. 3, Pl. IV.); and the place F being known where the equation of the centre is the greatest, the angle F S *r* will be the true angle described between F and *r*: then, from the time of describing this angle, compute the mean motion; and if the difference between the true and mean motions be *equal* to the greatest equation, then *r* is the aphelion, but if it be *less*, the earth is not arrived at the aphelion. Make another observation when the earth is at P, and if the difference between the true and mean motions is *greater* than the greatest equation, the earth has passed the aphelion. If this be not yet the case, continue the observations until it is; that is, till P be on a contrary side the aphelion from *r*. Then say, as the sum of the equations at *r* and P : the equation at *r* : : the angle *r* S P : the angle *r* S A, the distance of the point *r* from the aphelion. And the time at A may be found by the same analogy, having the respective times at *r* and P.

By one or other of these methods the earth's aphelion is now found to be when the sun is in 8° 40′ 12″ of ♋. In 1750, the place of the aphelion was, according to *Cassini*, 8° 27′ 23″; to *Halley*, 8° 28′ 43″; and to *La Lande*, 8° 37′ 16″.

316. If the place of the aphelion at present be compared with the place as determined several years ago, its motion will be known for that time; and, dividing the angle moved through by the number of years, the quotient will give the annual motion of the aphelion. This, according to the best observations, is about $1' \; 2''$. And the precession of the equinoxes being about $50\frac{1}{4}''$ annually; we shall have $11\frac{3}{4}''$ for the actual motion of the aphelion. The time required by the sun to pass over $11\frac{3}{4}''$ of longitude being added to the *sidereal year*, will give $365^d \; 6^h \; 14^m \; 2^s$ for the *anomalistic year*, or the time occupied by the earth in revolving from aphelion to aphelion.

317. M. *de la Lande* proposes to examine, and if need be correct the place of the aphelion by two observations; one near the aphelion, and the other near the time of the greatest equation. Calculate for each observation the equation of the centre from the supposed place of the aphelion, and take the difference of the equations, if the two observations be on the same side of the aphelion, but the sum if on different sides; and this difference or sum will show how much the true motion differs from the mean, the mean being known from the known interval between the observations. Hence, if the difference calculated agree with the difference observed, the place of the aphelion was rightly assumed; but if the true motion by calculation differ more from the mean motion than the true motion by observation does, the place of the aphelion was too near, or too far from, the observation made near the time of the greatest equation. Assume, therefore, another place for the aphelion, as may be judged proper from circumstances, and the true place will soon be found. But this mode of correction requires some knowledge of the calculations explained in the next problem.

318. PROB. VI. *Given the periodic time of the earth, the time from the aphelion, and the excentricity of the orbit;* to find *the angular distance from the aphelion, or the true anomaly, and the distance from the sun, in terms relative to the excentricity, or the major axis.*

Since the earth, as well as any other body moving in an orbit about a central point, describes areas proportional to the times (Art. 271.), the area ASP (fig. 4, Pl. IV.), moved over by the *radius vector*, while the earth passes from its aphelion A to the situation P, will be to the whole ellipsis, as the time of describing AP, to the whole periodical time; or (Art. 299.) as the mean anomaly at P, to 360°. The problem is therefore reduced to this: *Given the species of an ellipsis, to find the length and position of a line* SP *from one of the foci, which shall cut off a trilineal area* ASP, *in a given ratio to the whole ellipsis.* Under this form it has been considered in Dr. *Hutton*'s comprehensive and excellent *Treatise on Mensuration* (see p. 298, 2d ed.), where the solution is in substance as follows:

The area of an ellipsis, or any sector of it, not admitting of a correct expression in finite terms, recourse must be had to infinite series: let, therefore, CA or CQ $= r$, CG $= c$, and $z =$ PI; then S being the focus of the ellipse, SA $= r + \sqrt{r^2 - c^2}$, and the trilineal SAP $= \frac{1}{2} z \times \left(r + \sqrt{r^2 c_2} + \frac{r z^2}{3.2 c^2} + \frac{3 r z^4}{5.2.4 c^4} + \frac{3.5 r z^6}{7.2.4.6 c^6}, \text{ \&c.}\right) = \frac{1}{2} r z \times \left(1 + \sqrt{1 - \frac{c^2}{r^2}} + \frac{z^2}{3.2 c^2} + \frac{3 z^4}{5.2.4 c^4}, \text{ \&c.}\right)$. Putting T $=$ SAP $= \frac{1}{2} r z \times \left(a + \frac{z^2}{3.2 c^2} + \frac{3 z^4}{5.2.4 c^4}, \text{ \&c.}\right)$ a being $= 1 + \sqrt{1 - \frac{c^2}{r^2}}$, and $q = \frac{2\text{T}}{r} = z \times \left(a + \frac{z^2}{3.2 c^2} + \frac{3 z^4}{5.2.4 c^4} \text{ \&c.}\right)$ we shall have by reversion, $z = \frac{q}{a} - \frac{q^3}{6 c^2 a^4} + \frac{10 - 9a}{120 c^4 a^7} q^5$

$-\frac{280-504a+225a^2}{5040c^6a^{10}}q^7$, &c.) $=\frac{2T}{ra}\times(1-\frac{1}{6a^3}p^2+\frac{10-9a}{120a^6}p^4-\frac{280-504a+225a^2}{5040a^9}p^6$, &c.) where $p=\frac{2T}{rc}$, which latter series generally converges very fast. The application of this to the present problem will be very easy; only remarking, that when the earth is nearer the perihelion than the aphelion, or when the mean anomaly is between 3 and 10 signs, it will be better (if this method be used) to estimate the true anomaly from the perihelion: then SQ being $=r-\sqrt{rr-cc}$, a in the last series will be $=1-\sqrt{1-\frac{cc}{rr}}$.

Ex. Required the true anomaly of the earth, and its distance from the sun answering to the mean anomaly of 1 sign or 30°, the excentricity being ·01681, to the mean distance 1.

Here $AC=r=1$, $CS=\cdot01681$, $CG=c=\sqrt{1-\cdot01681^2}=\cdot9998587$, the area of a quadrant of the ellipsis $=\cdot785398$, &c. $\times rc=\cdot7852872$: then, as 3 signs : 1 sign : : ·7852872 : ·2617624 = T; hence $\frac{2T}{r}=2T=\cdot5235248$, and $\frac{2T}{rc}=\frac{2T}{c}=\frac{2}{3}\times\cdot78539$, &c. $=\cdot5235988=p$: also $a=1\cdot01681$, and $\frac{p^2}{a^3}=\cdot2589869$. Then,

Term.		
1st	$A=\frac{1}{a}=$	+ ·98346790
2d	$B=\frac{1}{6}A\times\frac{p^2}{a^3}=$	— ·04245088
3d	$C=\frac{10-9a}{20}B\times\frac{p^2}{a^3}=$	+ ·00046655
4th	$D=\frac{280-504a+225a^2}{42}C\times\frac{p^2}{a^3}=$	— ·00000044
	the sum =	·94148313

which drawn into $\frac{2T}{r}$ gives ·4928897 $= z =$ PI. But CI $= \frac{r}{c} \times \sqrt{c^2 - z^2} =$ ·87005233; therefore SI $=$ Sc + CI $=$ ·88686233. Consequently, SI : PI : : radius : tang. PSA $= 29{\cdot}063904^{\circ} = 29^{\circ}\ 3'\ 50''\ 3'''$, the true anomaly. And, radius : secant PSI : : SI : SP; $=$ 1·0146255, the distance of the earth from the sun, to the mean distance 1.

319. Dr. *John Keill*, in the 23d of his *Astronomical Lectures*, has given a direct method of finding the true anomaly; which, as it may be frequently applied with advantage, is here explained. Let ANQ (fig. 4, Pl. IV.) be a semicircle whose diameter is the major axis of the ellipsis, its centre C, and S the focus of the ellipsis in which the sun is placed. Through the place of the earth imagine a perpendicular NI to be drawn meeting the circle in N, then the arc AN will measure the excentric anomaly. The area ASN will be to the whole circle, as the given time from the line of apsides, to the earth's periodical time. Draw cN, and from S let fall upon it, produced, if required, the perpendicular ST: the area ASN is equal to the sum of the sector ACN and the triangle NSC; that is, equal $\overline{\tfrac{1}{2}\,CN \times AN} + \overline{\tfrac{1}{2}\,CN \times ST}$. And therefore, since $\frac{1}{2}$ CN is a constant quantity, the area ASN will be always proportional to the arc AN + the right line ST, when the motion is from the *aphelion* to the *perihelion*: but when the earth ascends from the *perihelion* to the *aphelion*, the area QSn is equal to the sector QCn — triangle CSn; and consequently will be proportional to the arc Qn — the right line St: Hence if we take the arc AD or Qd proportional to the time, AN + ST will be equal to AD, or Qn — St = Qd; for then AD and Qd will be proportional to the areas ASN, and QSn. Here, as the arc ND is equal to the right line ST, when the excentricity is very small ND may be reckoned

nearly rectilinear, and N C D, N C S, two equal rectilinear triangles: and, as N C the base of each is the same, the perpendiculars N D, S T may be considered equal and parallel, and the angle N C D = C D S. Then in the triangle C D S are known, CD, C S, and the included angle D C S, equal to the supplement of A C D, the mean anomaly: whence the angle D S C, or the arc A N, is easily found. But, to be more accurate, AN, and thence the true anomaly, must be determined as follows: Call the arc N D, y, the sine of A D call e, the co-sine f, and the excentricity C S, g. Then, as is shewn by the authors on trigonometry (See *Emerson*, *Mauduitt*, *Maseres*, &c.), the sine of A N $= y - \frac{y^3}{2.3} +$, &c. and co-sine $= 1 - \frac{y^2}{2} + \frac{y^4}{2.3.4} -$, &c. Hence the sine $= e - fy - \frac{ey^2}{2} + \frac{fy^3}{2.3} + \frac{ey^4}{2.3.4} -$, &c. Also, as rad. $= 1$: sine A N or Z S C T : : S C $(= g)$: S T = N D $= y = ge - gfy - \frac{gey^2}{2} + \frac{gfy^3}{2.3} +$, &c. Hence $ge = y + gfy + \frac{gey^2}{2} - \frac{gfy^3}{2.3} -$, &c. Put $ge = z$, $1 + gf = a$, $\frac{ge}{2} = b$, $\frac{gf}{2.3} = c$, $\frac{ge}{2.3.4} = d$, &c. then $z = ay + by^2 - cy^3 - dy^4 +$, &c. and by reversion of series $y = \frac{z}{a} - \frac{bz^2}{a^3} + \frac{2b^2 + ac}{a^5} \times z^3 - \frac{5abc - 5b^3 + a^2d}{a^7} \times z^4$, &c.; and, because of the values of b and d just given, we shall have $y = \frac{z}{a} - \frac{z^3}{2a^3} + \frac{cz^3}{a^4} - \frac{5cz^5}{2a^6}$, &c. If the arc A N be greater than 90° and less than 270°, the co-sine of A D will be negative; and then $a = 1 - gf$, and $y = \frac{z}{a} - \frac{z^3}{2a^3} - \frac{cz^3}{a^4} +$, &c. To express this value of y in degrees and parts of a degree, say, as radius is to this series, so are 57·29578 (the degrees of an arc equal in length to the radius) to a fourth

number: consequently, since the radius is unity, if we multiply the series by $57{\cdot}29578 = R$, we shall have y or N D in degrees and decimals $= \frac{Rz}{a} - \frac{Rz^3}{2a^3} + \frac{Rcz^3}{a^4} -$, &c. The very first term of this series will determine the excentric anomaly of the earth within a 10000th part of a degree, and will not cause an errour of the 200th part of a degree in any of the planets.

320. Having found N D by the above method, and consequently A N from the mean anomaly, we have next to determine the angle A S P: to this end, let w be the other focus. Then $PS - Pw^2 = wS_2 + 2wS \times wI$ (Pr. Euc. II. 12.) $= \overline{wS + 2wI} \times wS = 2Cw + 2wI \times 2CS = 2CI \times 2CS$: hence $SP + Pw : 2CI :: 2CS : SP - Pw$; or, since AC is denoted by 1, $2 : 2CI :: 2CS : SP - \overline{2 - SP}$; or $1 : CI :: CS : SP - 1$: therefore $SP = 1 + CS \times CI = 1 + CS \times$ co-sine $\angle ACN$. From the principles of plane trigonometry $\frac{1 - \text{co-s. } ASP}{1 + \text{co-s. } ASP} = \text{tang. } \overline{\tfrac{1}{2} ASP}^2$. But in the right-angled triangle S P I, we have S P, or, as above, $1 + CS \times$ co-sine ACN : rad. 1 :: S I, or $CS + CI$, or $CS +$ co-sine ACN : so-sine ASP $=$

$$\frac{CS + \text{co-sine } ACN}{1 + CS \times \text{co-sine } ACN}. \text{ Therefore } \frac{1 - \text{co-sine } ASP}{1 + \text{co-sine } ASP} = \left(\frac{1 + CS \times \text{co-sine } ACN - CS - \text{co-sine } ACN}{1 + CS \times \text{co-sine } ACN + CS + \text{co-sine } ACN} = \right.$$

$$\frac{1 - CS + \text{co-s. } ACN \times \overline{CS - 1}}{1 + CS + \text{co-s. } ACN \times CS + 1} = \frac{SQ - \text{co-s. } ACN \times SQ}{AS + \text{co-s. } ACN \times AS}$$

$$= \frac{SQ}{AS} \times \left.\frac{1 - \text{co-sine } ACN}{1 + \text{co-sine } ACN}\right) = \frac{SQ}{AS} \times \text{tang. } \overline{\tfrac{1}{2} ACN}^2 =$$

$\text{tang. } \overline{\tfrac{1}{2} ASP})^2$, as above. Therefore $\sqrt{AS} : \sqrt{SQ}$ :: tang. $\frac{1}{2}$ A C N : tang. $\frac{1}{2}$ A S P; whence A S P, the true anomaly, becomes known.

Ex. Required the true anomaly, when the excen-

tricity of the earth's orbit is as before ·01681, and the mean anomaly 30°?

Logarithms of $g = \cdot 01681$	8·2255677
Log. sine of $e = 30°$	9·6989700
Log. of R 59·29578	1·7581226
Log. of R g e, or of R z	9·6826603
Log. of a, or of $1 + gf$	0·0063137
Log. of $\frac{Rz}{a} = \cdot 47462$	9·6763466

The corresponding number ·47462 is a decimal of a degree, and answers to $28' 28'' = y =$ N D, true to a second; therefore, A N $= 29° 31' 32''$.

Hence, log. tang. ½ A C N, or of

14° 45′ 46″	9·4208078
½ log. S Q ·98319	9·9963187
	9·4171265
½ log. A S 1·01681	0·0036199
log. tang. 14° 31′ 40″	9·4135066

Hence the true anomaly, according to this method, is 29° 3′ 20″. Dr. *Keill*, taking ·01691 for the excentricity, finds $y = 28' 38''$, and the true anomaly $=$ 29° 2′ 54″. The distance from the focus, *i. e.* S P, may be found by means of the expression S P $= 1 +$ C S × co-sine A C N, in the foregoing investigation.

321. M. *de la Caille*, from a mode of investigation in many respects similar to what is here given, has drawn the following approximating rules (*Elements Astron.* p. 79):

I. As the aphelion distance is to the perihelion distance, so is the tangent of half the mean anomaly to the tangent of an arc, which added to that half, the sum is called the approximated excentric anomaly.

If the difference between the approximated and mean anomalies does not exceed 3°, its difference from the *true excentric anomaly* will not amount to a second. When this happens, which is always the case in the orbit of the earth, the next four articles become useless; nevertheless, as they are of utility in determining the true anomaly of the other planets, they are added.

II. As half the greater axis is to the excentricity, so are 57° 17′ 44⅘″ (or 206264⅘″ whose logarithm is 5·3144250) to a number of seconds, which call A.

III. As radius to the seconds A, so is the sine of the approximated excentric anomaly (I) to another number of seconds, which, taken from the mean anomaly, gives another approximated excentric anomaly.

IV. As radius to the seconds A, so is the sine of the new approximated excentric anomaly to a number of seconds, which, subtracted from the mean anomaly, gives also another approximated excentric anomaly.

V. This analogy must be repeated, always putting the sine of the last-found approximated anomaly for the third term, till two be found successively, which are equal; then either will be the true excentric anomaly.

VI. As the square root of the aphelion distance is to the square root of the perihelion distance, so is the tangent of half the true excentric anomaly to the tangent of half the true anomaly sought. If these anomalies should exceed 180°, their supplements, or half their supplements, must be used instead of these anomalies, or their halves.

VII. As radius to the co-sine of the true anomaly, so is the excentricity to a fourth quantity B; then, as half the greater axis, plus or minus the quantity B to the perihelion distance, so is the aphelion distance to the distance sought. B must be added in

the 3d, 4th, 5th, 6th, 7th, and 8th signs of the true anomaly, but subtracted in the other signs.

322. From what has been shewn in Art. 319, 320, we may deduce a method of determining the *greatest equation*, having given the proportion of the axis; for it has been shewn (Art. 312.), that when the equation is greatest $SP = \sqrt{AC \times CG}$; whence SP becomes known. But from Art. 320. $SP = 1 + CS \times$ co-sine ACN; therefore, co-sine ACN $= \frac{SP - 1}{CS} =$ co-sine SCT: hence, as SC is known from the proportion of the axes, ST becomes known, or its equal the arc ND: to convert this into degrees, say as rad. 1 : ND :: 57° 17′ 44⅘″ : the degrees in ND, which added to, or subtracted from, ACN, gives ACD the *mean anomaly*. And as ACN, the excentric anomaly, is known, we easily find the *true anomaly* by the analogy given in Art. 320. for that purpose. The difference between the mean and true anomalies thus found is the greatest equation required.

323. If the true anomaly were given, and it were required to find the mean, it might be readily done thus: the excentricity being known, and GS or AC being expressed by unity, we have $CG = \sqrt{GS^2 - SC^2}$, whence the ratio of AC to GC is known, and this is the ratio of NI to PI, by the nature of the ellipsis and circumscribing circle: hence the angle ASP being given, we have PI : NI :: tang. ASP : tang. ASN. And in the triangle NCS we know NC, SC, and the angle CSN; whence we find SCN, the supplement of which is SCT. Then in the triangle SCT we know SCT and SC, from which we get ST; which reduced to degrees, by reckoning 57° 17′ 44⅘″ as equivalent to radius, or 1, and added to ACN, gives the mean anomaly.

324. This important problem was first proposed by

Kepler, and is thence called *Kepler's problem.* The solution has been attempted by some of the ablest mathematicians in Europe. Dr. *Wallis* first gave the geometrical solution by means of the protracted cycloid; and Sir *Isaac Newton* has solved it in nearly the same manner, in the *Principia*, Prop. 31. lib 1. He has also given an excellent approximation in the subsequent scholium. Dr. *Matthew Stewart* likewise has given a very short, easy, and exact method of finding the place of a planet in its orbit, in the latter part of the 4th of his *Physical and Mathematical Tracts.* Dr. *Seth Ward*, in his *Astronomia Geometrica*, solved the problem on what has been since called the *simple elliptic hypothesis*, in which it is supposed that the angular velocity about the focus *w* is uniform: this, though not accurate, frequently serves as a useful approximation. M. *Cassini*, at p. 144 of his *Elem. Astron.* has given an ingenious method which may be sometimes adopted with advantage: he draws D *z* perpendicular to S T, and considers S *z* as the difference between the arc of the angle C D S, and its sine; he therefore computes the angle C D S, which is easily done, since C D, C S, and the mean anomaly, are known; and, taking the difference between the arc and its sine for S *z*, he uses this analogy, S D : S *z* : : radius : sine S D *z*, which, subtracted from the angle S D C, leaves *z* D C, or the alternate angle D C N; from which the true anomaly is soon found. To enter into a fuller description of this method is unnecessary, since the methods elucidated in the preceding articles are amply sufficient for every useful purpose.

325. Prob. VII. *To find the real distance of the earth from the sun.*

When treating on the subject of *parallax* (Art. 88.), it was shewn, that the distance of any heavenly body from the earth's centre might be easily found in terms of the earth's radius, provided its horizontal parallax

was known. Therefore, since the semidiameter of the earth is known, the only requisite, in the instance before us, is the sun's horizontal parallax. The distance of that luminary from us is so great, that the determination of the horizontal parallax requires considerable attention and skill. Many attempts have been made both by the ancients and moderns for this purpose; but the most accurate is that which depends upon the passage of *mercury* or *venus* over the sun's disc, called the *transit* of mercury or venus. The method, founded upon careful observations of such transit, gives as the mean result about 8″·65 for the sun's horizontal parallax at the time of the transit, or nearly 8″·75 as the mean distance of the sun from the earth (see Chap. XX.). We have, therefore, from the nature of parallax, this analogy:

As sine 8″·75 . . .	arith. comp.	4·3773324
To radius		10·0000000
So is 1 semidiameter		0·0000000
To 23841·43 semidiameters . .		4·3773324

This number of semidiameters multiplied into 3985, the number of English miles in the earth's semidiameter (taking it as a sphere), produces 95008098 miles for the earth's mean distance from the sun. Mr. *Short*, from his observations on the transit, stated the mean distance at 95173127 miles; D. *Hornsby* makes the distance 93726900 miles; while others make it 95834742. On the whole, the round number 95 millions of miles may be reckoned the mean distance, being undoubtedly as near the truth as can be reasonably expected in such an enquiry*.

* That excellent geometrician, Dr. *Matthew Stewart*, published, in 1763, a Supplement to his valuable *Tracts*, in which the distance of the sun from the earth was determined by *the theory of gravity*.

His method depended upon ascertaining the proportion of the centripetal force of the moon to the earth, to the mean solar force affecting the gravity of the moon to the earth; and this he effected by means of the motion of the moon's apogee. According to his determination, the sun's parallax is 6·9″, and mean distance 118541428 miles: but Mr. *Machin*, who had formerly endeavoured to ascertain the sun's distance on similar principles, only using the motion of the moon's nodes instead of that of the apogee, assigns 8″ for the parallax of the sun, which would give a distance much nearer the truth than Dr. *Stewart*'s. About five years after the publication of this supplement on the *sun*'s *distance*, a small pamphlet was published under the title of *Four Propositions*, in which several of the errours in Dr. *Stewart*'s investigation were pointed out, and it was sufficiently proved that the distance of the sun can scarcely ever be truly ascertained by the theory of gravity. This pamphlet was written by Mr. *John Dawson*, of Sedberg, in Yorkshire, who was then a very young mathematician, as observed by himself in No. 9 of the *Mathematical Repository* (where these Propositions are republished), but is now well known, and universally admired, as one of the most ingenious mathematicians and philosophers this country now possesses. The late Mr. *Landen* also published some *Animadversions* upon Dr. *Stewart*'s computation; but it is to be lamented, that they were written with much more of asperity and the spirit of exultation than ought ever to have been directed against a gentleman of such distinguished talents and worth as Dr. Stewart.

CHAPTER XII.

On determining, by Observation, the Magnitudes, Situations, &c. of the Orbits of the Planets.

ART. 326. IN the determination of the various particulars relative to the orbits of the planets, different authours have adopted different methods, each of which has probably its peculiar advantages: in this work the necessary elements will be determined by a process nearly similar to that which we had recourse to in the preceding chapters. We ſhall pursue the enquiry in what appears to be the natural order, by ascertaining,

1st, The times of the periodic revolutions of each planet.

2dly, The position of the line of the apsides, and an epocha of the passage of the planet through that line.

3dly, The excentricity, and thence the proportion of the axes of the orbit.

4thly, The position of the line of the nodes.

5thly, The inclination of the orbit to the ecliptic.

6thly, The relation of the principal axis of each orbit to that of the earth's, or the relation of their mean distances, and thence the absolute dimensions of each orbit.

These particulars are determined, according to the best methods now known, in the problems which immediately follow.

327. PROB. I. *To determine the time of a planet's periodical revolution.*

The periodic time of a planet may be determined either by means of observations upon it when in one of its nodes; or by knowing the place and time when it is in opposition to the sun, if it be a superiour planet, or in conjunction, if it be inferiour. As the methods depending upon the oppositions and conjunctions are best in practice, it becomes requisite to shew how to determine when a planet is in either of these situations.

328. The latitude and longitude of a planet may be found by observing its situation with respect to known fixed stars (Art. 190. 191.): and by comparing the longitudes deduced from such observations, with the longitude of the sun, either determined by the right ascensions and declinations (Art. 66.), or taken from the *Nautical Almanac*, or other *Ephemeris*, the time when the longitudes are the same, or when they differ 180°, may be discovered. But if there are opportunities of finding the longitude and latitude, by the passage of the planet over the meridian, such opportunities furnish us with a method which is more accurate. The time of the planet's culminating being carefully taken, its longitude may be known by finding the longitude of the culminating point of the ecliptic (Art. 160.) at that instant: for the longitude of the planet, and of that point of the ecliptic which culminates with it, are manifestly the same. Then, if it be a superiour planet, to ascertain the time of opposition, choose a day when the difference of the longitudes of the sun and planet is nearly 180°, and on that day, knowing the sun's longitude at noon, find by proportion, or by observation, the longitude at the exact instant (*i*) when the planet was on the meridian. Now in opposition the planet's apparent motion is retrograde (Art 254.); and consequently, the difference between the longi-

tudes of the planet and sun increases by the sum of their motions. Hence, subtract the sun's longitude from that of the planet at the instant *(i)*, to get the difference *(d)* of longitudes, estimated according to the order of the signs, and say, as the sum of the daily motions in longitude : the difference between 180° and *d* : : 24 hours : the interval between the the time *i* and the time of opposition. This interval must either be added to, or subtracted from, *i*, according as *d* is greater or less than 180°, and the time of opposition is obtained. As to the conjunction of inferiour planets, it must be found by means of neighbouring fixed stars, as first explained, unless their elongation at the time of conjunction be sufficiently great to render them visible.

329. Thus much being premised, we may find the periodic time of a superiour planet in this manner: observe when the planet is in opposition to the sun, also when it is next in opposition, and note its place at both times; for in these situations the heliocentric and geocentric longitudes are the same. (Art. 260.) Then will be known the arch the planet describes in the interval between the two observations; and it will be, *as that arch : the whole circumference : : the time elapsed : the periodic time nearly*; but this can only be exact, on the supposition that the orbits are circular.

330. For an inferiour planet, two observations might be made when the planet was stationary: and in the interval it would have described its whole orbit, together with an arch or angle, equal to that which the earth has described in the same interval. Then it will be, *as the whole circumference + that angle : the time elapsed : : 360° : the periodic time, nearly.*

331. The methods given in the two last articles will give us the periodic time only so far correct as the motion in the orbit is equable: but this approxi-

mation will be of considerable service, as it will enable us to tell nearly the number of a planet's revolutions in a given interval. Then to know the periodic time correctly, two observations must be chosen, which were made at the most distant times possible, when the planet was found in the same position relatively to some fixed stars, and at the same time in opposition or conjunction with the sun, according as it is a superiour or inferiour planet. The intervening time between these observations, being divided by the number of the planet's revolutions, will give the time of the periodic revolution respectively to the fixed stars. And as the motions of the planets are usually reckoned from the first point of aries, the periodic revolution with respect to the stars may be reduced to the revolution relatively to that point, by saying, *as* 360° : *the revolution found* : : *the precession of the equinoxes during the revolution* : *a fourth term* which must be deducted, to obtain the revolution with respect to aries. If the revolution, as thus determined, is not thought sufficiently accurate, it may be corrected after the other elements of the planets are determined, by allowing for the difference between the *equations of the centre*, at the two observations; but this will very seldom make any material difference.

332. For an example, we shall find the revolution of the planet *Mars*. On April 21, 1715, at 11^h, this planet was in opposition in ♏ 1° 9′ 30″. On June 11, 1717, the opposition happened at $9^h\ 11^m$ in ♐ 20° 17′ 15″. Now in this time, which was two years (one a bissextile) and $20^d\ 22^h\ 11^m$, mars had made one revolution, and 49° 27′ 45″ over; hence from these observations we get an approximation to a revolution, by saying, $360° + 49° 27' 45'' : 360° : : 781^d\ 22^h\ 11^m : 687^d\ 11^h\ 15^m$, the time of a revolution. From the observations of *Ptolemy*, it appears, that mars was in opposition on December 13,

at 11^h 48$'$ at Paris, 130 years after the christian æra, in ♊ 21° 22$'$ 50$''$. In 1709, January 4, at 5^h 48^m, mars was in opposition in ♋ 14° 18$'$ 25$''$. The interval between these observations was 1578^y 11^d 18^h; and allowing for the bissextiles, &c. we get 686^d 22^h 16^m, for the time of a *tropical* revolution. To obtain the sidereal revolution, say, *as the mean daily motion of the planet : the precession of the equinoxes in the time of a tropical revolution : : one day : a number of days*, which added to the tropical revolution, will give the *sidereal*.

333. After nearly similar methods, the tropical and sidereal revolutions of all the planets have been ascertained, and are very nearly as below:

Planets.	Tropical Revolutions.	Sidereal Revolutions.
	d. h. m. s.	d. h m. s.
Mercury	87 23 14 32·7	87 23 15 43·6
Venus	224 16 41 27·5	224 16 49 10·6
Mars	686 22 18 27·4	686 23 30 35·6
Jupiter	4330 14 39 2	4332 14 27 10·8
Saturn	10746 19 16 15·5	10759 1 51 11·2
Georgium Sidus .	30637 4 — —	30737 18 — —

From a comparison of the most ancient, with the modern observations, there is some reason to conclude, that the time of jupiter's revolution is decreasing, and that of saturn increasing, each very slowly.

334. Before we quit this problem, it may be necessary to say a few words, respecting what are called the *annual* and *secular* motions of the planets. If N (fig. 4, Pl. IV.) be the mean place of a planet in its orbit, at any given instant, and at the end of a year

from that instant the mean place of the planet be at V, then (rejecting the number of complete revolutions which the planet will have performed if it be inferiour) N V, reckoning according to the order of the signs, will be the mean *annual* motion of the planet. Or if, at a given instant, the mean place of a planet be at N, and at the end of 100 years, besides a certain number of complete revolutions, the planet be advanced to D or to V, then N D or N V will be the mean *secular* motion. The times of revolution of the planets being known, the mean annual motions may be nearly found by this proportion, *as the time of the planet's revolution* : 365 *days* : : 360° : *to a certain number of degrees*, which after rejecting 360°, or as many times 360° as this fourth number contains, leaves the mean *annual* motion. The mean *secular* motion may be nearly determined by making the number of days in a century (allowing for the bissextiles) the second term in the proportion. Hence, if to the mean place of a planet at the beginning of any year, the mean annual motion be added, the sum (rejecting 360°, if need be) gives the mean place at the beginning of the next year: if the year be a bissextile, the mean motion for a day must also be added. In a similar manner, the mean place at the end of 100 years may be found.

335. PROB. II. *To determine the position of the line of the apsides, and the epocha of the passage of a planet through that line.*

In order to find these particulars, we must first be able to reduce such observations as are made upon a planet from the earth, to those which would have been observed from the sun at the same times; the method of performing this, may be thus explained: In fig 5, Pl. IV. let S be the place of the sun, R the earth's place in its orbit R E *e*, when the planet's place reduced to the ecliptic is *p*, and if A be the first point of aries, S R *p* will be measured by the difference be-

tween the geocentric longitudes of the sun and planet. When the planet has gone through a complete revolution, and is again at P, or at p reduced to the ecliptic, let the place of the earth be E, and the angle p E S will be the observed difference of geocentric longitudes. Then, the theory of the earth being established, S R, S E, are known, and the angle E S R; whence the side E R, and the angles S E R, S R E, are readily determined. Also in the triangle p E R are known E R and the angles (for p E R $=$ p E S$-$S E R, and E R p $=$ S R p $-$ E R S), whence E p, R p, become known. And in the triangle p S R, the sides p R, R S, and the included angle S R p, are known; from which may be found S p the curtate distance (Art. 293.) of the planet, and the angle R S p, which is the difference between A S R the heliocentric longitude of the earth, and A S p that of the planet; whence A S p becomes known. Or, in a similar manner, A S p might be found by the triangle S E p. Then, if the geocentric latitude P E p of the planet be observed, which may easily be done by means of its right ascension and declination; we shall have tang. P E p : radius : : P p : E p; also rad. : tang. P S p : : p S : P p; therefore, tang. P E p : tang. P S p : : p S : E p : : sine S E p : sine E S p; that is, *as the sine of elongation in longitude, to the sine of the difference of heliocentric longitudes, so is the tangent of the geocentric latitude, to the tangent of the heliocentric latitude.* Again, in the triangle P S p, as sine S P p : sine S p P, or radius : : S p : S P; that is, *as the co-sine of the planet's heliocentric latitude to radius, so is the curtate distance to the planet's distance from the sun.*

336. If the situation of the line of the apsides be nearly known, find by means of observations reduced to the sun, as just explained, the points in the orbit diametrically opposite; in which points the planet is found at intervals equal to half its revolution. If the anomalistic revolution were exactly known (which it

cannot be, until the motion of the aphelion is ascertained), this method would be strictly accurate; or, at least, might be easily corrected to any required degree of accuracy.

337. Or, the place of the aphelion being nearly known, observe the places of the planet, and reduce them to the sun, at about the distance of two signs before the aphelion; do the same, for a few days, at nearly the same distance after the aphelion, and observe the times and places when the apparent motion of the planet is the same: then, as in Art. 314., the times and places of the planet at B and C (fig. 15, Pl. III.) are known; whence the position of A the aphelion, bisecting B C, is found; and the midway between the times is the epocha required.

338. But the following method appears preferable to either of the former, in point of accuracy: Find three places of the planet sufficiently distant from each other, and determine their heliocentric places, and distances from the sun, as shewn in Art. 335. Then, by the method given in Art. 310., find the position of the line of the apsides; and, knowing the position, continue the observations upon the planet till the exact instant when it is in that situation, and the *epocha* will be determined. When the excentricity and greatest equation are known, the position, as found by this method, may be corrected by *La Lande*'s method, explained in Art. 317. Or, from the same data, the place of the perihelion may be found by a theorem, the investigation of which was given by E. *Prosperin*, in vol. III, *Nova Acta Reg. Soc. Sci. Upsaliensis*: thus, let A C (fig. 1, Pl. IV.) $= 1$, S G $= a$, S H $= b$, S Q $= c$, $\angle$ G S H $= v$, G S Q $= y$, A S G $= x$, C S $= e$, the excentricity; and half the parameter $= r$: then, as is demonstrated by the writers on conics, $a = \frac{r}{1 + e \text{ co sine } x}$, $b = \frac{r}{1 + e. \text{ co-sine } \overline{v + x}}$, and $c = \frac{r}{1 + e. \text{ co-sine } \overline{y + x}}$; where-

fore $r = a + a\,e.\text{ co-sine }x = b + b\,e.\text{ co-sine }\overline{v + x} = c + c\,e.\text{ co-sine }\overline{y + x}$; whence, by reduction,

$$\frac{b - a}{a.\text{ co-sine }x - b.\text{ co-sine }\overline{v + x}} = e = \frac{c - a}{a.\text{ co-sine }x - c\text{ co-sine }\overline{y + x}}.$$

Substitute in this equation, for co-sine $\overline{v + x}$, and co-sine $\overline{y + x}$, the expressions co-sine v. co-sine x — sine v. sine x, and co-sine y. co-sine x — sine y. sine x, and there arises

$$\frac{b - a}{a.\text{ co-sine }x - b.\text{ co-sine }v.\text{ co-sine }x + b\text{ sine }v.\text{ sine }x} = \frac{c - a}{a\text{ co-sine }x - c.\text{ co-sine }y.\text{ co-sine }x + c.\text{ sine }y.\text{ sine }x}:$$

each denominator divided by co-sine x, gives

$$\frac{b - a}{a - b.\text{ co-sine }v + b.\text{ sine }v.\text{ tan. }x} = \frac{c - a}{a - c.\text{ co-sine }y + c.\text{ sine }y.\text{ tan. }x}.$$

From this equation, after a little reduction, we get

$$\text{tan. }x = \frac{b - \overline{c - a}.\text{ co-sine }v - c.\overline{b - a}.\text{ co-sine }y - a.\overline{c - b}}{b.\ \overline{c - a}.\text{ sine }v - c.\ \overline{b - a}.\text{ sine }y},$$

which determines the place of the perihelion.

339. In some cases, instead of making three distances from the sun, and the angles between them, the basis of the operations; it may be more convenient to find three heliocentric places, and the intervening times: when this is done, the calculations must be conducted in the manner pointed out in Art. 306., and the place of the aphelion will be ascertained.

340. The mutual attraction of the planets and satellites upon each other, causes a very minute motion in the places of their aphelia; to determine the quantity of such motion, we must know the exact instants of two passages of a planet through its aphelion, observed at the most distant times possible: then, dividing the interval between those two passages, by the number of revolutions of the planet relative to the stars; if the quotient be less than the periodic time, the motion of the aphelion is *retrograde*; if it be greater, the motion will be *direct*. And the difference between the quotient and periodic

time, reduced into motion, will be the motion of the apsides. Or, if the difference between the place of the aphelion, as determined many years ago, and as ascertained lately, be taken; the quotient of that difference, divided by the number of years, or the number of centuries, will be the *annual*, or the *secular* motion required.

341. According to the most accurate observations, ancient and modern, the places of the aphelia are found; and their motions in longitude having the secular precession of the equinoxes deducted, are all found to be *direct*.

The result of the whole is stated in the following table:

Planets.	Place of Aphelia, Jan. 1800.	Secular Motion Aph.
Mercury . . .	$8^s\ 14^\circ\ 20'\ 50''$	$1^\circ\ 33'\ 45''$
Venus	10 7 59 1	1 21 0
Mars	5 2 24 4	1 51 40
Jupiter	6 11 8 20	1 34 33
Saturn	8 29 4 11	1 50 7
Georgium Sidus .	11 16 30 31	1 29 2

342. PROB. III. *To find the excentricity, and greatest equation, of a planet's orbit.*

In the solution of this problem, we may either first determine the *excentricity*, and thence the *greatest equation*; or, first ascertain the *greatest equation*, and thence deduce the *excentricity*. If we first find the excentricity, we may do it by means of three relative distances of the planet from the sun, and the angles between them, as we determined the position of the apsides (Art. 338.); for by calculat-

ing, agreeably to the directions in Art. 310., S O, twice the excentricity becomes known, and consequently the excentricity itself. Or, if we adopt the method of *Prosperin*, x being found as in Art. 338., we substitute its value in the equation $e = \frac{b - a}{a.\ \text{co-sine}\ x - b.\ \text{co-sine}\ \overline{v + x}} = \frac{c - a}{a.\ \text{co-sine}\ x - c.\ \text{co-sine}\ \overline{y - x}}$, and thence obtain e the excentricity; also, $1 + e$, and $1 - e$, the perihelion and aphelion distances. Now, to discover the greatest equation, having C S and C Q, or G S (fig. 4, Pl. IV.), we find G C in relative terms; and then, knowing the proportions of the axes, we proceed as directed in the 322d article.

343. But it is frequently better to determine the greatest equation from observation, and to deduce the excentricity from it: for the unavoidable errors in observing will less affect the excentricity by this method, than by any other. To determine the greatest equation by observation, the times must be ascertained when the planet is in the two points of its orbit, where its distance from the sun is a mean proportional between the two semi-axes (Art. 312.); this may be readily done, since the mean and actual velocities in the orbit are then equal. The mean motion will be found for any given time, by saying, *as the time of a revolution* : 360° : : *the given time : its corresponding mean motion*. Then, to know the true angular velocity for a very short time, observe at nearly equal intervals, as day by day, &c. three or four positions of the planet, so that the differences between each successive observation shall not exceed 8° or 9°: these successive differences should then be interpolated, consistently with the rules of the *differential method*, and thus the corresponding motions will be found for shorter intervals of time, and the errours of observation, by being distributed into a great number of parts, become nearly insensible: on-

which account interpolations are applied with singular advantage to most astronomical observations. Finding, by this kind of process, the times when the mean and real motions correspond, and the planet's places at those times, the *mean* motion for the intervening period may be determined, and the difference of the places is the *true* motion; half the difference between these motions will be the greatest equation.

Ex. M. *de la Caille*, from a series of observations on the planet mercury in 1740, calculated the greatest equation of its orbit in the following manner:—the mean daily velocity was found to be $4° 5' 32\frac{1}{2}''$. The differences between the observations on the 13th, 14th, 15th, and 16th of July, were $4° 14' 33''$, $4° 6' 28''$, $3° 58' 44''$; by interpolating these differences, he found the instants between which the real motion in 24 hours corresponded with the mean, to be July 14th at $2^h 49^m$, and July 15th at $2^h 49^m$; therefore, on July 14th at $14^h 49^m$ mercury's distance from the sun was a mean proportional between the semi-axes, and the equation of the centre was a *maximum*. By interpolating mercury's true places on the 14th, 15th, and 16th of July, his true place on July 14th at $14^h 49^m$ was found in $5^s 22° 57' 27''$. In like manner, by interpolating the differences between the observations of the 3d, 4th, 5th, and 6th of September, and mercury's true places on the 3d, 4th, and 5th, the time of the greatest equation was found, Sept. 3d, at $22^h 14\frac{1}{2}^m$, and the planet's true place $11^s 4° 49' 10''$. The difference of the true places is $5^s 11° 51' 43''$, mercury's actual motion; and the mean motion, in the intervening period of $52^d 7^h 25\frac{1}{2}^m$, is $6^s 29° 58' 33''$. Half the difference between these two motions is $24° 3' 25''$, mercury's greatest equation. *Cassini* from other observations deduced $24° 2' 58''$ for the greatest equation; Dr. *Halley* makes it $23° 42' 36''$; and M. *de la Lande*, $23° 40'$.

344. The greatest equation being known, the ex-

centricity may be nearly found by this analogy; *as* $57^\circ\ 17'\ 44\frac{4}{5}''$: *half the greatest equation* : : *radius* : *the excentricity*; for in orbits nearly circular, the place of the planet, at the time of greatest equation, will be very near an extremity of the conjugate axis; consequently, S T is almost coincident with S C, and nearly equal to it, and D C N + S N C is nearly equal to 2 D C N; that is, the difference between the true and mean anomalies will be nearly equal to 2 D N, or 2 S T, or 2 S C. In orbits of great excentricity S C (fig. 4, Pl. IV.), when thus found, may be corrected, by computing the greatest equation to this value of it, and then using this analogy, *as the computed equation* : *the approximate excentricity* : : *given greatest equation* : *the excentricity*. Should this be thought not sufficiently correct, the greatest equation may be computed to this also, and the analogy repeated.

345. The excentricities and greatest equations, as determined by M. *de la Lande*, are given below.

Planets.	Excentricities. Mean dist. 100000.	Great. Equations.
Mercury	7955·4	23° 40 0″
Venus	498·	0 47 20
Mars	14183·7	10 40 40
Jupiter	25013·3	5 30 38
Saturn	53640·42	6 26 42
Georgium Sidus . .	90804·	5 27 16

In consequence of the mutual attractions of the planets, the excentricities and greatest equations are subject to a variation; it is therefore necessary, at the end of a few years, to institute another set of

observations, from which these particulars may be again ascertained.

346. Prob. IV. *To find the position of the line of the nodes of a planet's orbit.*

To determine the place of either of a planet's nodes, and, consequently, the position of the line of the nodes, various methods have been invented by astronomers: a few of these are here inserted.

1st Method. When a planet is in one of its nodes, it has no latitude; therefore, either by means of observations for the right ascension and declination, or from the position of the planet with respect to known fixed stars, find its place when it has no latitude; this place, reduced to that which would have been seen from the sun (Art. 335.), gives the place of the node. This method, when it can be applied, is the most simple.

347. *2d Method.* Find when the heliocentric latitude on one side of the node is equal to any certain number of degrees, &c.; then, after the planet has passed the node, observe when the heliocentric latitude is equal to the former, but of a contrary name: the middle point between the heliocentric longitudes, at the times of the observations, is the place of the node. Or, if the heliocentric longitudes of a planet be observed at two distant times, when the latitudes are both equal, and of the same kind, the middle point between these longitudes will be 90° from the nodes.

348. *3d Method.* Observe the time and place of a conjunction or opposition of a planet with the sun, when the latitude of the planet is but small; then determine, by observation, the time when the planet has no latitude, and calculate, by the theory of the planet (determined in the preceding problems), the true arc which the planet has run through in its orbit, between the time of its opposition or conjunction, and that of its passage through the node; this arc, added to, or subtracted from, the place wherein

the conjunction or opposition was observed, will give the node's true place.

349. *4th Method.* Find the heliocentric latitudes and longitudes of the planet just before and after it has passed the node, and say, *as the sum of the two latitudes : the difference of longitudes :: either latitude : the distance of the node from the longitude corresponding to that latitude.* This rule is on the supposition that the portions of the orbit, and of the orbit reduced to the ecliptic, contained between the two observations, differ not sensibly from right lines; for, on this hypothesis, let P and Q (fig. 6, Pl. IV.) be the two places of the planet in its orbit, p and q the places reduced to the ecliptic, and N the node; then, by reason of the similarity of the triangles NPp, NQq, we have $Pp + Qq : pq :: Pp : pN :: Qq : qN$. But if the difference of longitudes should exceed two or three degrees, the conclusion deduced by this rule will not be sufficiently accurate; however, PNp and QNq may then be considered as similar spherical triangles, and we shall have tang. $pN = \frac{\text{sine } pq \times \text{tang. } Pp}{\text{tang. } Qq + \text{cos. } pq \times \text{tan. } Pp}$.

350. *5th Method.* The place of the line of the nodes may be found by two observations, when the planet is in the same node. In figs. 7, 8, adapted to the two cases of a superiour and an inferiour planet, let the places of the earth in its orbit be E and e, when the planet is observed in the same node P; then, by means of the observations, the time elapsed between them, and the theory of the earth, we know in the triangle ESe, the angle ESe, and the sides SE, Se, whence Ee, and the angles SEe, SeE, become known; and the angles SEP, SeP, being determined by observation, we shall have the angles PEe, PeE; therefore, in the triangle PeE the angles and the side Ee are known, to find PE, or Pe, the planet's distance from the earth at either observa-

tion. Lastly, in the triangle P E S we have the sides P E, E S, and the angle P E S, from which the distance, S P, of the sun from the planet in the node may be found, and the angle ESP determining the position of the line of the nodes; or, from the triangle P *e* S, we could, having similar data, determine S P and the angle *e* S P. And if the position of the line of the nodes, thus ascertained, be compared with the position as found by the observations of the ancients, the motion of the nodes may be determined.

351. The longitudes of the ascending nodes, and their secular motions, in respect of the equinoxes, are, according to the most careful observations, as stated below:

Planets.	Long. ☊ 1750.	Secular Motion.
Mercury	1s 15° 20′ 43″	1° 12′ 10″
Venus	2 14 26 18	51 40
Mars	1 17 38 38	46 40
Jupiter	3 7 55 32	59 30
Saturn	3 21 32 22	55 30
Georgium Sidus . .	2 12 47 in 1788	1 44 35

352. Prob. V. *To find the inclination of a planet's orbit to the plane of the ecliptic.*

1st *Method.* The place of either of a planet's nodes being found by the last problem, begin to make a series of observations on the planet when it is near 90° from the node; deduce from these observations the heliocentric latitudes, and let them be continued till the latitudes diminish sensibly; then the greatest of these latitudes will be equal to the inclination of the orbit.

353. *2d Method.* Or the inclination of a planet's orbit may be found in a nearly similar manner to that by which we determined the obliquity of the ecliptic (Art. 37.). Thus the place of a node being known, from an accurate observation, find the heliocentric longitude and latitude at a considerable distance from it; then, in fig. 9, Pl. I. supposing E Q to represent a portion of the ecliptic, E S a portion of the planet's orbit; the planet being in S, we have E Q the longitude from the node, Q S the latitude, in the right-angled triangle E Q S; hence, as sine E Q: tang. S Q : : radius : tang. Q E S, the inclination of the orbit. In this method the farther the observations are made from the node, the more accurate the conclusion will be.

354. *3d Method.* If the sun and the place of a planet's node are found in the same point of the ecliptic, at the time that the geocentric longitudes of the sun and planet differ three signs, or 90°; then, the earth being in the intersection of the planes of the ecliptic and the planet's orbit, the plane of the angle formed at the observer's eye, between two lines, one passing to the planet, the other to its projection on the ecliptic, is perpendicular to this intersection; and, consequently, the angle thus formed at the observer's eye, is equal to the inclination of the planet's orbit to the ecliptic.

355. *4th Method.* Hence we may deduce another method, as follows: Let N S E (fig. 11, Pl. IV.) be the line of a planet's nodes, S the sun, E the earth, P the planet's place in its orbit; then P L being drawn perpendicular to the plane of the ecliptic, and P R perpendicular to the line of the nodes, P E L will be the geocentric latitude, P R L the inclination of the plane of the orbit to the ecliptic, and R E L the difference between the longitudes of the sun and planet, or the planet's elongation from the sun. Now, in the right-angled triangle R E L we have radius : E L

: : sine R E L : R L ; also, in the right-angled triangle P E L, radius : E L : : tang. P E L : P L ; hence, R L : P L : : sine R E L : tang. P E L. But, in the right-angled triangle P R L we have R L : P L : : radius : tang. P R L ; consequently, sine R E L : tang. P E L : : radius : tang. P R L. Therefore, observe very carefully the longitude and latitude of the planet when the sun passes through its node, and use this analogy, *as the sine of the observed elongation of the planet from the sun : the tangent of its geocentric latitude : : radius : the tangent of the inclination of the orbit.*

356. The inclination of the planets' orbits, as well as the places of the nodes and aphelia, the excentricity, &c. are subject to a variation arising from a similar cause, the mutual attractions of the planets: these variations have not been accurately determined from observations, but M. *de la Grange* has, by theory, found them to be as follows, for 100 years: saturn — $23''\cdot 11$, jupiter — $27''\cdot 19$, mars + $3''\cdot 45$, venus + $4''\cdot 47$, mercury + $20''\cdot 43$ The inclinations of the orbits to the ecliptic are, according to the determination of M. *de la Lande* for the year 1780, mercury $7°$, venus $3° \; 23' \; 35''$, mars $1° \; 51'$, jupiter $1° \; 18' \; 56''$, saturn $2° \; 29' \; 50''$, and georgium sidus $46' \; 20''$.

357. Prob. VI. *To determine the absolute dimensions of the planet's orbits.*

In order to find the absolute dimensions of the orbits of all the planets, it is necessary to have an accurate estimate of the dimensions of one of those orbits; and, as the nature and magnitude of the earth's orbit are known pretty nearly, we are thus furnished with data which will enable us to ascertain the orbits of the other planets. Now, by the method, described in Art. 335. we may, from two successive observations on a planet when in the same point of its orbit, determine its distance from the sun, either

in terms of the earth's distance at each observation, or in absolute terms, as in English miles, for instance; and the distance of the planet's place from the line of the apsides being found by some of the preceding articles, the proportion of the axes being known, we have given, from the theory of the planet (by Art. 318. 320.), the proportion of S P to A Q or S G; whence, S P being known in absolute terms, we know S G also. Or, the absolute distance of the sun from the planet, when in one of its nodes, may be found by Art. 350. and thence, the theory of the planet being established, the actual length of either axis of its orbit.

358. Or the distance of any planet from the sun may be determined by a single observation, in a manner which may be thus explained: in figs. 9, 10, Pl. IV. suiting the respective cases of a superiour and inferiour planet, let the line of the nodes be N S, and let the planet P be observed from the earth T, with the apparent latitude P T B. Let the plane of the apparent latitude be produced till it cuts the plane of the planet's orbit in P N, and the plane of the ecliptic in B N. From P demit P B perpendicularly on N B, and from T erect T O also perpendicular to N B, which, because the plane of the latitude P N B is perpendicular to the ecliptic, will be perpendicular to the plane of the ecliptic also. From T let fall T E perpendicular to N S, and join O E, which is likewise perpendicular to N S; and the angle O E T will be equal to the inclination of the planes of the ecliptic and of the planet's orbit. In the triangle T S N are given S T and angle T S N, by the theory of the earth and the known place of the node; and by observation the angle N T S, the elongation of the planet from the sun (computed in the ecliptic), or its supplement; therefore T N, N S, and T N S, will become known. In the right-angled triangle T E N, T N and T N E are given, and consequently T E.

In the triangle O T E, right-angled at T, there are known T E, and T E O, the inclination of the planet's orbit (found by the last problem), whence O T becomes known. In the triangle O T N, O T and T N are known, and the right angle T, whence we find O N T. In the triangle P N T, we know N T, T N P, and P T N, the apparent latitude of the planet, or its supplement; hence P N will be known, and, if required, P T the planet's distance from the earth. In the triangle N P B we know N P, the angle P N B, and the right-angle N B P, whence P B and N B will be found. Then in the triangle B N S, N B, and N S, with the angle B N S being given, N S B the heliocentric longitude of the planet from its node, and the side S B, will be known. Lastly, in the triangle P B S, right-angled at B, we know P B and B S; from which we find P S B the planet's heliocentric latitude, and P S its distance from the sun.

359. The distance of a superiour planet having a satellite may be nearly determined, as in the following instance for jupiter. Let S (fig. 1, Pl. V.) be the sun, E the earth, I jupiter: first observe the instant in which the satellite R disappears behind jupiter, and the instant in which it again appears; then dividing the intermediate time into two equal parts, we have the instant in which the earth E, jupiter I, and the satellite R, are in one right line E I D. Next observe the instant in which the satellite disappears behind the shadow of jupiter, and the instant in which it again appears (Art. 496.), and divide the time between these two instants into two equal parts, to find the instant in which the satellite is in the midst of the shadow; that is, in which the sun, jupiter, and satellite, are in a right line S I R; hence, the time taken up in passing from D to R is known: and if the time (t) of the entire revolution of the satellite be known, we have $t : 360° : :$ the time occupied in passing from D to R : the arc D R nearly. Thus

the arc D R, and the angles R I D, E I S, are found. Lastly, find by observation the angle I E S of jupiter's elongation from the sun; then, two angles of the triangle E I S being known, the third is known likewise. But the side E S, the earth's distance from the sun at the time of observation, is known by the earth's theory; consequently, as sine E I S : E S : : sine I E S : I S the distance of jupiter from the sun. The distance, E I, of jupiter from the earth might readily be found, by means of the same triangle.

360. According to one or other of these methods the real magnitudes of the planets' orbits may be determined, and the mean distances (or half the major axes), as deduced by these means, when compared with those calculated by Kepler's famous analogy (Art. 209.) between the squares of the periodical times, and the cubes of the mean distances, are found to correspond as accurately as can possibly be expected, when the unavoidable errours in observing are taken into consideration. The same results very nearly being deduced from operations in great measure, and, in some instances, altogether independent on each other, is a strong confirmation of the truth of the *Copernican System*, with the improvements of *Kepler* and *Newton*; and when, in addition to this, it is remarked, that the places and motions of the planets, as calculated from the theory of them, which is now adopted, are found to agree, we may almost say with *wonderful* minuteness, with the places actually observed, the whole demands so forcibly the assent of the mind to this system, that we think such assent can scarcely be withholden by any who have the least pretension to soundness of judgment and freedom from prejudice.

The mean distances of the planets from the sun are, in round numbers, nearly as follows: mercury, 37 millions of miles; venus, 68 millions; the earth, 95 millions, as before stated; mars, 144 millions;

jupiter, 490 millions; saturn, 900 millions; and georgium sidus, 1800 millions. Or, if these distances be required more accurately, the mean distance of the earth being known, they may be found by the following numbers, which have the same relation as the respective mean distances, 38710, 72333, 100000, 152369, 520279, 954072, and 1908352.

361. PROB. VII. *To explain the process in calculating the longitude, latitude, and other requisites, of a planet seen from the sun, and from the earth, for a given instant.*

As it may be useful to throw together in one view a general exposition of the process used in calculating the planet's places, we lay before the reader, in the following fifteen articles, the rules given by M. *de la Caille* for this purpose.

362. RULE I. Reduce the given time to mean time (Art. 117.).

363. II. Take the interval between the given instant reduced to mean time, and the instant of the epocha of the earth's preceding passage through its aphelion: then say, as the time of the earth's whole revolution, is to the interval found; so are 360°, to the earth's mean anomaly.

364. III. Reduce the mean anomaly to the true anomaly (Art. 321.), which, added to the earth's true aphelion place, gives the earth's true heliocentric place in some point of its orbit, as in T (figs. 12, 13, Pl. IV.), or reduced to the heavens in H; so that the earth's longitude will be the arc ♈ D H of the ecliptic.

365. IV. Adding six signs, or 180°, to the earth's true place, we have the true place of the sun seen from the earth at the point O of the ecliptic, so that the sun's geocentric longitude will be the arc of the ecliptic ♈ D H N O.

366. V. Compute (Art. 321.) the distance S T of the sun from the earth.

367. VI. For the planet, say, as the time of its annual revolution, is to the interval of time between the instant given and the epocha of its preceding passage through the aphelion; so are 360°, to the planet's mean anomaly.

368. VII. Reduce the mean anomaly to the true anomaly, and add it to the place of the planet's aphelion, the sum is the planet's true place as seen from the sun, either at P in the orbit, or *p* in the heavens; so that the planet's heliocentric longitude in its orbit is the arc ♈ D *p*.

369. VIII. Calculate the distance S P of the planet from the sun, and reduce the distances S T, S P, to one measure, such as that of which the earth's mean distance from the sun contains 100000 equal parts.

370. IX. The heliocentric longitude ♈ D of the ascending node, taken from ♈ D *p* the heliocentric longitude of the planet in its orbit, leaves D *p* the argument of latitude.

371. X. Say, as radius, to the sine of the inclination of the planet's orbit; so is the sine of the argument of latitude D *p*, to the sine of the heliocentric latitude *p l*.

372. XI. Reduce the argument of latitude D *p* to D *l* the true distance from the place of the node to the point whereon the place of the planet is projected, by this analogy, as radius, to the co-sine of the inclination of the orbit; so is the tangent of the argument of latitude D *p*, to the tangent of the reduced distance D *l*.

373. XII. The distance D *l*, added to the longitude ♈ D of the ascending node, gives ♈ D *l* the heliocentric longitude of the planet.

374. XIII. Reduce S P the distance found (Art. 369.) to S L the curtate distance, by computing the side S L of the rectiline triangle S P L, whereof S P is the hypothenuse, and the angle P S L equal to the planet's latitude: thus, as radius, to the co-sine of the

planet's latitude seen from the sun; so is the distance S P, to the curtate distance S L.

375. XIV. Take the difference between ♈ D H the heliocentric longitude of the earth, and ♈ D L the reduced heliocentric longitude of the planet; and in the right-lined triangle S L T there are given the angle T S L, with the including sides T S, S L; calculate the angle S T M or O T M between O the place of the sun seen from the earth, and M the place of the planet seen from the earth, and the geocentric longitude ♈ D M of the planet, and even its distance T P or T L, may be easily found.

376. XV. Lastly, say, as the sine of the angle T S L at the sun, to the sine of the angle L T S at the earth; so is the tangent of the heliocentric latitude P S L, to the tangent of the geocentric latitude P T L.

All the foregoing rules are either evident consequences of what has been previously demonstrated, or are readily deduced by an application of the principles of plane and spherical trigonometry, to the respective triangles in the figures referred to.

377. These calculations being very tedious, and requiring a considerable portion of care and patience to perform them accurately, astronomers have constructed tables, by the assistance of which much of this labour is saved, and the place of a planet found with comparatively little trouble. It is unnecessary to explain the different methods which are resorted to, in order to facilitate the computation of such tables; nor is it necessary to give precepts for the use of them when computed, since they are generally accompanied by proper directions. The tables of the planets which appear to be most useful, are: the mean motions in current years; the mean motions in Julian years; the mean motions in months; the mean motions in days; and the same in hours, minutes, and seconds: the equations of the centre to different degrees of mean anomaly; the logarithms

of the distances from the sun to different degrees of mean anomaly; the inclination of the orbit, its reduction to the ecliptic; and the logarithm of curtation, calculated to the differences between the heliocentric longitude, and the longitude of the ascending node. A very useful set of tables of this kind, together with tables of the mean motions of the sun and moon, and some proper subsidiary ones, are given in small compass by Mr. *Alexander Ewing*, at the end of his *Practical Astronomy*. It is, however, to be lamented, that a complete and extensive set of tables, agreeably to the late improvements in the science, is still a *desideratum*.

378. SCHOLIUM. It would be unpardonable to close a chapter on the determination of the orbits of the planets, without taking specific notice of a discovery which does so much honour to the present age; we allude to that of the GEORGIAN PLANET, or URANIUS, by Dr. *Herschel*, a gentleman whose character for persevering diligence and skill will ever rank highly in the annals of science. It is needless to enter into the history of this brilliant discovery; but, as it led to various investigations and calculations by different astronomers, to determine the elements of this planet's orbit, and as an account of some one of these methods may tend to illustrate different problems in this and the preceding chapters, a concise description of the investigation pursued by Mr. *Robison*, professor of natural philosophy in the university of Edinburgh, is here added. Mr. *Robison* founded his investigation upon observations of the times of five successive oppositions, the first happening on Dec. 21, 1781, and the last on Jan. 8, 1786; for, from the heliocentric longitudes and latitudes at those times he determined the place of the node, and the inclination of the orbit; and thence he found the arcs described in the orbit between the respective oppositions. Then he took the time of the third

opposition, which was Dec. 31, 1783, as an epocha to which the other observations were to be reduced, and fixing upon four equal intervals of time, two before and two after this epocha, he calculated the planet's places in its orbit for the terms of these intervals; but, upon comparing their differences, found that there were some irregularities inconsistent with the motion of a body in an ellipsis: these he naturally ascribed to some inaccuracy in the observations, and as they might lead to great errours in ascertaining the elements of the orbit, he thought it necessary to apply a correction. This was done very ingeniously, by finding, as nearly as circumstances would admit of, the part of the orbit in which the planet was at the times of the observations, and then by determining the law followed by the first and second differences of the angles described in equal times by other planets in similar parts of their orbits, which it was fair to conclude would be the law of the differences of the new planet. Now he found that in similar parts of the other planets' orbits, in equal intervals of time the first differences decreased very slowly, and the second differences increased very slowly; he therefore gave to the first differences of the new planet's motions a small diminution, and to the second a minute increase, and this without altering any of the longitudes more than $3''$, which must be allowed to be within probable limits. In this manner he obtained the times corresponding to the above-mentioned equal intervals, with their corrected longitudes, and thence deduced four intercepted arcs, with their first and second differences; from which he thus constructed the orbit:—

379. Let A C P (fig. 14, Pl. IV.) be the orbit, P the perihelion, S the focus, A, B, C, D, E, the places of the planet at the five oppositions; and suppose the chords and radii drawn. Now, if we conceive the chords A C, C E, to be bisected by the radii S B, S D, in x and g, then by Euc. I. 38. the triangle A S x =

C S x, and the triangle B x C = B x A; and in proportion as the ellipsis approaches to a circle, S x becomes nearer a perpendicular to C A; consequently, the chords B C, B A, and the two segments, will be very nearly equal; and these segments being extremely small compared with the triangles C S B, A S B, the sectors C S B, A S B, will be very nearly equal; whence the times from A to B, and from B to C, may be considered as equal, without any material errour, and therefore B will be the place of the planet at the second observation. In like manner it may be shewn that D will be the place at the fourth observation. Let the given angles be denoted thus: A S B by u, B S C by v, C S D by x, D S E by y, A S E by w, and C S E by z; then A S : A x : : sine A x S : sine u; also C x or A x : C S : : sine v : sine C x S, or A x S; hence, A S : C S : : sine v : sine u. In a similar way it may be proved, that E S : C S : : sine x : sine y. Hence we know the ratio of A S, C S, and E S, and the angles between them; consequently, the species and position of the ellipsis may be found as in Articles 310. or 338. 342.

380. Although the errour arising from considering the chords as bisected, is so very minute that it might safely be neglected; yet, as the greatest accuracy is always desirable in mathematical enquiries, Mr. *Robison* has given the following method of correction: Bisect A E in F, A C in H, C E in G, and draw S H b, S F c, S G d, O F k, O G v, O being the centre of the ellipsis. Since the angles k O v, c S d, are very small, the triangles c F k, d G v, are nearly similar; and c F, d G, being considered as versed sines, they will be very nearly as the squares of the chords; hence the area c F k : d G v : : $c\,F^2$: $d\,G^2$: : A E : C E. Now by the property of the ellipsis, the area E F k = A k F, also E F S = A F S; consequently, S F k E = A k F S; add F k c to each, and S c E = A k F S + F k c = S c A, + 2 F k c; therefore S c E − S c A = 2 F k c: but as S C E = S C A,

therefore S *c* E — S *c* A = 2 S C *c*, and of course F *k c* = S C *c*. For similar reasons, *d* G *v* = S D *d*; but since S D, S C, are nearly equal, C *c* : D *d* : : area S C *c* : S D *d* : : F *k c* : *d* G *v* : : A E^4 : C E^4. And as the arcs A C, C E, are very nearly equal, therefore *t* F = C *c*, and G *g* = D *d*, very nearly; also A E : C E : : 2 : 1, very nearly; and consequently *t* F : G *g* : : 16 : 1 nearly. And in the same way it might be shewn that *t* F : H *x* : : 16 : 1.

Now, C S : C *g* : : sine *g* : sine *x*
C *g* : E *g* : : C *g* : E *g*
Also, E *g* : E S : : sine *y* : sine *g*
∴ C S : E S : : C *g* × sine *y* : E *g* × sine *x*.
Let C S : *e* S : : sine *y* : sine *x*
Then E S : *e* S : : C *g* : E *g*
and E S : E *e* : : E *g* : C *g* — E *g* : : E *g* : 2 G *g*.

In like manner make C S : *a* S : : sine *w* : sine *v*, then A S : A *a* : : A *x* : 2 *x* H : : E *g* : 2 G *g*, and E *e* : A *a* : : E S : A S nearly; but E S = A S nearly, hence E *e* = A *a* nearly.

Assume A S : S *o* : : sine *z* : sine *w*
but S E : A S : : E *t* × sine *w* : A *t* × sine *z*
∴ S E : S *o* : : E *t* : A *t*.
And S E : E *o* : : E *t* : A *t* — E *t* : : A *t* : 2 *t* F nearly
or S E : E *o* : : 2 E *g* : 32 *g* G : : E *g* : 16 G *g* nearly
but E *e* : S *e* : : 2 G *g* : : E *g*
hence E *o* = 8 E *e* nearly.

Make A S : S *i* : : sine *z* : sine *w*; then *a* S : S *i* : : A S : S *o*, and A *a* : *io* : : A S : S E ∴ *io* = A *a* or E *e* nearly, and *e i* = 6 E *e* nearly.

Hence, for approximating to the true ratios of A S and G S to C S,

Make C S : *a* S : : sine *w* : sine *v*
C S : *e* S : : sine *y* : sine *x*
e S : *r* S : : sine *w* : sine *z*
a S : *i* S : : sine *z* : sine *w*

Make $AS = aS + \frac{ar}{6}$, and $ES = eS - \frac{ei}{6}$. Then the points A, C, E, will be in the circumference of an ellipsis, whose focus is S, centre O, and having the sectors A S C, C S E, nearly equal. The approximation will be much easier, and almost as accurate, by adding $\frac{1}{6}$ (log. a S — log. r S) to log. a S, for log. A S, and subtracting $\frac{1}{6}$ (log. i S — log. e S) from log. e S, for log. E S. Or, by adding $\frac{1}{6}$ (log. e S — log. i S) to log. a S, and subtracting it from log. e S. Mr. *Robison* calculated the aphelion and perihelion distances, by finding the value of $1 + e$ and $1 - e$ in Arts. 338. 342. : and by their means obtained the mean anomalies corresponding to the true anomalies O S A, O S E. Then, *the difference of these two mean anomalies* : 360 : : *the time between the appulses of the planet to the points* A *and* E : *the time of a sidereal revolution.* And, *the square of a sidereal year* : *the square of a sidereal revolution* : : 1 : *to the cube of the planet's mean distance from the sun.* From this process were deduced the following elements :

Mean distance	. . .	19·08247
Excentricity	. . .	0·9006
Periodic time	. . .	83·359 years.
Mean anomaly at E	. .	$4^s\ 0^\circ\ 32'\ 51''$
Longitude of aphelion	for epocha, Decem. 31, 1783	11 23 9 51
Long. of node		2 12 46 14
Inclination of the orbit	. .	— — 46 14

Those who wish to see the whole of Mr. *Robison's* ingenious paper, are referred to vol. I. of the *Edinburgh Transactions.*

381. These elements do not differ widely from those deduced from the investigations of other astronomers : *Mechain* made the semi-axis major, 19·07904; excentricity, ·82034; inclination of orbit, 43′ 35″. *Hennert*, semi-axis major, 19·02766; greatest equa-

tion, 4° 53′ 16″; inclination of orbit, 42′ 13″. *De la Place*, semi-axis major, 19·0818; excentricity in seconds, 9815″; inclination of orbit, 46′ 12″. Dr. *Zach*, semi-axis minor, 19·0602; excentricity, ·90804; annual heliocentric motion in orbit, 4° 18′ 57″·148; inclination of orbit, 46′ 12″.

382. This new planet has been distinguished by different names. Dr. *Herschel*, in honour of his present majesty, called it *georgium sidus*: the French, and indeed most English astronomers, call it *herschel*, in honour of the discoverer: other names have been proposed, such as *neptune*, *cybele*, *astrea*, &c. *Bode* gave it the name of *ouranus*, who was the father of saturn, as saturn was of jupiter. Different marks have been proposed for it. In France, $\frac{1+1}{0}$; in England, ♅. *Koehler* proposed the sign of the new metal platina del pinto; which sign, with the name *ouranus*, have been adopted by the Academy royal of Prussia, and introduced into its astronomical ephemerides. An interesting history of the discovery was presented to the *Brussels Academy of Sciences*, in May, 1785, by Dr. *Zach*, and is inserted in the first volume of the Memoirs of that academy. And a curious paper on the secular variations of the elements of this planet, by M. *du Val le Roi*, may be found in the *Memoirs of the Berlin Academy of Sciences*, from 1786, to the end of 1787; published in 1792.

⁂ Although the nineteenth century is yet in its infancy, a circumstance of such importance in the history of science has already taken place, as induces us to hope that this century also will pay its proportional tribute to the fund of human knowledge. On the 1st of January, 1801, M. PIAZZI, astronomer royal, at *Palermo*, in Sicily, discovered *another* PRIMARY *planet!* In order to preserve the honour of this discovery, as well as the observations, to himself, he kept it secret, till on the 11th of February he was obliged by illness to discontinue his observations. This

celestial phenomenon is an intermediate planet between the orbits of mars and jupiter, and appears as a star of the eighth magnitude, being probably about the size of the moon. Its distance from the sun is about $2\frac{3}{5}$ times that of the earth, and its periodical time nearly four years and two months. At the present time (August 25), it is very near the star ξ ♋; and will soon be in a good situation to be observed. . . . . . . Since the arc of its orbit through which this planet ran during the period it was observed by *Piazzi* was but small, no great degree of accuracy can be expected in stating the elements of its theory; the following, however, are the most exact yet known:

" Place of the ascending node . . .	$2^s\ 20^\circ\ 58'\ 30''$
" Inclination of the orbit . . .	10 47 0
" Place of the aphelium . . .	2 8 59 37
" Time of the passage through the aphelium, January, 1801 . . .	1·3328
" Excentricity	0·0364
" Log. of the greater semi-axis . .	0·4106386
" Time of the sidereal period . .	4·13 years."

The authour begs to return his sincerest thanks to his learned and excellent friend Dr. *Hutton* of Woolwich, for communicating to him a paper, whence the above particulars were extracted.

Mr. *Maclaurin*, and other philosophers, expected, nearly one hundred years ago, that such a discovery as this of M. *Piazzi's* would be made by some diligent astronomer: and the opinion has been lately revived by Mr. *Capel Lofft*, a gentleman well known for his attachment both to the sciences and the muses. In the *New London Review*, for March, 1800, this gentleman, in a critique on the Athenian letters, ventured to offer some conjectures respecting an intermediate planet between mars and jupiter, the coincidence of which with the new discovery is very remarkable. He supposed that the distance of the intermediate planet from the sun, would be to that of mars, either as thirty-three to fifteen, or as twenty to fifteen; the midway between which, corresponds nearly with the fact: with respect to its diameter, he conceived it might be to that of mars, as that of mars to the diameter of the earth; and then, being not much more than half the diameter of mars, and at five times the perigean distance, it would be seen from the earth under an angle of $2\frac{1}{2}''$ or $3''$; while georgium sidus would appear under an angle of $4''$. These lucky conjectures were drawn from a certain kind of *Pythagorean harmony*, and are ingeniously defended in the Review just mentioned.

CHAPTER XIII.

On the apparent and real Diameters of the Sun and Planets.

ART. 383. IT is obvious from the principles of optics, that the determination of the real diameters of the heavenly bodies will depend conjointly on a knowledge of their apparent diameters, and their real distances from the earth. For, suppose A P B (fig. 2, Pl. V.) a section of such body made by a plane passing through the place O of the eye, and the centre C of the body: then A O B will be the angle, which will measure the apparent diameter of the body; this being known, A O C, its half, will be known. And A O being a tangent to the surface, the angle C A O will be a right angle; whence, co-sine A O C : sine A O C : : A O : A C, the semidiameter of the body. Now A O or O C may be found by articles 358. 359. 375.; or, when the body is in opposition or conjunction with the sun, by taking the difference, or sum of its distance from the sun, and the earth's distance from that luminary, according to the respective cases; taking care to attend to the different operations required for a superiour and inferiour planet. And as to the apparent diameters, it may be worth while to point out a few methods of ascertaining them.

384. The sun's vertical or perpendicular diameter may be found by two observers taking, the one, the

height of the upper edge of the disc, the other, that of the lower, at the same instant. This is most conveniently done when the sun is at or near the meridian; because there is then no sensible change in his altitude during the space of two or three minutes. The height of each edge must be corrected, by allowing for parallax and refraction; and the apparent diameter will be equal to the difference of the corrected altitudes of the upper and lower edge. This method is very simple, and gives the apparent diameter, with exactness proportional to the accuracy of the instruments made use of.

385. Another method of determining the sun's apparent diameter, is to observe by a good clock the time in which the sun's disc passes over the plane of the meridian, or some other hour circle. At, or very near, one of the equinoxes, when the sun's apparent diurnal motion is in the equator, or a parallel very near it, *say, as the time between the sun's leaving the meridian, and returning to it again* : 360° : : *the time in which he transits the meridian* : *his apparent semidiameter*. At any other time of the year, when the sun is in a parallel at some distance from the equator, his diameter measures a greater number of minutes and seconds in that parallel, than it would do in a great circle (Art. 150.), and takes up proportionally more time in passing over the meridian; we may then use this analogy, *as radius* : *co-sine of the sun's declination* : : *the time in which the sun transits the meridian, converted into motion, at the rate of four minutes in time to* 1°. : *the arc of the great circle which measures the sun's apparent horizontal diameter*. This method may be easily put in practice by two observers; or indeed by one, if he have an half-second pendulum placed near enough for him to hear the beats of it, whilst he observes the transit.

386. Or the sun's apparent diameter may be measured by means of the projection of his image; thus,

let the sun's rays be admitted into a dark room, through a circular aperture *c d*, the diameter of which is accurately known; the rays passing perpendicularly through this orifice, will fall perpendicularly on a parallel plane (whose distance *i s* from the former must also be known), and form an image, the diameter of which is *a b* (fig. 3, Pl. V.). Now, the diameter *c d* being taken from *a b*, leaves *r r*, half of which is *s r*: hence, in the right-angled triangle *i s r* we know *i s*, and *s r*, and from thence may determine the angle *s i r*, which will be equal to the apparent semidiameter of the sun. According to some of these methods the sun's diameter has been measured, and may be stated at a mean to be $32' \, 1\frac{1}{2}''$. The greatest and least diameters have been given before (Art. 304.).

387. To determine the apparent diameter of a planet, one way is to place between it and the eye a thin plate of metal with a small circular orifice in it, at such a distance from the eye that the entire disc of the planet may appear exactly to fill the hole; and then, by measuring the diameter of the hole and the distance of the plate from the eye, to determine by trigonometry the visual angle subtended by the diameter of the aperture, which will evidently be the same as the apparent diameter of the planet. Another method similar to this is, to suspend a thread or wire of known diameter, at such a distance from the eye that it may just hide the disc of the planet, and then, after measuring the distance of the thread from the eye, determine trigonometrically the visual angle. A third method is, to measure the picture of a planet cast through a telescope upon white paper in a dark room, and compare it with that of the sun or moon projected in the same manner. But to mention no more such uncertain methods, the diameters of the planets are best taken by the micrometer, an instrument so contrived that two parallel wires being

placed in the focus of a telescope, one fixed and the other moveable, or both moveable, they may be made to approach or recede one from the other till they appear to touch exactly two opposite points in the disc of the planet, and then the index shews the apparent diameter in minutes and seconds. The apparent diameters of the planets when at about their respective mean distances from the earth, are as follow: mercury, 11″; venus, 58″; mars, 27″; jupiter, 39″; saturn, 18″; georgium sidus, 3″ 54‴. And from these apparent diameters, and the respective distances from the earth, the diameters of the sun and planets have been determined in English miles as here stated: mercury, 3224; venus, 7687; mars, 4189; jupiter, 89170; saturn, 79042; georgium sidus, 35112; the sun, 883246. Observations upon the planets herschel, saturn, jupiter, and mars, prove that there is a sensible difference between their equatoreal and polar diameters; and it is probable that there is a like difference between the diameters of the other planets, but this has not yet been determined by observation.

388. Since the apparent diameters of distant bodies vary inversely as their distances, we may, having the distances from the earth at which the respective planets subtended the above angles, and knowing their mean distances from the sun, find the mean apparent diameters of all the planets, as seen from the sun; they have been thus given: mercury, 20″; venus, 30″; earth, 17″; mars, 10″; jupiter, 37″; Saturn, 16″; georgium sidus, 4″.

389. To measure the quantity of matter in distant bodies appears a problem of insuperable difficulty: but this has been effected to a considerable degree, by the principles of the Newtonian philosophy. Since the quantity of matter in a globe is proportional to the mean density multiplied into the cube of the diameter, and the diameters of the pla-

nets are known, the mean densities are all that are required for the solution of the problem. Now, in homogeneous, unequal, spherical bodies, the gravities on their surfaces are as their diameters, when the densities are equal; or the gravities are as the densities, when the bulks are equal: therefore, *in spheres of unequal magnitude and density, the gravity is in the compound ratio of the diameters and densities; or the densities are as the gravities divided by the diameters.* But the diameters are known, and the gravities at the surface are nearly found, either by means of the revolutions of the satellites, or by calculations deduced from the effects the planets are found to produce upon each other; consequently the relation of the densities becomes known. The mean density of the earth was calculated by Dr. *Hutton*, from observations made by Dr. *Maskelyne* at the mountain *Schehallien*; he made it to that of water as 9 to 2, and to common stone, as 9 to 5, on the supposition that the hill is only of the density of common stone. He also states the mean densities of the sun and planets to that of water, thus: sun, $1\frac{2}{15}$; mercury, $9\frac{1}{6}$; venus, $5\frac{11}{15}$; earth, $4\frac{1}{2}$; mars, $3\frac{2}{7}$; jupiter, $1\frac{1}{24}$; saturn, $0\frac{3}{32}$; and georgium sidus, $0\frac{99}{100}$.* These densities are such as the bodies would have if they were homogeneal; and may be admitted as a fair estimate of the whole, although the density of each planet may vary considerably at different distances from the surface. From the densities, as thus estimated, and the known diameters, we may readily find the proportions of the quantities of matter; they are as under: the sun, 333928; mercury, ·1654; venus, ·8899; the earth, 1; mars, ·0875; jupiter, 312·1; saturn, 97·76; georgium sidus, 16·84.

CHAPTER XIV.

On the Rotations of the Sun and Planets, and the Inclinations of their axes.

ART. 390. BESIDES that motion of the planets by which they move round the sun in elliptical orbits, most of them, and from analogy we conclude all of them, have another motion, which causes them to turn round on their axes: this motion is called their *rotation*. After attentive observations on the planets, and indeed on the sun also, it will be found that there are dark spots on their discs, of different shapes and sizes; it will be seen likewise, that these spots change their apparent places, and pass from one side of the body to the other; that, after disappearing for a rather longer time than they were visible, they again become visible on the side where they were first seen, and again pass over the disc, as before; that their apparent velocities continue to increase till they arrive at the middle of the disc, after which they gradually decrease, and are slowest just before they are hidden from view; that these spots appear to widen as they advance from the margin towards the middle of the disc, where they appear most round and broad, and as they approach the edge they become oblong and slender: from these observations it is concluded, that the spots adhere in some manner to the sun and planets, and that their apparent motion and change of magnitude are occa-

sioned by the real motion of the bodies on which they are.

391. The physical nature and cause of these motions, or, as they are called, *diurnal rotations*, will be in some degree evinced from the following considerations: If a sphere of homogeneous matter, placed in an unresisting medium, receive at the same time one or several impulses in the plane of the same great circle, and in directions oblique to its surface; these impulses will give that sphere two motions, the one a uniform rotation about the axis of that great circle, which preserves a constant situation; the other, an equally uniform translation in the plane of that great circle. For, let E B N (fig. 4, Pl. V.) be the plane of a great circle of a sphere (which call the *plane of its equator*), and let A B represent the direction and action of a power in that plane: it is manifest, that if A B had been in the direction of one of the axes of the globe, and consequently perpendicular to the surface, the action of that power would have caused the globe to move uniformly in that direction; but A B being oblique to its surface, it follows, from the composition and resolution of forces, that its effort must be divided into two, the one B O perpendicular to its surface, the other B H a tangent thereto; so that the two right lines B O, B H, become sides of a parallelogram H O, the diagonal of which is B I = A B. Now B O expresses that part of the effort A B which impels the globe uniformly in the direction B L, which is in the plane of the equator (Art. 267.); and B H expresses the part of the effort A B that imparts to the point B of the globe's surface a tendency to recede from the centre C along the tangent B H. This effort B H, then, is like a uniform projectile force impelling B towards H. But, because of the connection and reaction of all the parts of the globe, the point B returns as much towards the centre C,

as the effort B H tends to make it depart from it: therefore the point B must turn in a circular uniform manner about the centre C, or, which amounts to the same, about the axis passing through C, perpendicular to the plane of E B N; hence the action of the power A B causes the globe to turn uniformly about its axis perpendicular to the plane of the equator, and at the same time to advance uniformly in the direction of the same plane. But if the globe in this situation receive another impulse, expressed both with regard to force and direction by D E, in the plane of the same equator; this new force must also be divided into two, the one E P impelling the globe uniformly in the direction E K with a velocity expressed by E P; the other E F tending to make the globe turn uniformly about the axis of that equator. Now, making C K = E P, and C L = B O, the globe is urged by two uniform forces of translation, expressed by C K, C L: therefore it will move uniformly through the diagonal C M, which is in the plane of the equator; turning at the same time uniformly on the axis of the equator with a motion imparted by the force E F + B H, expressing the sum of E F, B H, because those forces conspire to produce the same effect, and are situated in the same manner with respect to the surface of the globe. Hence it appears that the more oblique the direction of the acting power is, that is, the more the angle A B H exceeds a right angle, the swifter is the motion of rotation in proportion to that of translation.

392. It may be easily shewn, that the different impulses of a central force acting on a globe that already has a motion of rotation, and a uniform motion of translation, affect only the latter, in whatsoever plane of a great circle the central force acts: thus, if a globe P I Q K (fig. 5, Pl. V.), while revolving on its axis P Q, should be impelled by a central force in any

direction C S passing through its centre, and consequently perpendicular to its surface, it is evident that its whole effect on the two equal hemispheres I Q K, K P I, whereon it acts equally, is only to direct them towards the point S. Therefore that force must necessarily accelerate or retard the motion of translation, according as it conspires with it, or acts contrary to it, and may continually change its direction and cause the globe to describe a curve without altering the motion of rotation, which continues uniform in the same plane about the same axis situated as before. Hence it appears that a uniform projectile force, and a constant central force, are all that are necessary to account for both the diurnal and annual revolutions of the planets; it also appears, that the inequalities of the annual motions do not prevent or impede the uniformity of the diurnal motions; these inequalities arising only (Art. 281.) from a central force varying inversely as the squares of the distances.

393. To determine the periods in which the diurnal rotations of the sun and planets are performed, observations must be made upon the spots on their respective surfaces: and the times of rotation may be found, either by observing the arc of the small circle described by any spot in a given time, and then by proportion finding how long time would be occupied in describing the whole circle; or, by observing the return of a spot to the same position with respect to the earth. The position of any one spot, observed at three separate times, will determine the position of the great or less circle in which it moves, and thence the position of the axis. The method of determining these particulars may be exemplified, by pointing out the steps which have been pursued to ascertain the time of the sun's rotation, and the inclination of its axis. Here, in the first place, it is necessary to find, from the right ascension and declination of a spot on

the sun, also the right ascension and declination of the sun's centre, the *geocentric longitude and latitude of the spot*, and thence its heliocentric position. To perform this, suppose E C (fig. 6, Pl. V.) to be the ecliptic, E Q the equinoctial, S the centre of the sun's disc B D, *e* a spot thereon: then drawing *o* S and *i a* secondaries to the equator, and S *n* parallel to it; also letting fall *e u* perpendicular to the ecliptic, and joining S *e*, we know *i o* the observed difference of the right ascensions of the spot and sun's centre, and *e a* the difference of their declinations; and from these we are to find *e u* the latitude of the spot, and S *u* the difference between its longitude and that of the sun's centre. Now it is evident, from the principles of spherics, that S *a* = *o i* × co-sine S *o*; wherefore, in the triangle S *e a*, right-angled at *a*, we know S *a* and *e a*, from which S *e* and the other angles are found. Also, in the triangle E S *o*, right-angled at *o*, E *o* and *o* S are known, the one being the sun's R. A., the other his declination, from which, either with or without the angle S E *o* the obliquity of the ecliptic, we find the angle *o* S E, and this taken from the right angle *o* S *n*, leaves *u* S *a*; but *e* S *a* is known from the triangle S *e a*, consequently *e* S *u* is known. Therefore, in the triangle *e* S *u*, right-angled at *u*, we know S *e* and the angle *e* S *u*; from which we find S *u*, the difference of the geocentric longitudes, and *e u* the difference of the geocentric latitudes; and thus may the position of a spot be ascertained at any time, as seen from the earth. But we must find the position *as seen from the centre of the sun*; in order to which, let B *b* D *m* (fig. 7, Pl. V.) be the projection of the sun's disc, S its centre, *e* the spot on its surface, and B D the ecliptic; then *e l* is the observed geocentric latitude of the spot, and *l* S the difference between its longitude and that of the sun's centre, corresponding with *e u* and S *u* in fig. 6. Now the arc of a great circle between S, the point on

the sun's surface where the earth is vertical, and e the place of the spot, is projected into its sine $S e$. To find this arc, suppose E the place of the earth, $E b$ a tangent to the sun's disc; then $a b$ will be the apparent semidiameter, and $e d$ being drawn parallel to $l D$, $c d$ will be the apparent distance of the spot from the centre: the triangle $S E b$ is right-angled at b, consequently the angle $b S E$, or the arc $b D$ which measures it, is the complement of the sun's apparent semidiameter; and therefore, as $a b$ the observed semidiameter : the sine of the arc $b D :: c d$, the observed distance of the spot from the centre : the sine of the arc $d D$; that is, the arc whose sine is $S e$. The angle $e S l$ in the projection is evidently equal to that formed by the great circles upon the surface of the sun; and therefore, as this angle may be easily computed by the rectiline triangle $e S l$, by adding $m S l = 90^\circ$ we get $m S e$, m being the pole of the ecliptic on the sun's surface: we also have $m S = 90^\circ$, and $S e$ the arc found by the above analogy; whence, knowing two sides of a spherical triangle $m S e$, and the angle they include, we find $m e$ the third side, and the angle $S m e$: but the difference between $m e$ and $m v$ ($= 90^\circ$) is $v e$ the heliocentric latitude of e; and S being the point on the disc to which the earth is vertical, its longitude seen from the sun's centre is the same as that of the earth (*i. e.* differing by six signs from the longitude of the sun's centre at the earth), it is therefore known; and being increased or lessened by the angle $S m e$, according as l is to the east or west of S, we have the longitude of e seen from the sun's centre. After a similar manner we may determine the heliocentric position of any spot.

394. The heliocentric position of the same spot being known at three separate times, we may from thence find *the inclination of the sun's axis, or its equator, to the ecliptic, and the ascending node of*

the equator: thus, let Q n R N (fig. 9, Pl. V.) be the sun's equator, P its pole, E S N the ecliptic, p its pole on the sun's surface, N and n the nodes of the sun's equator; through p P describe the great circle p P t, which passing through the poles of the ecliptic and solar equator, must intersect each at the distance of 90° from either node N or n; therefore, when the longitude of P or t is found, that of N or n becomes known. Now, by the method explained in the last article, pM, p N, p O, will be found; as also M p N, N p O: and in the triangles, P p M, P p N, P p O, P O, P N, P M, are equal to each other, and P p common; therefore two sides of each triangle are constant, and the third varies by a finite quantity, in which case M. *Cagnoli* gives the following solution*. Taking any two of these triangles, P p M, P p N, he proves that, $\overline{\text{sine } \tfrac{1}{2} \times \overline{pN - pM}}$: $\overline{\text{tang. } \tfrac{1}{2} \text{M}p\text{N}}$: : sine $\tfrac{1}{2} \times \overline{Np + Mp}$: co-tan. $\tfrac{1}{2} \times \overline{PMp + PNp}$; in like manner, from the triangles P p N, P p O, we get $\overline{PNp + POp}$. Therefore, if we put L, L′, L″, for the observed longitudes of the spot at M, N, O, and D, D′, D″, for $\overline{PM}$, $\overline{PN}$, $\overline{PO}$; also $a = \tfrac{1}{2}.\overline{PMp + PNp}$, $b = \tfrac{1}{2}.\overline{PMp + POp}$, and $c = \tfrac{1}{2}.\overline{PNp + POp}$; then

$$\text{tang. } a = \frac{\text{sine } \tfrac{1}{2} \times \overline{D' - D} \times \text{co-tan. } \tfrac{1}{2} \times \overline{L' - L}}{\text{sine } \tfrac{1}{2} \times \overline{D' + D}}$$

$$\text{tang. } b = \frac{\text{sine } \tfrac{1}{2} \times \overline{D'' - D} \times \text{co-tan. } \tfrac{1}{2} \times \overline{L'' - L}}{\text{sine } \tfrac{1}{2} \times \overline{D'' + D}}$$

$$\text{tang. } c = \frac{\text{sine } \tfrac{1}{2} \times \overline{D'' - D'} \times \text{co-tan. } \tfrac{1}{2} \times \overline{L'' - L'}}{\text{sine } \tfrac{1}{2} \times \overline{D'' + D'}}$$

Also, $\overline{\text{tang. } \tfrac{1}{2} \text{O}p\text{N}}$: tang. $\tfrac{1}{2} \times \overline{PNp - POp}$: : tang. $\overline{Pp\text{O} + \tfrac{1}{2}\text{O}p\text{N}}$: tang. $\tfrac{1}{2} \times \overline{PNp + POp}$, where all the terms are known, except the third, of

* See Mr. Vince's *Complete System of Astronomy*, vol. I. p. 222.

which one part OpN is $L'' - L'$; hence tang. $PpO + \frac{1}{2} \times \overline{L'' - L'} =$ tang. $\frac{1}{2} \times \overline{L'' - L'} \times$ tang. $c \times$ co-tan $\overline{a - b}$, which put $=$ tang. x, and there arises $PpO + \frac{1}{2} \times \overline{L'' - L'} = x$; to each side of this equation let there be added $\frac{1}{2} \times \overline{L'' + L'}$, and the sum gives $PpO + L'' = x + \frac{1}{2} . \overline{L'' - L'}$, the longitude of the pole P or of t, to which add 90°, the sum is the longitude of the node N.

To determine Pp, put $P =$ the longitude of the pole P, then $PpO = P - L'' \pm S$, and $POp = b + c - a = d$; consequently the tangent of half the difference of PM and Pp is $\frac{\text{tang.} \frac{1}{2} D' \times \text{sine} \frac{1}{2} \overline{S - d}}{\text{sine} \frac{1}{2} \overline{S + d}}$ $=$ tang. y, and the tangent of half the sum is $\frac{\text{tang.} \frac{1}{2} D'' \times \text{co-sine} \frac{1}{2} . \overline{S - d}}{\text{co-sine} \frac{1}{2} . \overline{S + d}} =$ tang. z: then $z + y =$ PM (PM being greater than Pp), and $z \backsim y = Pp$ the arc measuring the inclination of the solar equator to the ecliptic. If s be greater than 180°, instead of s use $360° - s$, and the same for d. But if d be less than 90°, then $Pp = 180° - \overline{z + y}$, and $PM = 180° - \overline{z \backsim y}$.

395. As it is difficult to determine exactly the situation of the spots, it is advisable, in order as far as possible to reduce the errours, to take the inclination, and the place of the node, at a mean of the results deduced from a great number of observations. In this way, the nodes are found to be in $2^s\ 17°\ 53'$, and $8^s\ 17°\ 53'$, and the inclination of the equator to the ecliptic 7° 16′, or inclination of the axis to the ecliptic 82° 44′. When the earth is in the nodes of the sun's equator, which it is about the beginning of June and December, it being then in its plane, the spots appear to describe right lines; but as the earth recedes from the nodes, the path of a spot becomes more and more elliptical, until the distance of the

earth from the nodes is 90°, when the minor axis of the ellipsis is a *maximum*; this happens about the beginning of March and September.

396. The time of rotation has been sometimes determined in the following manner: The interval of time in which a spot, after having left any point on the disc, arrives at the same point again, has on a mean of many observations been found to be $27^d\ 12^h\ 20^m$; and in this interval the earth was found to describe the angular motion of 26° 22′ about the sun's centre: therefore, as $360° + 26°\ 22' : 360° :: 27^d\ 12^h\ 20^m : 25^d\ 15^h\ 16^m$, the time of a complete rotation on its axis. But M. *Cassini*, to ascertain the time, proceeded in a manner somewhat different: he denoted the interval in which a spot returns to the same situation upon the sun's disc by t, the true motion of the earth in that time by m, and its mean motion by n; then $360° + m : 360° + n :: t$: the time of return, had the motion been uniform; this is set down at $27^d\ 12^h\ 20^m$ as above. In this period the earth's mean motion is 27° 7′ 8″: wherefore $360° + 27°\ 7'\ 8'' : 360° :: 27^d\ 12^h\ 20^m : 25^d\ 14^h\ 8^m$ the time of rotation. The time might also be ascertained, by finding the degrees in the arc M N O of the small circle, and the time of describing that arc; then, the arc M O : 360° :: time of describing M O : the time of a revolution about the axis. But, on account of the difficulty of determining accurately the arc M O, the method of *Cassini* just before given is probably more to be relied upon.

397. Respecting the nature of these spots on the surface of the sun, many widely different opinions have been entertained: *Scheiner*, who gave an account of them in his *Rosa Ursina*, published in 1612, supposed them to be bodies of very irregular figures revolving about the sun, very near to it. He has given a description of near 2000 observations on these spots, from which we learn that he

frequently saw 50 of them at once, yet for nearly 20 years scarcely any appeared. So that his theory is not very consistent with his own statement of the facts; which was speedily shewn by *Galileo*. M. *de la Lande* supposes that the sun is an opaque body covered with liquid fire; and that the spots arise from the dark parts, like rocks, which, by the alternate flux and reflux of the liquid igneous matter, are sometimes raised above the surface. Dr. *Wilson* opposes this hypothesis, and gives it as his opinion that the spots are excavations in the luminous matter of the sun. But Dr. *Herschel*, with a view of ascertaining more accurately the nature of the sun, made frequent observations upon it from the year 1779 to the year 1794. He imagines that the dark spots on the sun are mountains on its surface, which, considering the great attraction exerted by the sun upon bodies placed at its surface, and the slow revolution it has upon its axis, he thinks may be more than 300 miles high, and yet stand very firmly. He says that in August, 1792, he examined the sun with several powers from 90 to 500; and it evidently appeared that the dark spots are the opaque ground or body of the sun; and that the luminous part is an atmosphere, which, being intercepted or broken, gives us a view of the sun itself. Hence he concludes that the sun has a very extensive atmosphere, which consists of elastic fluids that are more or less lucid and transparent; and of which the lucid ones furnish us with light. This atmosphere, he thinks, is not less than 1843, nor more than 2765 miles in height; and he supposes that the density of the luminous solar clouds need not be much more than that of our *aurora borealis*, in order to produce the effects with which we are acquainted. The sun then, if this hypothesis be admitted, is similar to the other globes of the solar system, with regard to its solidity—its atmosphere—its surface diversified with moun-

tains and valleys—its rotation on its axis—and the fall of heavy bodies on its surface; it therefore appears to be a very eminent, large, and lucid planet, the primary one in our system, disseminating its light and heat to all the bodies with which it is connected. The doctor has assigned several reasons for this hypothesis, which may be seen at large in the *Philosophical Transactions* for 1795 *.

398. Astronomers have not been able to determine decisively either the times of rotation, or the inclination of the axes, of *mercury* † and *georgium sidus*: for no spots have been discovered in the former, to lead to the determination; and the latter is at so great a distance, that even the best telescopes have not enabled any person to ascertain these particulars.

399. Both bright and dark spots have been discovered on the disc of *venus*: from several successive observations on theses pots, *Bianchini*, concludes that the planet's rotation is performed in $24\frac{1}{3}$ days; while, on the contrary, *Cassini* and *Campani* stated the time of rotation, $23^h\ 16^m$. This is nearly confirmed by *Schroeter*, who makes the time $23^h\ 21^m$. But Dr. *Herschel*, from the result of a long series of observations, between the years 1777 and 1793, observes, that the time of the planet's rotation is uncertain; that the position of its axis is also very un-

* Or the reader may be referred to Part I. of *Mathematical and Philosophical Tracts and Selections*, lately published, where this and many other valuable papers extracted from expensive works, may be procured at a moderate rate.

† M. *Jerome de la Lande* in his *History of Astronomy for the year* 1800, informs us, that M. *Schroeter* of Lilienthal has some observations of *Mercury*, of which he believes the period of rotation to be 24 hours 5 minutes. And with respect to georgium sidus, Dr. *Herschel*, in *Phil. Trans.* 1798, remarks, that many observations with different instruments ascertain the flattening of the poles; whence we may conclude without hesitation, that this planet also has a rotation on its axis of a considerable velocity, although the precise time of rotation is not yet known.

certain; that its atmosphere is very considerable; that it has probably inequalities on its surface, but that no eye which is not considerably better than his, or assisted by better instruments, will ever get a sight of its mountains: he also infers from the apparent diameter of this planet, that it is somewhat larger than the earth, instead of being less, as is generally imagined.

400. M. *Cassini* observed several spots on *mars*, which he found made one revolution in $24^h\ 40^m$: this determination is confirmed by *Maraldi*. Dr. *Herschel* finds the time to be $24^h\ 39^m\ 22^s$. In the *Philosophical Transactions* for 1784, the same gentleman has given a paper on the remarkable appearances at the polar regions of this planet, &c. He observed several remarkably bright spots near each pole which had some small motion; the results of his observations are as follow: inclination of axis to the ecliptic 59° 22′; the node of the axis is in ♓ 17° 47′; obliquity of the planet's ecliptic 28° 42′. The point aries on mars's ecliptic, answers to our ♐ 19° 28′. The equatoreal is to the polar diameter, nearly as 16 to 15.

401. *Jupiter* is surrounded by faint substances, called zones or *belts*, in which so many changes appear, that they are generally supposed to be clouds: they have sometimes been observed of different breadths, and afterwards have all become nearly of the same breadth: large spots have been seen in these belts; and when a belt vanishes, the contiguous spots disappear with it. The broken ends of some belts have often been observed to revolve in the same time with the spots; only those nearer the equator in somewhat less time than those nearer the poles. Dr. *Smith* states the periodical time of the spots near the equator, at $9^h\ 50^m$, but of those near the poles, at $9^h\ 56^m$: this corresponds very nearly with the determination of *Cassini*. Dr. *Herschel* also found

the time of rotation of different spots to vary; and that the time of rotation of the same spot diminished: thus, a spot he observed in 1778 revolved as follows: from February 25, to March 2, in $9^h\ 55^m\ 20^s$; from March 2 to 14, in $9^h\ 54^m\ 58^s$; from April 7 to 12, in $9^h\ 51^m\ 35^s$. This, he observes, is agreeable to the theory of equinoctial winds, as it may be some time before the spot can acquire the velocity of the wind; and if jupiter's spots should be observed in different parts of its year to be accelerated and retarded, it would almost amount to a demonstration of its monsoons and their periodical changes. *Schroeter* makes the time of rotation $9^h\ 55^m\ 37^s$. The ecliptic and equator of jupiter are nearly parallel to each other, that is, its axis is nearly perpendicular to its orbit: hence this planet has no sensible change of seasons. This is wisely ordered by the Author of Nature, for if the axis of this planet were inclined any considerable number of degrees, just so many degrees round each pole would, in their turn, be almost six years together in darkness. The difference between the polar and equatoreal diameters of jupiter is upwards of 6000 miles, the former being to the latter as 13 to 12; and either of them more than 80000 (Art. 387.).

402. *Saturn* also, as well as jupiter, has *belts*, but besides these, this planet is surrounded with a double ring, which renders it perhaps one of the most engaging objects which astronomy presents to our view. *Galileo* was the first who observed this ring; he advertised his discovery in 1610, in a sentence which signified "*I have observed saturn to* "*have three bodies*:" but in a few years, as telescopes became improved, the nature of this appearance was better known. In 1656 *Huygens* gave the result of his observations on saturn in the letters of this sentence transposed—"*Annulo cingitur tenui,* "*plano, nusquam cohærente, ad eclipticam inclina-*

"*to.*" But leaving what we have to remark with respect to this ring till we have treated upon the satellites, we shall give Dr. *Herschel*'s determination of this planet's rotation. He found that the arrangement of the belts always followed the direction of the ring; that, as the ring opened, the belts began to shew a corresponding curvature, and *vicè versa.* He conjectured, therefore, that saturn revolved about an axis perpendicular to the plane of its ring: another reason in support of this conjecture is, that the planet is an oblate spheroid, having the diameter in the direction of the ring, to the diameter perpendicular to it, nearly as 11 to 10. The truth of his conjecture he has now verified, having found that saturn revolves about its axis, according to the order of the signs, in $10^h\ 16^m\ 2^s$ very nearly.

CHAPTER XV.

On the Phases, Stations, and other Appearances, of the Planets.

ART. 403. IN the seventh chapter, where we took notice of the most striking appearances exhibited by the planets, it was observed (Art. 199. 200.), that the planets mercury and venus, when viewed through a telescope, seemed to undergo changes in their phases similar to those of the moon: the occasion of these changes we shall now explain.—The planets being opaque bodies, without any light of their own, shining only, or at least chiefly, by the borrowed light of the sun, that face of either of them will alone appear bright which is turned towards the sun, while the opposite part, for want of light, is altogether invisible; consequently, if the situation of any of the planets be such with respect to the earth, that its dark side be towards us, then is that planet invisible; but if the whole illuminated face be towards us, then the appearance is that of a full shining orb; and, for the same reason, all the intermediate phases between these extremes arise from the different situations of the earth and planet, with respect to the sun. For instance, let us suppose A B C D F G H (fig. 10, Pl. V.) to be the orbit of *venus*, E R a portion of the earth's orbit, S the sun; let E be the place of the earth, when venus is at A in her superiour conjunction with the sun: here it is plain that the face of venus, which is illuminated by the sun, is directed towards the earth, and, of course, the planet appears to us as a full lucid circle, like the moon at full. But

if venus were at B, and the earth at E, then *n q*, part of her darkened hemisphere, will be turned towards the earth, and the arc of the circle *r q*, dividing the illuminated from the dark part, will be projected in an arc of an ellipsis upon the disc; whence the planet will appear gibbous. When venus is at C, if the earth be at E, *m q*, that part of the enlightened hemisphere which is towards us, will not much exceed 90°, of course we shall not see much more than half her disc. At the position D, *mq*, the part enlightened, is 90°; then *r q*, the circle dividing the illuminated from the darkened hemisphere, will be projected into a right line, and half the disc will be seen. Between D and F less than half the enlightened hemisphere would be directed towards the earth at E, and venus would appear horned. And in the position F, when she is in her inferiour conjunction with the sun, her dark side is entirely turned towards the earth, and she quite disappears, unless she happen to be in or near one of her nodes, when she will appear like a black spot to pass over the body of the sun, and is then said to *transit* his disc. In a similar manner the appearances in the positions G, H, I, might be traced out; and the same delineation and explanation will serve for the planet *mercury*. *Mars* also, it might be shewn, exhibits some of these phases; and jupiter, saturn, and the georgian, in a less degree; but in the latter three, the changes are scarcely ever perceptible *.

* When tracing out the progress of astronomy, and the invention of different systems, in Chap. VII. an account was given of an objection urged against Copernicus by the followers of Ptolemy, drawn from the phases of the inferiour planets (Art. 207.), and of the reply made by Copernicus, together with the supposed confirmation of the Copernican system by Galileo's discovery of the phases of venus. It might then have been remarked, had the subject been much reflected upon, that the mere changes in the discs of the inferiour planets, as urged by the Ptolemaists, proved nothing at all as to the truth or falsehood of the Copernican hypothesis.

404. From these remarks it will be easy to delineate the appearance of an inferiour planet in any given situation: thus, when the planet is in the position C, the earth being at E, and the sun at S, $r\,q$ will be the circle of illumination perpendicular to S O, and $m\,n$ perpendicular to E O, the plane of vision; then O q, part of the circle $r\,q$, being seen obliquely from E, at an immense distance compared with the size of the planet, will, according to the principles of orthographic projection, be projected into an elliptic arc on the plane $n\,m$, and the point q will appear at s, so that $n\,s$ will be the versed sine of the arc $n\,q$, or $m\,s$ the versed sine of $m\,q$. But the angle q O m is equal to P O S, for q O p is equal to P O r, and r O S equal to m O p, both being right angles; therefore, $q\text{O}p + p\text{O}m = \text{PO}r + r\text{OS}$, or $q\text{O}p = \text{POS}$; consequently, $m\,s$ is the versed sine of P O S. And in the same manner it may be proved, that in every position the greatest breadth of the illuminated part of the disc is equal to the versed sine of the exteriour angle S O P of the triangle S O E, formed by lines to the planet from the earth and sun respectively. Let, therefore, B O C P (figs. 8, 12, Pl. V.) be the projected hemisphere, or disc of the planet, and F P the

For, from a little consideration, it will appear that, according to the Ptolemaic system, *all* the planets undergo mutations in their phases similar to those of the moon; so that the objection would militate with more force against that system than against the Copernican. Besides, venus, in the latter system, may appear round without being in opposition to the sun; whereas in the former, in order to appear round, she must be in opposition; but, in fact, venus is never seen in opposition to the sun. If, therefore, in addition to the mere changes in the discs of the inferiour planets, we consider their situations in respect of the sun when these changes occur, we shall have a satisfactory proof of the fallacy of the Ptolemaic system. In this system, venus could not be dichotomized, unless she was 90° from the sun; nor gibbous, unless her distance from the luminary was more than 90°; nor full, unless she was in opposition; whereas, in reality, venus undergoes all these changes, though her elongation never exceeds $47\frac{1}{2}$°, and though she is never in opposition, nor even in quadrature.

versed sine of the exteriour angle to the radius L P, set off upon O P a diameter perpendicular to B C; then describe a semi-ellipse, having B C for its transverse diameter, and F L for the semiconjugate, and B F C P will represent the enlightened part as it appears at the earth. Now, from the nature of the circle and the ellipsis, we have L P in a constant ratio to F P, wherever the line P O is drawn perpendicular to B C; therefore, 2 L P or P O is also in a constant ratio to F P: but (by Euc. V, 12.) the sum of all the lines P O (equal to the area of the circle) is to the sum of all the lines F P (equal to the area of the illuminated part), as the diameter of the circle O P to the part F P; that is, from what goes before, to the versed sine of the exteriour angle S O P; and, all other circumstances remaining the same, the total illumination of the planet will be to the illumination at the given position in the same ratio.

405. But, since the intensity of light decreases in the duplicate proportion of the distance from the centre whence it proceeds, a planet may not appear with its greatest lustre when at A (fig. 10, Pl. V.), although its whole enlightened disc be then towards us. On this account, Dr. *Halley*, in the *Philosophical Transactions*, No. 349, gave a solution to a problem, in which it was required *to determine the position of venus when she appeared with the greatest lustre.* This may be determined for either mercury or venus, in the following way: put $E S = a$, $S v = b$, $E v = x$, S, E, and v, being the situation of the sun, earth, and venus, when the required effect takes place; from the point v with the radius $v E$ describe the arc $E a$, and let fall the perpendicular $E b$: put $b v = d$, then $a b = x - d = v$ the versed sine of the angle $a v E$. Now (by Euc. II, 12.) we have $a^2 = b^2 + x^2 + 2 b d$, whence $a^2 - 2 b d = b^2 + x^2$, and, adding $2 b x$ on each side, $a^2 + 2 b x - 2 b d = x^2 + 2 b x + b^2$; that is, $a^2 + 2 b v = x^2 + 2 b x + b^2$:

for the last part of this equation put r; then $r - a^2 = 2bv = s$, and multiplying by $2x$, we have $4bxv = 2xs$; whence, converting this equation to an analogy, we get $4bx : s :: 2x : v$; that is, $4bx : x^2 + 2bx + b^2 - a^2 :: 2\text{E}v : ab$. But it has been shewn, that *the area of the whole disc of the planet is to the area of the enlightened part, as the diameter of a circle to the versed sine of the exteriour angle*, therefore they are as $4bx : x^2 + 2bx + b^2 - a^2$; and the area of the whole disc varies as $\frac{1}{xx}$; hence, $4bx : x^2 + 2bx + b^2 - a^2 :: \frac{1}{xx} : \frac{b^2 + 2bx + x^2 - a^2}{4bx^3}$, which in all cases will express the absolute lustre of the planet. To determine this a *maximum*, put its fluxion $= 0$, then $\overline{2b\dot{x} + 2x\dot{x}} \times 4bx^3 = 12bx^2\dot{x} \times \overline{b^2 + 2bx + x^2 - a^2}$, from which we readily find $x = \sqrt{3a^2 + b^2} - 2b$*. The same conclusion might have been obtained in a more concise manner; but the reasons of the process would have appeared less obvious. In this solution we have supposed the orbits to be circular, and to apply it to *venus* we have $a = 1$, $b = \cdot 72333$ (Art. 360.), whence $x = \cdot 43036$, the angle $\text{ES}v = 22^\circ\ 21'$, and $\text{SE}v = 39^\circ\ 44'$. Also the angle $\text{S}v\text{P} = \text{SE}v + \text{ES}v = 62^\circ\ 5'$, and its versed sine is $\cdot 5318$ to the radius 1; therefore, the enlightened part visible (Art. 404.) : whole disc :: $\cdot 5318 : 2 :: \cdot 2654 : 1$; consequently, venus appears rather more than a fourth part illuminated. When venus is situated agreeably to this solution, her beauty and brightness are very admirable, for she then shines

* It may not be amiss to note, that the equation $x = \sqrt{3a^2 + b^2} - 2b$ has a limit; for, if b were equal to $\frac{1}{4}a$, the point v would fall on A, and the whole disc of the planet would be the *maximum*, when in its superiour conjunction with the sun: and it would be the same if b were less than $\frac{1}{4}a$; the arch described from the centre E, with the radius Ev, in such case not intersecting the semicircle ACF.

with such lustre that she is often seen in the day-time, even in full sun-shine. If we apply this solution to mercury, then, *a* being as before, *b* is $\cdot 3871$, and $x = 1 \cdot 0006$; the angle $ESv = 78° \ 55'$, $SEv = 22° \ 19'$, and $SvP = 101° \ 14'$, the versed sine of which is $1 \cdot 1948$; therefore, mercury appears nearly $\frac{3}{5}$ths illuminated, when his light is greatest.

406. It was observed in Art. 199. that when venus appears to the east of the sun, she sets after him, and is called the *evening star*; but, on the contrary, when she is westward of that luminary, she shines before he rises in the morning, and is called the *morning star*. We may now, since the terms inferiour and superiour conjunction are explained (Art. 250.), remark that, from the inferiour to the superiour conjunction, venus is a *morning* star; and, from the superiour to the inferiour conjunction, an *evening* star. For, let E be the earth (fig. 14, Pl. V.), *t s n*, *r i m*, two tangents thereto, representing the horizon at each place, S the sun, V R *v* O the orbit of venus, V and *v* her places at the superiour and inferiour conjunctions; then venus, in any point of the portion *i* O *m* (or *v* O B because of the smallness of the earth with respect to S *v*), is above the horizon of a spectator at *r*, while the sun at S is below it; hence venus, from the inferiour to the superiour conjunction, shines before the sun rises. Again, when the earth, by its rotation in the orbit *r o t*, is carried to *t*, the sun is set; but venus, in that part of her orbit from *n* to *s* (or V R *v*), will still be above the horizon *t n*; consequently, from her superiour to her inferiour conjunction, she shines after the sun sets.

407. When treating on the apparent motion of the planets, in Chapter IX. it was shewn, that the superiour planets are stationary some time before and after their oppositions, and the inferiour planets stationary some time before and after their inferiour conjunctions: we might, from the theory there laid down,

determine *the relative positions of any two planets, when one as seen from the other appears stationary;* but this may be done, perhaps, more easily as follows:—To facilitate the calculations, we suppose the orbits to be circular, and in the same plane; for this, though not strictly true, may be allowed us in the present instance, which is rather a matter of curiosity than utility; then let A R and C B (fig. 11, Pl. V.) be parts of the orbits of the two planets, A and B their respective places, S the place of the sun; and it is manifest, that the parts *a* A, *b* B, described in the respective orbits during the time that the planets appear stationary to each other, must be such that the lines *a b*, A B, may be parallel, for then both those lines will be directed to the same point among the fixed stars, at an exceedingly great distance. Now *a* A and *b* B, being described in small equal times, denote the velocities of A and B, and these are reciprocally as the square roots of S A and S B, as is proved by the authors on mechanics, and might be deduced from Art. 283. Produce *a* A and *b* B till they intersect in F; or, which amounts to the same, draw A F and B F perpendicular to the radii S A, S B: then, since the angles at A and B are right angles, the figure F B A S may be inscribed in a semicircle having F S for its diameter; therefore, the angle BFA = BSA, AFS = ABS, FSB = FAB, and BFS = supplement of B A S. Let S B, the radius of the exteriour orbit, be denoted by r, S A that of the interiour being represented by unity; also put F B = x, and B D = y, then D S = $r - y$; and since *a b*, A B, are parallel, F B (x) : F A : : B *b* : A *a* : : 1 : $\surd r$; therefore, F A = $x \surd r$, and $FA^2 = r x^2$. Also in the right angled triangles FBS, FAS, $FS^2 = FA^2 + AS^2 = FB^2 + BS^2$, or $FA^2 + 1 = x^2 + r^2$, and $FA^2 = x^2 + r^2 - 1$: but $FA^2 = r x^2$, wherefore $r x^2 = x^2 + r^2 - 1$. This reduced gives $x =$

$\sqrt{\frac{r^2-1}{r-1}} = \sqrt{r+1}$; consequently, F A $= x\sqrt{r}$ $= \sqrt{r^2+r}$. Again, by the similar triangles S A D, F B D, F B (x) : BD (y) : : S A (1) : A D $= \frac{y}{x}$; also, S A (1) : S D $(r-y)$: : F B (x) : F D $= rx - yx$; whence F A = F D + A D $= \frac{rx^2 - yx^2 + y}{x} = x\sqrt{r}$, by the above. From this equation we find $y = \frac{r^2-1}{r+\sqrt{r}}$, by substituting for x its equal $\sqrt{r+1}$: hence S D $= r - y = r - \frac{r^2-1}{r+\sqrt{r}} = \frac{\overline{r+1}\sqrt{r}-r}{r}$. Then S D : radius : : S A (1) : sine S D A $= \frac{r}{\overline{r+1}.\sqrt{r}-r}$ $= \frac{\sqrt{r}}{r+1-\sqrt{r}} =$ cosine A S B.

408. The sine of A S B $= \frac{\overline{r-1}-\sqrt{r+1}}{r\sqrt{r+1}}$. For, in [illegible] F B D, F B (x), sine D : : B D (y) : sine [illegible] $\frac{\overline{r-1}-\sqrt{r}\times\overline{r+1}}{r\sqrt{r+1}} \times \frac{\sqrt{r}}{r+1-\sqrt{r}}$, which [illegible]s the value of the sine of D F B, or its [illegible]al A S B, as just stated.

409. The tangent of the angle ASB $= \frac{\overline{r-1}\sqrt{r+1}}{r+\sqrt{r}}$. For, in the triangle F B D, F B (x) : radius : : B D (y) : tang. B F D or A S B $= \frac{y}{x} =$ (Art. 407.) $\sqrt{r+1} \div \frac{r^2-1}{r+\sqrt{r}} = \frac{\overline{r-1}\sqrt{r+1}}{r+\sqrt{r}}$.

410. The tangent of the angle A B S at the upper orbit is $\frac{1}{\sqrt{r^2+r}}$. For in the triangle A F S, A F (Art. 407.) $= \sqrt{r^2+r}$) : radius : : S A (1) : tang. A F S $= \frac{1}{\sqrt{r^2+r}} =$ tang. A B S.... And the sine of

$ABS = \frac{1}{\sqrt{r^2+r+1}}$. For in the triangle AFS, $FS^2 = FB^2 + BS^2 = r + 1 + r_2$ (Art. 407.); therefore $FS = \sqrt{r_2 + r + 1}$: then FS : radius :: AS (1) : sine AFS = $ABS = \frac{1}{\sqrt{r^2+r+1}}$.

411. The tangent of the angle BAS at the lower orbit is $= \frac{r}{\sqrt{r+1}}$, and its sine $= \frac{r}{\sqrt{r^2+r+1}}$. For, in the triangle SBF, FB $(\sqrt{r+1})$: radius :: BS (r) : tang. BFS, or its supp. BAS $= \frac{r}{\sqrt{r+1}}$. And in the same triangle, FS $(\sqrt{r^2+r+1})$: radius :: SB (r) : sine BFS = sine of its supp. BAS $= \frac{r}{\sqrt{r^2+r+1}}$*.

412. When any planet is stationary, *the tangent of the angle at the earth is to the tangent of the angle at the planet, as the periodic time of the planet [illegible] to that of the earth.* For, tang. B A [illegible] A and b B [illegible] amounts to the same $\frac{r}{\sqrt{r+1}} : \frac{1}{\sqrt{r^2+r}} :: \frac{r}{1} : \frac{1}{\sqrt{r}} :: r\sqrt{r}$ [illegible] periodic time of B : periodic time of A.

413. Thus have we determined the expressions for the sines and tangents, of the angles at the sun, at the earth, or at the planet, whether inferiour or superiour : it must be evident from this investigation, that if the lower planet had been at E, making ER = RA, the two planets would still have appeared stationary to each other ; for the positions of all the lines would be the same relatively at E and B, as at A and B. It also appears, that *if the orbits be ellipses or any other curves, the planets moving in them*

* This expression for the sine of the elongation of a superiour planet from the sun, when stationary, agrees with that deduced from a different method by Mr. *Vince*, at p. 177 of his *Complete System of Astronomy*.

will appear stationary, when the tangents A F, B F, *are proportional to the velocities at* A *and* B: for if A F, B F, are as the velocities in A and B, they will be as *a* A and *b* B; and therefore *a b*, A B, will be parallel; consequently the planets will appear stationary to each other.

414. With respect to the *time* when any of the planets appear stationary, if the orbits were exact circles it might easily be found; but as they are excentric and elliptical, we can only approximate to the time by trials. Having the mean distances of the earth and planet, find the helicentric angle (a) between the earth and planet when stationary, by some of the preceding articles (407. 408. 409.) Then, by means of the *Nautical Almanac*, or *White's Ephemeris*, find when the difference (d) between the heliocentric places of the earth and planet is nearly equal to this angle (a): and to know when they are exactly equal, put P for the periodic time of the earth, p for that of the planet, and x for the difference between the time when the angle at the sun is d, and when it is a. Then $P : 360^\circ :: x : \frac{360\,x}{P}$, mean motion of the earth in the time x; and $p : 360^\circ :: x : \frac{360\,x}{p}$ mean motion of the planet in that time. The difference of these mean motions must be put equal to the sum or difference of d and t, as the case requires, and x may be found by reduction, and thence a nearer approximation to the time. Then find the geocentric longitude of the planet for that day, and for the preceding or following day; if they are the same, the planet is stationary; but if not, having the planet's geocentric longitude for three or four days, the method of interpolation will give the exact time of the station.

415. If when a planet is stationary, the elongation from the sun be observed, the planet's distance

from the sun may be known in terms of the earth's distance. Thus, for a superiour planet, we have (Art. 411.) the tangent of B A S $= \frac{r}{\sqrt{r+1}}$; this being known by observation, we denote by t, then $t^2 = \frac{r^2}{r+1}$; whence $r^2 = t^2 r + t^2$, and $r^2 - t^2 r = t^2$; from which, by completing the square, &c. we find $r = \frac{1}{2} t^2 + t\sqrt{1 + \frac{t^2}{4}}$*. Again, for an inferiour planet we have tang. A B S $= \frac{1}{\sqrt{r^2+r}} = t$ (Art. 410.); whence $1 = t^2 r^2 + t^2 r$, and $r^2 + r = \frac{1}{t^2}$; from which, by reduction, we have $r = \sqrt{1 + \frac{1}{t^2}} - 1$. And, as the value of r thus determined, is to unity; so is the distance of the earth from the sun, to the distance of the planet from the sun; or, if unity be put for the third term of this analogy, then $\frac{1}{\sqrt{1+\frac{1}{t^2}} - 1}$, or $t^2 + t^2\sqrt{1 + \frac{1}{t^2}}$, to which it will reduce, will express the distance of the planet from the sun, that of the earth being one.

416. *To find the time of the heliocentric conjunction of any two planets*; let P and p be the periodic times, P representing the greater; then P : one day : : 360° : $\frac{360°}{P}$ the degrees passed over in a day by the superiour planet; also, p : one day : : 360° : $\frac{360°}{p}$ the degrees described by the inferiour

* The expression given by Mr. *Vince*, at page 180 of his *System of Astronomy*, is not general; for it will only answer when applied to the case of superiour planets.

planet in one day. Therefore $\frac{360^\circ}{p} - \frac{360^\circ}{P}$ = the degrees gained in a day by the inferiour planet. Whence $\frac{360^\circ}{p} - \frac{360^\circ}{P} : 360^\circ :: \text{one day} : \text{the time of gaining}$ the whole circumference $= \frac{360^\circ}{\frac{360}{P} - \frac{360}{p}} = \frac{1}{\frac{1}{p} - \frac{1}{P}} = \frac{Pp}{P - p}$. This will be the mean time between one conjunction and another, when seen from the sun: or the mean time between two oppositions, or between any two similar situations. Then, if the mean time of one heliocentric conjunction be known, the mean time of the next, and all the succeeding ones, will be known. Therefore, find the heliocentric places of the two planets to that time, and note the errour; and by observing whether the inferiour or superiour planet is foremost, it will be seen whether the true time is past or not yet arrived. Let a new time be assumed accordingly, for which, let the planet's places be found, and again note the errour; from these two errours, and respective times, the true time may be ascertained by the common methods. But with regard to the *geocentric* conjunctions and oppositions of the planets, they cannot be determined easily except by repeated trials, because those bodies do not revolve regularly about the earth, as they do about the sun. As the three highest planets approach nearest to a regular motion about the earth, the geocentric conjunctions or oppositions of any two of them will differ but little from their heliocentric ones; happening a *little* sooner or later, according to the situation of the earth in its orbit.

417. *To know in what period any planet and the earth return to the same point of the heavens;* find, from a table of their mean motions, a number of years agreeing to a complete number of revolutions of the planet. According to M. *de la Lande, mer-*

cury, in thirteen years (of which three are bissextiles) and three days, makes fifty-four revolutions, and 2° 55′; in the same time the earth makes thirteen revolutions, and 2° 49′ over; therefore, in this period, the earth and mercury return to very nearly the same relative situation in the heavens. *Venus*, after a space of eight years, is found within 1° 32′ of the same place, and the earth within 4′. In seventy-nine years and four days (supposing there are twenty bissextiles), *mars* returns to the same situation within 3° 39′, and the earth within 3° 48′. *Jupiter*, in eighty-three years, returns to within 12′ of the same point, and the earth within 6′. *Saturn*, in fifty-nine years and two days, returns to within 1° 45′ of the same situation, and the earth to within 1° 41′. M. *de la Lande* also informs us, that on May 22d, 1702, jupiter and saturn were within 1° 4′ of each other: On February 11, 1524, venus, mars, jupiter, and saturn, were very near each other, and mercury not above 16° from them: On November 11, 1544, mercury, venus, jupiter, and saturn, were within the space of 10°: On March 17, 1725, mercury, venus, mars, and jupiter, were so near each other, as to be all seen through the same telescope without changing its position: On December 23, 1769, venus, mars, and jupiter, were within 1° of each other.

418. Since the theories of the planets are established, and their geocentric places may be ascertained for any time, with considerable accuracy, by means of the *Nautical Almanac*, we may, after having found their right ascensions and declinations (Art. 168.), determine the time of their rising, culminating, setting, and their altitude, azimuth, &c. at any time, by the same methods which were made use of to determine these particulars with respect to the fixed stars, in *Chap*. VI., *Probs*. 17. 19. 20. 21. &c. And with the assistance of a celestial globe, we

may very readily find their situations in the heavens with regard to the fixed stars; or indeed their altitudes and azimuths, generally, within a degree at any given time.

For example, it is required to shew the positions of all the planets, with their altitudes and azimuths at Greenwich, on March 13, 1801, at seven o'clock P. M. . . . . Their geocentric longitudes and latitudes, on March 13th, at *noon* (which are near enough for our present purpose, since the daily variation in longitude, even in *mercury*, does not exceed a degree, and the latitudes vary still less), are as follow: *mercury*, longitude ♈ 10° 46′, latitude 2° 9′ N.; *venus*, long. ♉ 8° 32′, lat. 2° 33′ N.; *mars*, long. ♊ 12° 8′, lat. 1° 47′ N.; *jupiter*, long. ♋ 24° 36′ lat. 0° 37′ N.; *saturn*, long. ♌ 18° 10′, lat. 1° 22′ N.; *georgium sidus*, long. ♎ 0° 33′, lat. 0° 47′. N. By these geocentric longitudes and latitudes, mark on the globe the respective places of the planets; then rectify the globe for the given latitude and day, and turn it about till the index points to the given hour; the particulars required for each planet will then be as below: *mercury*, about the same altitude as γ pegasus, and 12° westward of that star; altitude 6½°, azimuth 3° from the west towards the south. *Venus*, about 5° to the west of the star in the ram's north eye; altitude 31°, azimuth 14° from the west southward. *Mars*, nearly midway between *aldebaran* and the bull's north horn; altitude 55°, on the south-west azimuth circle. *Jupiter*, nearly in a direct line with *castor* and *pollux*, about 8° below the latter; altitude 57°, azimuth 60° from the east towards the south. *Saturn*, about 8½″ to the west of *regulus* or *cor leonis*, somewhat higher than that star; altitude 42°, azimuth 30° from the east southerly. *Georgium sidus*, almost in opposition to mercury, very near η in the virgin; altitude 5°, azimuth 3° from the east towards the south. From

the above, it appears that, at the time proposed, all the planets might be seen at one instant; which is far from a usual circumstance. It also appears from the ephemeris, that mercury is at his *greatest elongation* from the sun, venus nearly so, and both mercury and jupiter within ten days of one of their *stations*. In a similar manner may the relative situations, and various other particulars respecting the planets, be ascertained for any given time, with tolerable accuracy, with the help of a celestial globe and an ephemeris.

CHAPTER XVI.

On the Moon.

ART. 419. IT must be evident from what was observed in the 198th article, that the motions of the moon are very irregular; the irregularities are indeed so considerable, that the determination of the theory of the moon from observation, is a matter which requires more than usual care; and on this account it has been deferred till the methods of ascertaining the planets' orbits and motions were explained.

420. The orbit of the moon is changeable in point of situation, and does not always persevere in the same figure; for though it be elliptical or nearly so, with respect to the earth, which is one of its foci, the excentricity of the ellipsis is varied, being sometimes increased, and sometimes diminished. The particular irregularities in the moon's motion, to which the earth and other planets are not subject, arise from the action of the sun, which disturbs her ordinary course through her orbit, and are all mechanically deducible from the same great law that directs her general motion; namely, the law of gravitation and attraction. For a familiar idea of this, it must be considered, that if the sun acted equally on the earth and moon, and always in parallel lines, this action would only serve to restrain them in their annual motions round the sun, and no way affect their actions on each other, or their motions about their common centre of gravity. But because the

moon is nearer the sun, in one half of her orbit, than the earth is, but farther off in the other half thereof; and because the power of gravity is always less at a greater distance; it follows, that in one half of her orbit, the moon is more attracted than the earth towards the sun, and less attracted than the earth in the other half; and hence irregularities in her motions necessarily arise; the excess of attraction in the one case, and the defect in the other, becoming a disturbing force. And besides, the action of the sun on the earth and moon is not directed in parallel lines, but in lines that meet in the centre of the sun; which makes the effects of the disturbing force still more complex and embarrassing. Hence, conjointly with the various situations of the moon relatively to the earth and sun, arise the numerous irregularities in her motions, and the consequent difficulty of establishing the elements of her theory.

421. NEWTON was the firſt who computed the irregularities from their causes. In many parts of his great work, *The Principia*, he applies, with admirable sagacity and penetration, to the determination of the motions of the moon. The result of his investigations on this subject, are condensed into small compass in Dr. *Hutton's Mathematical and Philosophical Dictionary* (Art. *Moon*), nearly as in the eighteen subsequent articles.

422. Newton finds that the disturbing force added to the gravity of the moon in her quadratures (Art. 250.) is to the gravity with which she would revolve in a circle about the earth, at her present mean distance, if the sun had no effect on her, as 1 to $178\frac{29}{40}$: he finds that the force subducted from her gravity in the conjunctions and oppositions, is double of this quantity; and that the area described in a given time in the quarters, is to the area described in the same time in the conjunctions and oppositions, as

10973 to 11073: he finds also, that in such an orbit her distance from the earth in her quarters, would be to her distance in the conjunctions and oppositions, as 70 to 69.

423. The plane of the moon's orbit is inclined to that of the ecliptic, making with it an angle of about five degrees; but this inclination varies, being greatest when she is in the quarters, and least when in her syzygies.

424. The apogee of the orbit in the moon's syzygies goes forward, in respect of the fixed stars, at the rate of 23′ each day; and backwards in the quadratures $16\frac{1}{3}'$ each day: the mean annual motion is therefore estimated at 40°.

425. The gravity of the moon towards the earth is increased, by the action of the sun, when the moon is in the quadratures, and diminished in the syzygies: and from the syzygy to the quadrature, the gravity of the moon towards the earth is continually increased, and she is continually retarded in her motion; but from the quadrature to the syzygy, the moon's gravity is perpetually diminished, and the motion in her orbit is accelerated.

426. The moon is less distant from the earth at the syzygies, and more at the quadratures.

427. As radius is to $\frac{3}{2}$ of the sine of double the moon's distance from the syzygy, so is the addition of gravity in the quadratures, to the force which accelerates or retards the moon in her orbit.

428. And as radius is to the sum or difference of $\frac{1}{2}$ the radius, and $\frac{3}{2}$ the co-sine of double the distance of the moon from the syzygy, so is the addition of gravity in the quadratures, to the decrease or increase of the gravity of the moon at that distance.

429. The apsides of the moon go forward when she is in the syzygies, and backwards in the quadratures (Art. 424.). But in a whole revolution they go forward with the greatest velocity, when the line

of the apsides is in the nodes; and if they are going back when in the nodes, their regression is then slowest of all, in the same revolution.

430. When the line of the apsides is in the quadratures, the apsides are carried, *in consequentia,* with the least velocity when the moon is in the syzygies; but they return the swiftest in the quadratures; and in this case, the regress exceeds the progress, in one entire revolution of the moon.

431. The excentricity of the orbit undergoes various changes every revolution. It is the greatest of all when the line of the apsides is in the syzygies, and the least when that line is in the quadratures.

432. Considering one entire revolution of the moon, *cæteris paribus,* the nodes move, *in antecedentia,* with the greatest velocity when she is in the syzygies; then slower and slower, till they are at rest, when she is in the quadratures.

433. The line of the nodes acquires successively all possible situations in respect of the sun; and every year it goes twice through the syzygies, and twice through the quadratures.

434. In one whole revolution of the moon, the nodes go back very fast when they are in the quadratures; then slower till they come to rest, when the time of the nodes is in the syzygies.

435. The inclination of the plane of the orbit is changed by the same force as that by which the nodes are moved; being increased as the moon recedes from the node, and diminished as she approaches it.

436. The inclination of the orbit is least of all when the nodes are come to the syzygies. For, in the motion of the nodes from the syzygies to the quadratures, and in one entire revolution of the moon, the force which increases the inclination, exceeds that which diminishes it; therefore, the inclination is increased; and it is the greatest of all when the nodes are in the quadratures.

437. The moon's motion being considered in general, her gravity towards the earth is diminished coming near the sun, and the periodical time is the greatest; as also the distance of the moon, *cæteris paribus*, the greatest when the earth is in the perihelion.

438. All the errours in the moon's motion are something greater in the conjunction than in the opposition.

439. All the disturbing forces are inversely as the cube of the distance of the sun from the earth; when *it* remains the same, they are as the distance of the moon from the earth. Considering all the disturbing forces together, the diminution of gravity prevails.

Most of these conclusions, drawn by Newton from the nature of forces, are confirmed by actual observations on the moon.

440. Prob. I. *To determine the time of the moon's revolution.*

The period of the moon's revolution about the earth, is called its *periodic revolution*; and the period between one opposition or conjunction and another, the *synodic revolution*: the latter of these is most easily determined by observation, and thence the former. In the middle of a *central* lunar eclipse (Art. 519.), the moon is in direct opposition to the sun; and though not exactly so in other eclipses of the moon, is yet so nearly in opposition, as to cause no sensible errour in a long interval. Compute therefore the time between two lunar eclipses, at a considerable distance from each other, and divide this by the number of lunations that have passed in the interval; the quotient will be the quantity of a synodic revolution. Compute the sun's mean motion during the time of the synodic revolution, and add it to 360°: then, as the sum is to 360°; so is the time of the synodic, to the periodic revolution. The

number of lunations in any given interval, may be found by dividing it by $29\frac{1}{2}$ days, the time of a lunanation according to the most common observations.

For example: Copernicus observed two lunar eclipses, the one at Rome on November 6, 1500, at twelve at night; the other at Cracow, on August 1, 1523, at 4^h 25^m; the difference of meridians being 0^h 29^m: hence, to determine the synodic revolution,

2d observation . . .	1523^y	237^d	4^h	25^m
1st do.	1500	310	0	29
Difference	22	292	3	56
Intercalary days add .		5		
Exact interval . . .	22	297	3	56

Or 8327^d 3^h 56^m; which divided by 282, the number of lunations, gives 29^d 12^h 41^m for the time of the synodic revolution.

From two other observations of eclipses, the one at Cracow, by *Copernicus*, the other at Babylon, in the year 721, before the christian æra, as recorded in *Ptolemy's Almagest*, the former-mentioned person, determines more accurately the quantity of the synodic revolution to be 29^d 12^h 43^m. And M. *de la Lande*, from a mean of a great number of observations, makes it 29^d 12^h 44^m 3^s.

Now, the sun's mean motion in this interval is $29°$ $6'$ $24''$ $18'''$; let this be added to $360°$; then, as the sum $389°$ $6'$ $24''$ $18'''$: $360°$: : 29^d 12^h 44^m 3^s : 27^d 7^h 43^m 5^s, the mean periodical revolution with respect to the equinoxes.

441. As the precession of the equinoxes is $50\frac{1}{4}''$ in a year, or about $4''$ in a month, the mean revolution relatively to the equinoxes must be less than that with respect to the fixed stars, by the time the

moon is describing 4″ with its mean motion; this time is very nearly 7^s, therefore the time of a *sidereal revolution* of the moon is $27^d\ 7^h\ 43'\ 12''$.

442. Hence, the time of the periodical revolution being known, the moon's diurnal or horary motion, &c., may be found by the rule of three: and thus may tables of the mean motion of the moon be constructed. Also, if the mean diurnal motion of the sun be subtracted from that of the moon, the remainder will shew the moon's diurnal motion from the sun: and thus may a table of this motion be constructed.

443. The nodes of the moon's orbit, and the place of the apogee, being subject to a motion, the times of revolution with respect to the nodes, or to the apogee, are not exactly the same as the sidereal revolution; thus, by comparing the mean motions of the moon and of the nodes, it will appear that they are, as 1 : ·00402185; and as the motion of the nodes is retrograde, we have 1 + ·00402185 : 1 : : $27^d\ 7^h\ 43^m\ 12^s$, the time of a sidereal revolution : $27^d\ 5^h\ 5^m\ 36^s$, time of a revolution in respect of its nodes. The mean motion of the apogee is to that of the moon, as ·0084522 : 1; and the motion of the apogee (on the whole) being direct, we have 1 — ·0084522 : 1 : : $27^d\ 7^h\ 43^m\ 12^s$: $27^d\ 13^h\ 18^m\ 34^s$, the time of a revolution with respect to the apogee, or of an *anomalistic revolution*. These agree very nearly with the determinations of M. *de la Caille*.

444. Dr. *Halley*, when comparing the ancient eclipses observed at Babylon, between 700 and 800 years before Jesus Christ, and those observed by *Albategnius* in the ninth century, with some of his own time, was led to the suspicion, that the moon's mean motion from the sun, compared with the diurnal motion of the earth, is somewhat quicker than it was formerly: this increase of the moon's motion is called *her acceleration*. The doctor could not ascer-

tain the quantity of this; because the longitudes of Bagdat, Alexandria, and Aleppo, where the observations had been made, were not then precisely known. But since his time, these longitudes have been ascertained, and Mr. *Dunthorne*, by comparing the recorded times of ancient and modern eclipses, with the calculations of them by his own tables, found, that the same tables gave the moon's place more backward than the true place in ancient eclipses, and more forward than her true place in later eclipses; whence he confirmed the opinion of *Halley*, that the moon's motion was slower in ancient times, and in modern times quicker than the tables gave it. Also, by means of the most authentic account now remaining of an eclipse observed at Babylon, in the year 721, before Jesus Christ, he found, that the observed beginning of this eclipse was about an hour and three quarters sooner than the beginning by the tables; and that, therefore, the moon's true place preceded her place by computation at that time, by about 50′ of a degree. Then, admitting the acceleration to be uniform, and the aggregate of it as the square of the time, it will be at the rate of about 10″ in 100 years. M. *de la Lande* makes the secular variation 9″·886, from eclipses in 977 and 978. And in *Mayer*'s tables it is set down at 9″, beginning at 1700.

445. Dr. *Long*, in the second vol. of his *Astronomy*, has enumerated some of the causes from which this acceleration may arise. Either, the annual and diurnal motion of the earth continuing the same, the moon is really carried about the earth with a greater velocity than formerly; or, the diurnal motion of the earth and the periodical revolution of the moon continuing the same, the annual motion of the earth about the sun is retarded; or, 3dly, the annual motion of the earth and the periodical revolution of the moon continuing the same, the rotation of the earth

upon its axis is a little retarded: in this case, the day and all its subdivisions being lengthened, the moon's period being measured thereby, will appear to be shortened, though it may be in reality the same. But M. *de la Place*, in the *Mém. de l'Acad. Roy. des Scien.* for 1786, has assigned another cause for this *acceleration*. The excentricity of the earth's orbit is known to be diminishing, in consequence of the action of the planets upon the earth: but the mean force of the sun to dilate and contract the orbit of the moon is proportional to the square of the excentricity of the earth's orbit; the major axis remaining the same: therefore, by the diminution of the excentricity, the moon's mean motion is accelerated. When the excentricity has arrived at its *minimum*, according to this principle of *La Place*, the acceleration will cease; and when the excentricity of the earth's orbit again increases, the moon's mean motion will be proportionally retarded.

446. PROB. II. *To determine the place of the moon's nodes, and the time when the moon is in that place.*

1*st Method.* The moon's place at the middle of a central lunar eclipse (Art. 440.) is directly opposite to the sun, and the moon is then in one of her nodes; as will be shewn when we treat on eclipses: therefore, calculate the sun's true place at the middle of such eclipse, or deduce the sun's place at that time from observations; the opposite point will be the true place of the moon's node.

By this method, M. *Cassini* found the time and place of the moon's node in April, 1707. A central eclipse was observed at Paris, on April 16: the middle of which was found to be at $13^h\ 48^m$ apparent time. The true place of the sun calculated for that time, was ♈ 26° 19′ 17″: the opposite point to which is ♎ 26° 19′ 17″, or 6^s 26° 19′ 17″, the place of the moon's node. Previous to the eclipse

the moon had north latitude, and afterwards south latitude; this was, therefore, the *descending* node.

447. 2d *Method.* The place of the moon's node and corresponding time may be found by a method similar to the fourth method we gave to find the nodes of a planet (Art. 349.). Thus, let *qp* (fig. 6, Pl. IV.) be the ecliptic, PQ the moon's orbit, Q the place of the moon therein when on the meridian the day before she passed the node, P her place in her orbit when on the meridian the day after the node: then, by means of the moon's right ascensions and declinations, find her latitude and longitude at both times of passing the meridian, and then determine the place of the node by either of the rules given in Art. 349. The time may be found, by saying, as *qp* the difference of longitudes between the two observations : *q*N the difference between the longitude of the first observation and the place of the node : : the interval of time between the two passages over the meridian : the interval from the first passage to the time of the node.

Ex. On March 1, 1801, the moon came to the meridian at $13^h\ 4^m$, and on the 2d of March at $13^h\ 48^m$; the longitudes and latitudes at culminating, as deduced from the observed right ascensions and declinations (Art. 66.), were, on the first day, long. $5^s\ 29°\ 17'\ 40''$, lat. $1°\ 1'\ 22''$ N.; on the 2d day, long. $6^s\ 13°\ 42'\ 49''$, lat $0°\ 12'\ 33''$ S. Therefore, $1°\ 13'\ 55''$ (sum of the latitudes) : $44°\ 25'\ 9''$ (difference of longitudes) : : $1°\ 1'\ 22''$ (first latitude) : $36°\ 52'\ 38\frac{1}{2}''$ difference of longitude between the first observation and the place of the node: hence $5^s\ 29°\ 17'\ 40'' + 36°\ 52'\ 38\frac{1}{2}'' = 6^s\ 6°\ 10'\ 18\frac{1}{2}''$ longitude of the moon's descending node in the beginning of March, 1801. Again, as $44°\ 25'\ 9'' : 36°\ 52'\ 38\frac{1}{2}'' : : 24^h\ 44^m$ (interval between the times of passing merid.) : $20^h\ 32\frac{1}{2}^m$; which, added to the time of coming to the meridian on the first day, gives (after deducting 24^h)

$9^h\ 36\frac{1}{2}^m$ for the time on March 2, when the moon is in the node. This conclusion is not quite accurate; for, in the first place, the triangles Q N q, P p N, are spherical, but the calculation is on the supposition that they are plane; and, in the next, we have supposed the motion of the moon to be *equable* during the interval between the observations, which is not the case. When the moon's longitude is required for any time between two known longitudes, it is best done by the equation of second differences, annexed to Mr. *Taylor's Sexagesimal Tables*, according to the method explained by Dr. *Maskelyne*, in the *Nautical Almanac;* and the longitude thus found, being compared with the longitude as deduced above, will enable us to approximate to the true longitude of the node, to any desirable degree of accuracy. But, after all, the longitude found by a lunar eclipse is most to be depended upon.

448. By comparing the places of the nodes at a small interval of time, their motion for that interval is obtained; and then, by extending the comparison to larger intervals, the mean motion may be determined pretty accurately. The motion of the nodes for a short period is very irregular (Art. 432. 434. &c.); but, by comparing together very distant observations, the mean annual motion is found to be about 19° 19′ 44″ *in antecedentia;* so that the nodes make a complete retrograde revolution, with respect to the equinoctial points, in about $18^{yrs}\ 228^d\ 5^h$. As this motion is not uniform, certain equations are given in Astronomical Tables, to be applied to the mean place at any time, in order to obtain the true place.

449. Prob. III. *To determine the inclination of the moon's orbit.*

After the places of the nodes are determined, the inclination of the orbit may be ascertained for that period, by finding the moon's right ascension and declination when she is 90° from either node, which

will be at the distance of $13^d\ 14^h\ 32^m\ 48^s$ in time; for from these the latitude is readily found, and this latitude is the measure of the inclination. But as this inclination is liable to considerable changes, it is necessary to have a long-continued series of observations, for every possible situation of the line of the nodes, of the sun and moon with respect to that line, and of the earth with respect to its distance from the sun; and it will appear that the least inclination is about 5°, and the greatest about 5° 18′ 6″: the variations corresponding with what has been already stated in Art. 435. 436.

450. PROB. IV. *To determine the place of the moon's line of apsides, and the excentricity and greatest equation of her orbit.*

To find the place of the moon's apogee, take with a micrometer a great many measures of the moon's diameter when at or near the full; when those diameters are at the least, the moon is in apogee; and when they are greatest, she is in perigee. From the moon's right ascensions and declinations at these times, her longitudes may be found; and from the apparent semidiameters of the moon we have (Art. 304.) the ratio of her distance from us in perigee to her distance in apogee; and thence we may deduce the excentricity, as in the article just referred to. Or, if two diameters are found equal to each other, very near together, the moon must have been in either the perigee or apogee at the middle point of time: but if the diameters are found equal at an interval of eight or ten days, the middle point may shew either the place of the apsides, or a point at 90° distance from that line.

451. The diameter of the moon may, in general, be best measured by noting the time of her passage over the meridian: call this time t, d the horizontal diameter of the moon, s = secant of her declination, and l the time from the moon's passage over the

meridian, on the day we want the diameters, to her passage on the following day; then the moon's diameter in right ascension (Art. 150.) will be sd, and we shall have $360° : sd :: l : t$; consequently $d = \frac{t}{sl} \times 360°$ for the diameter required. It must here be observed, that as the moon is at different distances from the spectator when she is at different altitudes, the apparent diameters must be reduced to the same as they would have at any common altitude; and this may be done by saying, as the co-sine of the observed altitude to the co-sine of the *common altitude*, so is the observed horizontal diameter to the diameter at the common altitude. Where great accuracy is required, a previous analogy must be made use of, taking the co-sine of the true altitude as corrected for refraction, and that of the apparent altitude for the first antecedent and consequent, and thus obtaining a more correct horizontal diameter.

452. Another, and perhaps the best, method of determining the particulars required in this problem, is the following: Observe, with all possible precision, the place of the moon, as frequently as it can be done, throughout the whole of her orbit; let the true motions be compared with the mean ones, and since, in the course of a revolution through her orbit, the moon must have passed twice through the place of the greatest equation, half the greatest difference will be the greatest equation: and the two observations where the mean horary motion differs not from the true horary motion, will give either the place of the apsides, or the place where the equation is greatest. If the moon has moved through six signs of her orbit between the observations, they determine the place of the apsides. The excentricity may be deduced from the greatest equation, by the method given in Art. 344.

453. The greatest equation, and, consequently, the

excentricity, of the moon's orbit, undergo variations; sometimes the greatest equation is found to be about $4° 58' 46''$, at others nearly $7\frac{1}{2}°$. The excentricities vary from ·04333 to ·06677, the mean distance being 1. According to M. *de la Caille* the mean excentricity is ·05505. *Mayer* makes the mean excentricity ·0550357, and corresponding greatest equation $6° 18' 31''·6$.

454. By comparing the place of the moon's apogee at different times, its mean motion is determined; and the more distant the times are chosen, the more accurately will that motion be known: thus *Cassini*, by comparing the place of the apogee on Nov. 29, 1686, at $11^h\ 7^m\ 18^s$ apparent time, with the place on Dec. 10, 1685, at $10^h\ 38^m\ 10^s$ apparent time, found that a revolution of the apogee was performed in about $8\frac{3}{4}$ years. The time being ascertained thus nearly, he computed the mean motion from a mean of several distant observations, and found it to be such, that a complete revolution of the apogee, *in consequentia*, would be performed in eight common years, $311^d\ 8^h$; hence the mean annual motion is $1^s\ 10° 39' 52''$, and the daily motion $6' 41'' 1'''$: this is in respect of the equinoxes. M. *de la Lande* states the tropical revolution of the apogee at $8^y\ 311^d\ 8^h\ 34^m\ 57^s$, the sidereal revolution at $8^y\ 312^d\ 11^h\ 11^m\ 39^s$, and the diurnal motion at $6' 41''·0698$. According to *Ptolemy*, the mean diurnal motion of the apogee is $6' 41'' 2'''$; and his determinations of the other motions and irregularities of the moon are, in many respects, so conformable to the determinations of the modern astronomers, as prove him to have been a very diligent and accurate observer of that body.

455. Prob. V. *To determine the parallax, and distance, of the moon.*

Some methods of ascertaining the parallax of the heavenly bodies were given in the fifth chapter; by

one of which the parallax of the moon might be found; but it is sometimes proposed to be determined in the following manner: If there were no parallax, or refraction, the difference of the moon's meridian altitudes, when at her greatest north and south latitudes, would, according to the proposers of this method *, be equal to the sum of the two latitudes of the moon; consequently, the difference between the sum of the two latitudes, and the difference of the altitudes, corrected for refraction, will be the difference between the parallaxes at the two altitudes. To find from thence the parallax at the greatest altitude, put S and s for the sines of the greatest and least apparent zenith distances, X and x the corresponding parallaxes; then, since small arcs are nearly as their sines, when the distance is given the parallax varies as the sine of the zenith distance (Art. 83.), we therefore have $S : s :: X : x$; hence, by division, $S - s : s :: X - x : x = \frac{s \times \overline{X - x}}{S - s}$: but $X - x$, being the difference of the parallaxes, is given from the observations; therefore x, the parallax at the greatest altitude, becomes known. This method is liable to many objections; for, it is therein supposed that the distance of the moon from the earth is the same at both observations, which will probably not be the case; it will, therefore, need correction on this account. It is also supposed, that the difference of the greatest and least altitudes, corrected for refraction and parallax, is equal to the sum of the two latitudes of the moon; which is not true *generally*, because the longitude of the moon, the inclination of the ecliptic to the horizon, and the ascending or descending point, are far from being always the same, when the moon is at her greatest north and south latitudes: but, besides these, the inclination of the moon's orbit is

* See Mr. *Vince's Astronomy*: *System*, p. 61; *Principles*, p. 76.

assumed as known, and this cannot be precisely determined until the proper allowances for parallax are applied; so that, in this method, we are under the necessity of supposing that to be known, which the operation is designed to determine.

456. But the method described in Art. 90. is not liable to these objections; nor is the following, which is given because it is peculiarly applicable to the moon. In an eclipse of the moon, observe when both horns of the enlightened part are in the same vertical circle, and at that moment take the altitude of each; correct these altitudes for refraction, and half their sum is the altitude of the moon's centre clear of refraction. Now the true altitude at this time (as will appear from the doctrine of eclipses) is very nearly equal to the altitude of the centre of the earth's shadow; and, knowing the sun's place in the ecliptic, together with the time, we can find the sun's depression below the horizon, and this is equal to the altitude of the opposite point of the ecliptic, in which is the centre of the shadow. The difference between the true altitude, and the altitude freed from refraction, is manifestly equal to the parallax at that altitude. Then, as co-sine of the apparent altitude, to the sine of the parallax at that altitude; so is radius, to the sine of the horizontal parallax. Or the moon's parallax might be determined, by observing her altitude and right ascension, when her enlightened cusps in an eclipse are in a line parallel to the equator; but the operation would be more prolix, and the conclusion less accurate, than that deduced from the above method.

The moon's horizontal parallax is found to vary from about 61′ 32″ to 53′ 52″ (Art. 93.). According to M. *de la Caille*, the mean horizontal parallax is 57′ 5″; M. *le Monnier* states it at 57′ 12″; others have made it 57′ 18″. Variations like these arise

from the spheroidal form of the earth, and the frequent changes in the moon's distance.

457. When the moon's parallax is known, her distance from the earth may be found in the same way as we ascertained that of the sun (Art. 325.). M. *de la Hire*, by this method, found her distance when in the perigee to be about 56 of the earth's semidiameters, but in her apogee nearly $63\frac{1}{2}$: hence he states the mean distance at $59\frac{3}{4}$ or 60 semidiameters. M. *de la Caille*, from the variations in the parallax, deduces $64\frac{2}{3}$ for the greatest distance, $55\frac{3}{4}$ for the least, and $60\frac{1}{5}$ for the mean; corresponding nearly with the former. We may therefore, since the earth's semidiameter is nearly 4000 miles, state the moon's mean distance at about 240000 miles: and by comparing this with the distance of the sun from the earth, we shall find that the one is to the other nearly as 1 to 390.

458. PROB. VI. *To determine whether the curve described by the centre of the moon in one revolution, is any-where convex towards the sun.*

In order to simplify this problem, we will suppose the earth to revolve in a circular orbit round the sun as its centre, and the moon to move round the earth in the same manner; also, that the orbits are both in the same plane, and that the radii of the respective orbits are as 390 to 1. Then, by dividing the time of the earth's revolution through its orbit, by the respective times of the moon's periodic and synodic revolution, it will appear that there are 13·36881 of the former, and 12·36881 of the latter, performed in the time of one revolution of the earth. Let S (in fig. 1, Pl. VI.) represent the sun, E the earth, E H a portion of its orbit passed through during one synodic revolution of the moon; make S A : E A : : 12·36881 : 1; then the arc A K described from the centre S with the radius S A, will be equal to the circum-

ference of the circle E A F; and when the moon is in conjunction with the sun, the distance S M between the sun and moon will be much greater than S A. Now the curve described by the centre of the moon M, E M being the semidiameter of the moon's orbit, is the same as would be described by a point M in the radius E A of the circle E A F which rolls along the arc A K; it is therefore of the *cycloidal* kind. To determine whether it has a point of inflection, put S A or S R $= a$, E A or e R $= b$, E M or $e\,m = c$, R $m = r$, R $d = s$; and let m C be the *radius of curvature* at any point m, which, it is well known, must pass through the point of contact R. Suppose the point n indefinitely near to m; then R r on A K, and R r on $a\,r$ R, being the indefinitely small contemporary arcs with $m\,n$, and consequently the triangles R $m\,r$ and R $n\,r$ equal in all respects, if we consider the said little arcs R r, R r, as little right lines perpendicular to the radii $e\,r$ and S r, we shall have the angle m R $n = r$ R r = R $e\,r$ + R S r, because the angles e R r and S R r added to either m R n or r R r, make two right angles. Now S R : e R : : R $e\,r$: R S r, very small angles varying as their sines; hence S R : e R + S R : : R $e\,r$: R $e\,r$ + R S r, that is, $a : a + b$: : R $e\,r$: m R $n = \frac{a+b}{a}$ R $e\,r$. Again, in any triangle, as $d\,m\,r$, if the angles $m\,d\,r$, $m\,r\,d$, and R $m\,r$ the complement of the obtuse angle to two right angles, be indefinitely small, they will be proportional to the opposite sides $m\,r$, $m\,d$, and $d\,r$ *;

* For, let the triangle be circumscribed by the circle $r\,m\,d$ (fig. 2, Pl. VI.), then will the arches $d\,m$ and $m\,r$ differ infinitely little from their chords $d\,m$ and $m\,r$, which therefore may be taken as equal to them. And since (by Euc. III. 20.) the angle at the centre of a circle is double the angle at the periphery, the arches or chords $r\,m$ and $m\,d$ are the measures of double the angles $m\,d\,r$ and $m\,r\,d$ respectively; or the angles $m\,d\,r$ and $m\,r\,d$ are to each other as their opposite sides $r\,m$ and $m\,d$: and, because (by Euc. I.

that is, $dr : md :: Rmr : mrd$; and $dr - md : dr :: Rmr - mrd : Rmr$; or $mR : dR :: Rdr : Rnr$; that is, $r : s :: \frac{Rer}{2} : Rnr = \frac{s}{2r} Rer$. And again, $RCn : RnC :: Rn : RC$, that is, $mRn - Rnr : Rnr :: Rm : RC$, or $\frac{a+b}{a} - \frac{s}{2r} \times Rer : \frac{s}{2r} Rer :: r : RC = \frac{ars}{2ar + 2br - as}$. Consequently $mR + RC = mC = \frac{2ar^2 + 2br^2}{2ar + 2br - as} = \frac{r^2}{r - \frac{as}{2a + 2b}}$ = the radius of curvature at any point m.

Now, it is evident, that at the point of inflection, the radius of curvature must be infinite; or that, on one side of the said point, the expression for the radius of curvature must be *affirmative*, and on the other *negative*; therefore r must be *greater* than $\frac{as}{2a+2b}$ on one side of the said point, and on the other fide, *less*, and consequently *at* the point of inflection, $r = \frac{as}{2a+2b}$; which, substituted for r, makes $(dm \times mR =)\ rs - r^2 = \frac{2abs^2 + a^2s^2}{\overline{2a+2b}|^2} = b^2 - c^2$, because $dm \times mR = fm \times ma$; from this equation we have $s = \frac{\overline{2a + 2b}\sqrt{b^2 - c^2}}{\sqrt{2ab + a^2}}$. Or, to find r, since $r = \frac{as}{2a + 2b}$, $s = \frac{2ar + 2br}{a}$, and $(dm \times mR =)\ rs - r^2 = \frac{ar^2 + 2br^2}{a} = (fm \times ma =)$

32.) the angle Rmr is equal to the sum of the angles mdr and mrd, the measure of it is the sum of the arches or chords rm and md; which differing infinitely little from the side or chord rd, may be considered as equal to it; therefore the angles mdr, mrd, and Rmr, will be proportional to rm, md, and rd.

$b^2 - c^2$: from which we get $r = \sqrt{\frac{ab^2 - ac^2}{a + 2b}}$, when the point m becomes a point of inflection.

Now, as mR ($= r$) must, by the nature of the circle, always be greater than ma; that is, as $\sqrt{\frac{ab^2 - ac^2}{a + 2b}}$ must always be greater than $b - c$, and consequently $\frac{ab^2 - ac^2}{a + 2b}$ be greater than $\overline{b - c}|^2$, or $\frac{ab + ac}{a + 2b} \times \overline{b - c}$ greater than $\overline{b - c} \times \overline{b - c}$; but $\frac{ab + ac}{a + 2b} \times \frac{ac + 2bc}{ab + ac} = c$, and $\overline{b - c} \times \frac{ac + 2bc}{ab + ac} = \frac{b^2}{a + b}$; therefore c must always be greater than $\frac{b^2}{a + b}$; that is, E M must be *greater* than a third proportional to E S and E A, in order to have a point of inflection in the curve. But in the present instance E S, E A, and E M, are as 13·36881, 1, and $\frac{13 \cdot 36881}{390}$, or ·03428; consequently E M is considerably *less* than the said third proportional, and therefore the curve M mn I described by the centre of the moon, has *not* a point of inflection, or is *no-where convex towards the sun.*

459. But a solution to this problem may be deduced from physical principles, in a much shorter compass, and in a manner perhaps more satisfactory to the mind, as below: Let tT (fig. 13, Pl. V.) represent a portion of the earth's orbit, the circle L M the orbit of the moon, S the sun, and L the moon when in syzygy; let T represent the periodical time of the earth, and t that of the moon; then it is shewn by the authours on central forces (see *Emerson*, prop. 5. sect. I.) that the ratio of the forces by which the body L is attracted towards T, and towards S, is, at a mean, as $\frac{\text{TL}}{t^2} : \frac{\text{TS}}{\text{T}^2}$, or (multiplying both terms by t^2) as

TL : TS $\times \frac{T^2}{t^2}$; take therefore LA = TS $\times \frac{t^2}{T^2}$; and then, the force by which the body L is drawn towards T being represented by TL, the force by which it is drawn towards S will be represented by LA. Now these forces acting upon the body L (by its *vis insita** tending towards B along the tangent LB, drawn perpendicular to TA) in opposite directions, the greater force must necessarily draw it from the tangent BL towards itself, by a force which, at the instant the body moves from L, is equal to the difference betwixt TL and LA; and it is manifest, that according as the path of the body falls *above* or *below* the tangent BL (with respect to T), it will be *concave* or *convex* towards S. Now, in the proposed case of the moon, LA = TS $\times \frac{t^2}{T^2}$ = TS $\times \frac{1}{(13 \cdot 36881)^2}$ = TS $\times \frac{1}{178 \cdot 72}$; and TL being about $\frac{1}{390}$ $\times$ TS; it is manifest, therefore, that at L the path of the moon is considerably *concave* towards S; and if it be concave at L, it must necessarily be more so at any other point *l* where the force TL = *tl* is divided into two, *tc* and *cl*, and only the one *cl* which is perpendicular to the path has any power to bend it; therefore the path of the moon, being *concave* at L, is so throughout †.

* "The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line." *Motte's Translation of Newton's Principia, book* I. *definition* 3.

† This problem is considered very ingeniously by Mr. *Maclaurin*, in his *Account of Newton's Philosophical Discoveries*, book IV. ch. 5, where he also applies the theory of gravity to the determination of the absolute paths of some of the satellites of jupiter and saturn. And Mr. *Ferguson*, in his *Astronomy*, describes an instrument, which he calls the *Trajectorium Lunare*, by which the respective paths of the earth and moon may be delineated.

460. Because the gravity of the moon towards the sun is found to be greater, at her conjunction, than her gravity towards the earth, some have apprehended that the moon ought necessarily to abandon the earth. And, indeed, if the absolute velocity of the moon, at the conjunction, were less than that which is requisite to carry a body in a circle there around the sun, supposing this body to be acted on by the same force which acts there on the moon (*i. e.* by the excess of her gravity towards the sun, above her gravity towards the earth), then the moon would really abandon the earth. But though the absolute velocity of the moon at the conjunction be less than that of the earth in the annual orbit, and her gravity towards the sun, greater than that towards the earth, yet the former gravity is so much diminished by the latter, that her absolute velocity is still much superiour to that which is requisite to carry a body in a circle, there, about the sun, that is acted on by the remaining force only. Therefore, from the moment of the conjunction, the moon is carried without such a circle, receding continually from the sun to greater and greater distances, till she arrive at the opposition; where, being acted on by the sum of those two gravities, and her velocity being now less than what is requisite to carry a body in a circle, there, about the sun, which is acted on by a force equal to that sum, the moon thence begins to approach to the sun again; and thus recedes and approaches alternately, during every revolution about the earth.

461. Prob. VII. *To determine the moon's magnitude and time of rotation.*

The apparent diameter of the moon varies from about $29\frac{1}{2}'$ to $34'$; when the moon is at her mean distance from the earth, the diameter measures $31'\ 8''$ very nearly: then, as radius : 240000 miles (the moon's mean distance from the earth) : : sine of $31'\ 8''$: 2173·486 miles, real diameter of the moon. But as 240000 miles is probably somewhat greater

than the moon's mean distance (Art. 457.), being estimated from the earth's diameter in round numbers, the diameter above may be rather too small; it is, therefore, generally reckoned about 2180 miles. This is to the diameter of the earth nearly as 20 to 73; therefore, the surface of the moon is to that of the earth (being as the squares of their diameters) nearly as 1 to $13\frac{1}{3}$. And, admitting the moon's density to be to that of the earth as 5 to 4, their respective quantities of matter will be as 1 to 39 very nearly.

462. From the most careful observations upon the moon, it appears, that she has always very nearly the same side turned towards the earth: hence, if she turn upon her axis, *the time of rotation must be equal to that of the moon's synodic revolution* (Art. 440.), *or nearly* $29\frac{1}{2}$ *days*. Therefore, though the solar year be of the same absolute length, both to the earth and moon, yet the number of days in each is very different; the former having nearly $365\frac{1}{4}$ natural days, whilst the latter has only about $12\frac{7}{18}$; every day and night on the moon being as long as $29\frac{1}{2}$ on the earth. . . . . . It is also found by observations upon the moon's spots, and calculations, on nearly the same principles as those used to determine the inclination of the sun's axis (Art. 394.), that the axis of the moon is inclined to the ecliptic in an angle of 88° 17′ very nearly: and M. *Cassini* concludes, that the nodes of the lunar equator agree with the mean place of the nodes of the lunar orbit, and, consequently, their mean motions are the same.

463. Although nearly the same parts of the moon are turned towards the earth at all times, this is not so uniformly the case, but that she turns sometimes a little more of the one side, and sometimes of the other towards us; this is called the *moon's libration*, and is owing to her equable rotation about her own axis once in a month, in conjunction with her unequal motion in the perimeter of her orbit round the

earth. For, if the moon moved in a circle whose centre coincided with the centre of the earth, and turned round her axis in the precise time of her period round the earth, the plane of the same lunar meridian would always pass through the earth, and the same face of the moon would be constantly and exactly turned towards us. But since the real motion of the moon is in an orbit nearly elliptical, having the earth in one of its foci, and the motion of the moon about her axis is equable; that motion, as seen from the earth, must be unequal; for every meridian of the moon by this rotation, describing angles proportional to the times, the plane of no one meridian will constantly pass through the earth. Dr. *Gregory*, in his *Elements of Astronomy*, divides the *libration of the moon* into three kinds:

464. I. Her *libration in longitude*, or a seeming motion to and fro, according to the order of the signs of the zodiac. This libration is nothing, twice in every periodical month; viz. when the moon is in her apogee, and when in her perigee; for in both these cases, the plane of her meridian, which is turned towards us, is directed alike towards the earth.

465. II. Her *libration in latitude*, which arises from hence, that her axis not being perpendicular to the plane of her orbit, but inclined to it, sometimes one of her poles, and sometimes the other will nod, as it were, or dip a little towards the earth; and consequently she will sometimes shew more of her spots, and sometimes less of them, towards each pole. This libration, depending on the position of the moon, in respect to the nodes of her orbit, and her axis being nearly perpendicular to the plane of the ecliptic, is properly said to be in *latitude*. It is completed in the time in which the moon returns again to the same position in respect of her nodes.

466. III. There is also a third kind of libration;

by which it happens, that although another part of the moon be not really turned to the earth, as in the former libration, yet another is illuminated by the sun. For, since the moon's axis is nearly perpendicular to the plane of the ecliptic, when she is most southerly, in respect to the north pole of the ecliptic, some parts near to it will be illuminated by the sun, while, on the contrary, the south pole will be in darkness. In this case, therefore, if the sun be in the same line with the moon's southern limit, then, as she proceeds from conjunction with the sun towards her ascending node, she will appear to dip her northern polar parts a little into the dark hemisphere, and to raise her southern polar parts as much into the light one. And the contrary to this will happen two weeks after, while the moon is descending from her northern limit; for then her northern polar parts will appear to emerge out of darkness, and the southern polar parts to dip into it. And this seeming libration, or rather these effects of the former libration in latitude, depending on the light of the sun, will be completed in the moon's synodical revolution.

467. PROB. VIII. *To explain the phases of the moon.*

If the moon's surface were smooth and polished like a mirrour, it would not then reflect light upon all sides, and every way; but it would merely ſhew us, in some positions, the image of the sun, no larger than a kind of point, but with an immense lustre. But the moon being, like a primary planet, an opaque, rough, spherical body, reflecting the sun's rays falling upon it, it is evident that the half of it, turned towards the sun, is illuminated and bright, while the other half, which is turned from the sun, continues obscure and dark. Now, only that hemisphere of the moon which is towards the earth, can be seen by an inhabitant of the earth; consequently,

the phases of the moon will be various, according to the various habitude of the enlightened hemisphere to that which is turned towards the earth, as was shewn with respect to the phases of the inferiour planets. (Art. 403.). Thus, let Q T L (fig. 3, Pl. VI.) be a part of the earth's orbit, S the sun, T the earth, A C E G the moon's orbit. When the moon is in conjunction with the sun, as at E, because she is between the sun and earth, her illuminated hemisphere will be wholly turned from the earth, consequently her disc will be dark: this is the time of *new moon*. At A, being in opposition her illuminated hemisphere will be wholly towards the earth, and her whole disc visible: this is the time of *full moon*. At C and G, the apparent distance of the moon from the sun will be about 90°; whence the moon, at each of those situations, will be about half way between the conjunction and opposition, that is, about the middle state between the complete illumination and the entire obscuration of the disc; consequently, the disc will be about half enlightened: here, G is nearly the situation of the moon at the *first quarter*, and C, at the *third quarter*. In passing from G to A, and from A to C, the disc will be more than half illuminated, and will appear *gibbous*. In passing from C to E, and from E to G, the disc will be less than half illuminated by the sun, and will appear *horned*: but, when the moon is at D or F, near her conjunction, a considerable portion of the enlightened hemisphere of the earth will be towards the moon; and as the surface of the earth is more than thirteen times greater (Art. 461.) than that of the moon, the latter must in these situations receive a large degree of reflected light from the former; so that the darkened part of the moon is rendered visible, though it is by no means so brilliant as the part of the disc illuminated by the direct rays from the sun.

468. As to the quantity of the disc illuminated by

the sun, in any given position of the earth and moon, it may be determined by a process nearly similar to that which we had recourse to in finding the illumination of the inferiour planets (Art. 404.). Thus, for instance, when the moon is at F, the earth and sun being at T and S, the circle of illumination will be represented by *n m*, and the plane of vision will cut the moon in *q r*; *n q* is the arc of the illuminated part which is visible at T, and this will be projected into its versed sine, upon the circle of vision *q r*. The angle *n o q* = T *o* P; for *n o* P and T *o q* are right angles, and T *o n* is common to each, therefore the remainders, *n o q* and T *o* P, are equal; but the external angle T *o* P of the triangle T *o* S is equal to the sum of the angles S T *o* and T S *o*; therefore, if the latter angle be very minute, S T *o* may be considered as equal to T *o* P or *n o q*; and their versed sines will be equal. Now, in the present case, S T : T *o* : : 390 : 1 (Art. 457.), and the angle T S *o* will be the greatest the conditions of the enquiry will admit of, when S P is a tangent to the moon's orbit, that is, when T *o* S is a right angle; then T S : T *o* : : 390 : 1 : : radius : sine T S *o* = 8′ 48″. Hence, since the angle T S *o* never exceeds 8′ 48″, and is generally much less, the angles *n o q* and S T *o* may be looked upon as equal in delineating the moon's phases; and, following the same steps as were traced out in the latter part of Article 404. it would appear, that *as the diameter of a circle, to the versed sine of the moon's elongation from the sun; so is the whole disc of the moon, to that part of it which is illuminated by the sun;* which part might be laid down from the elongation being known, in the same manner as was done in figs. 8, 12, Pl. V. from the given exteriour angle S O P in fig. 10.

469. According to the mode of delineation here pointed out, the outward arc B P C of the enlightened part of the disc is a semicircle; and it is in

general *nearly* so: yet, strictly speaking, it is generally *greater*, and sometimes *less* than a semicircle. The moon's disc, as seen from the earth, is not bounded by a great circle of the moon, but by a parallel or less circle; and the visible segment is less than an hemisphere. The circle separating the enlightened and darkened parts of the moon, is likewise a parallel circle; the dark segment being less, and the light one greater, than the hemisphere. Therefore, when the dark segment of the moon is greater than the segment visible at the earth (which happens when the moon is in perigee), then two extremities of some diameter of the visible disc may be involved in the dark segment, and consequently the outward enlightened arc will at that time be *less* than a semicircle; but this can only take place about the change. And at almost every other time the outward enlightened arc will be *greater* than a semicircle. But the difference either with respect to excess or defect is so trifling, that it may be safely neglected in a construction which is designed not to ascertain, but to exemplify, a fact deduced from observation.

470. With respect to *the position of the horns of the moon*, it may be observed, that *before* the conjunction, or new moon, they are turned towards the *east*; *after* it, towards the *west*: for the sun, after the conjunction, setting before, that is, to the west of the moon, illuminates the west side of the disc, whence the horns, which are towards the dark part of the disc, are towards the east; the reverse takes place after the opposition, when the moon is seen in the east, before the sun rises. It will also be observed with respect to the moon's horns, that the line joining their *cusps*, or extreme points, varies much in its position relatively to the horizon, appearing at times nearly in a vertical direction, at others, so much reclined from the perpendicular as to be nearly horizontal; when, as the country people express it,

the moon lies on her back. To explain these changes of position, it must be considered, that the cusps of both horns will fall in the plane which divides the illuminated from the dark part of the moon, and that the situation of this plane, with respect to us, will always depend upon the relative situations of the sun and moon: thus, suppose C O H, in fig. 4, Pl. VI. to be the earth, *e q* its equator, E L the ecliptic, in which the sun is at S, M the place of the moon in her orbit M *m*; then *r s*, perpendicular to S M, will be the diameter of the moon's circle of illumination, or the line which connects the cusps of her horns; and it is manifest that the inclination of this line to the horizon will vary, as the places of M and S change with respect to C; that is, while the angle M C R decreases, and R C S increases, the angle O *u r* will decrease, and *z u r* increase; and when M C R increases, and R C S decreases, O *u r* will increase, and *z u r* decrease. Or we may, without any material errour in this elucidation, suppose the place of the moon to be at L in the ecliptic; for the planes of illumination *a i*, *r s*, will only differ from parallelism by the angle at S contained between the lines S L, S M: now, if we imagine the moon at M to be 90° from her nodes, the arc L M measuring the angle L C M will never exceed 5° 18′, and it will be very nearly as S L (391) : C L (1) : : sine 5° 18′ : sine $48''\frac{2}{3}$ = L S M; or if the moon were 90° from her nodes in *m*, then S E (389) : E C (1) : : sine 5° 18′ : sine $48''\frac{8}{9}$ = *m* S E. And stating the parallax O M C at 1°, it would, by a like analogy, be found to affect the angle at S not more than $\frac{12}{13}$ of a second: so that, whether we consider the moon as at M in her orbit, or L in the ecliptic, and the observer at O or at C, it can never cause a difference of 1′ in the position of the line *a i* or *r s* connecting the cusps. Suppose, therefore, the moon at L, and the

observer at C; then, $a\,i$ being the line of the cusps, and $p\,n$ parallel to C Z, or perpendicular to the horizon, Z C L will be equal to C $c\,n$, and Z C R and C $c\,i$ being both right angles, the angle $n\,c\,i$, measuring the inclination of the cusps to the vertical line, will be equal to the angle c C R, which is the inclination of the ecliptic of the horizon: hence it appears, that the inclination of the line joining the cusps, to the horizon, is nearly equal to the complement of the inclination of that part of the ecliptic in which the moon is at any time to the horizon;—and conversely, the inclination of that part of the ecliptic (or of the ecliptic itself) to the horizon may be ascertained, within a minute of a degree, by determining the deviation of the line joining the moon's cusps, from a vertical line: this, of course, furnishes us with an easy method of finding the altitude (Art. 155.) of the *nonagesimal degree*, whenever the cusps of the moon's horns can be clearly distinguished*.

471. Prob. IX. *To find the altitude of the lunar mountains.*

The face of the moon is greatly diversified with inequalities, and parts of different colours, some brighter and some darker than the other parts of her disc; and indeed there is abundant reason to conclude that there are very high mountains, and deep

* It may be observed, that the line which connects the cusps of the moon, is sometimes, though but seldom, *perpendicular* to the horizon: when this happens, the moon is in the nonagesimal degree; and this can never occur when she is on the meridian, except when she is in the first point of cancer or capricorn. For the line connecting the cusps is, manifestly, perpendicular to the plane of the triangle formed by lines joining the centres of the sun, earth, and moon, that is, by what has been shewn above, nearly perpendicular to the ecliptic: and it is evident from the nature of the sphere, that a line which intersects both the horizon and the ecliptic perpendicularly, must pass through the nonagesimal degree; and can only coincide with the meridian, when the first points of aries and libra are in the horizon, that is, when the first point of either cancer or capricorn is upon the meridian.

vales, upon the moon's surface. For, if there were no parts in the moon which were prominent or higher than the rest, then a right line in the *dichotomy* or quadrature, and an elliptic line in all the other phases, would terminate the light and dark parts of the disc: but when the moon is viewed with a telescope, we find that the confines of those parts appear, as it were, toothed, and cut with innumerable notches and breaks; and even in the dark parts, near the borders of the lucid surface, there are seen some apparently minute specks enlightened by the sun's beams. These shining points, situated without the confines of the illuminated surface, are, we conceive, the tops of high mountains, which rising far above the other parts of the surface, are sooner reached by the solar rays, and are therefore longer in the light than those parts which are lower. In all situations of the moon, the elevated parts are constantly found to cast a triangular shadow in a direction from the sun; and, on the contrary, the cavities are always dark on the side next the sun, and illuminated on the opposite one; all which is exactly conformable to what we observe of hills and valleys on the earth. Several astronomers, who have diligently observed the face of the moon, have concluded, that some of the mountains are volcanic: thus, Dr. *Herschel* informs us, that on April 19, 1787, he discovered three volcanoes in the dark part of the moon; two of them seemed to be almost extinct, but the third exhibited an actual eruption of fire or luminous matter, resembling a small piece of burning charcoal, covered by a very thin coat of white ashes; it had a degree of brightness about as strong as that with which such a coal would be seen to glow in faint day-light.

472. Some astronomers, as *Florenti*, *Langreni*, *Hevelius*, *Ricciolus*, *Cassini*, *De la Hire*, and others, have given maps of the face of the moon, as viewed through telescopes; of these the most celebrated are

those of *Hevelius*, in his *Selenographia*, in which he has represented the appearance of the moon in her different states, from the new moon to the full, and from the full to the new. Some of these authours called the spots and shining parts of the surface after the known names of different countries, islands, and seas, on the earth; while others, as *Langrenus*, and *Ricciolus*, designated them after the names of philosophers, mathematicians, and other celebrated men, giving the names of the most eminent characters to the largest spots: the latter mode of distinction is now generally adopted. Of these spots, forty whose situations are well ascertained, and which are supposed to be mountains, are noted by the names, *Archimedes*, *Plato*, *Aristarchus*, &c.; and about eight, being dark and very extensive, are supposed to be seas, and are called *Mare Nubium*, *Mare Nectaris*, *Mare Tranquilitatis*, &c.

473. But it is time to shew the method of measuring the altitude of the lunar mountains; and first we will describe that used by *Ricciolus*. Let D G E be the disc of the moon (fig. 5, Pl. VI.), E G D the boundary of light and darkness, and B A a mountain in the dark part, the top of which just begins to be illuminated: he measured, with a micrometer, or some other instrument, the proportion of A E to the diameter E D; then there were given the two sides A E, E C, of the right-angled triangle A E C, from which he found A C the hypothenuse, by taking the square root of the sum of the squares of the other two sides: thus having A C, either in terms of the moon's diameter, or in absolute numbers, and knowing B C, he, by subtraction, obtained A B, the altitude of the mountain. In this way, having observed the top of the mountain called *St. Catherine*, on the 4th day after the new moon, to be illuminated when it was distant from the confines of the enlightened hemisphere about a sixteenth part of the moon's diameter,

he determined its height to be nearly nine miles. *Galileo* makes A E only one twentieth of E D, and *Hevelius* only one twenty-sixth; the former of these gives about $5\frac{1}{2}$ miles, and the latter $3\frac{1}{4}$ miles, for A B the height of the mountain; and, probably, the smallest of these results is too large.

474. The preceding method will only give the true result when the moon is in quadratures, or the apparent disc dichotomized: the following general method is, therefore, given by Dr. *Herschel*, in the *Phil. Trans.* 1781. Suppose R C B the moon (fig. 6, Pl. VI.), F the place of the earth, A B the mountain: draw F A n and E o perpendicular to the moon's radius R C; also A e perpendicular to S A, and E r perpendicular to n F, or parallel to R C. Now E A, when seen from the earth at e, at the time of the moon's quadratures, would measure its full length; but it gives E r for its measure, when seen from F. The plane, in which are S A, F A, being perpendicular to a line joining the cusps, the circle R E B may be looked upon as a section of the moon perpendicular to that line; then it is manifest that the angle S E o, or E C R, is very nearly equal to the elongation of the moon from the sun (Art. 468.); and the triangles E r A, E C o, being similar (for o E R, C E A, are right angles, from each of which taking C E r, the remainders C E o, r E A, are equal, and, consequently, the right-angled triangles to which they belong are equi-angular), we have, therefore, E o :

$$EC :: Er : EA = \frac{EC \times Er}{Eo} = Er \text{ divided by the}$$

sine of elongation, radius being unity: then E A being determined, we find A B as in the method of *Ricciolus*. Dr. *Herschel* gives these directions for measuring the line E r; set the immoveable hair of the micrometer parallel to I K, then moving the other hair parallel to it from E to r on the disc, it gives the measure of E r. Or, observe some spot near to E,

to which the line E r is directed, or take a view of the shadow of some neighbouring mountains; either of these will point out a line perpendicular to that which joins the cusps, by which the micrometer may be set. Or, if the micrometer be furnished with a hair perpendicular to the moveable wire, and that hair be made to coincide with E r, it at once gives the position of the mountain.

475. Dr. *Herschel* observed a mountain in the south-east quadrant, and found the angle under which E r appeared to be 40″·625, when the sun's distance from the moon was 125° 8′: hence 40″·625, divided by ·8104 (the natural sine of the angle of elongation), gives 50″·13, the angle under which E A would appear, if seen from e. Now the semidiameter of the moon was 16′ 2·6″, and, taking its real length to be 1090 miles, we have E A = 56·73 miles, and A B = 1·47 miles.

476. Many of the lunar mountains were measured by Dr. *Herschel*, and the admeasurements led him to the remarks below: "From these observations I believe "it is evident, that the height of the lunar mountains, "in general, is greatly overrated; and that, when "we have excepted a few, the generality do not "exceed half a mile in their perpendicular eleva-"tion."

477. It is rather extraordinary, that, if we apply the mode of calculation given by Dr. *Herschel*, to the admeasurement of *Ricciolus*, the result will give the altitude of the mountain considerably greater than it was thought to be, even by *Ricciolus* himself. For, when the moon is four days old, her elongation from the sun will be about 48° 48′; therefore, ·0625 (E r) divided by ·7524 (nat. sine of elong.), gives ·083 for A E; whence A B is found to be nearly fifteen miles! This altitude considerably exceeding the bounds of probability, we cannot help concluding that some great errour attended the observation of

Ricciolus. Since *micrometers* were not invented, or, at least, the invention was not publicly known*, till about ten years before the death of *Ricciolus*, it is probable, either that he took the measure without a micrometer, or that the one he used was very incomplete, and in either case a material errour might take place †.

478. It has been a long controverted question, Whether the moon have an atmosphere, or not? But the very careful and accurate observations of modern astronomers have now nearly placed the matter beyond all doubt. Since the moon has mountains and valleys like the earth, and appears nearly the same with respect to shape, and the nature of her motions, a strong argument for the existence of a lunar atmosphere might be drawn from analogy: but we may appeal to more weighty evidence, the evidence of facts. Thus it has been observed, that the moon

* For, although Mr. *Gascoigne* invented the micrometer before the civil wars in the reign of *Charles* I. his invention was probably not at all known till after the publications of *Azout* and *Huygens*, in 1666 and 1659, wherein their respective inventions are described.

† Since the above was written, the authour has learnt that M. *Schroeter*, the learned and diligent astronomer of Lilienthal, published some time ago, a work on the heights of the lunar mountains, as compared with those of the earth; and lately he has published a new work on the height of the mountains of venus. He deduces his conclusions from the projection of the shadows formed by these mountains when they begin to appear on their horizon in regard to us, or when they are about to disappear below the horizon. He has found that the moon, which is about thirty-nine times smaller than the earth, has mountains more than four thousand toises in height; while that of Chimboraco, one of the Andes in South America, the highest mountain of our globe, is little more than three thousand. Venus, which is less than the earth by a ninth (according to his admeasurement), has mountains twenty-three thousand toises in height. The French toise is to the English fathom as 1· to 1·0664. The highest mountains on these three globes appear all to be volcanic. See *Journal de Physique, Prairial, An.* 7.

does not always appear with the same brightness, even when our atmosphere is equally clear. *Hevelius* relates, that he has several times found, in skies perfectly clear, when even stars of the 6th and 7th magnitude were visible, that at the same altitude of the moon, with the same elongation from the sun, and the same telescope, the moon and her maculæ did not always appear equally lucid and conspicuous, but were much more so at some times than at others. M. *Cassini* says, he often observed that saturn, jupiter, and the fixed stars, had their circular figures changed into elliptical ones, when they approached either to the moon's dark or illuminated limb. It is also found, that in total lunar eclipses the moon has very different appearances, in respect of colour; and, in the time of total solar eclipses, a luminous circle round the moon has been observed by many astronomers. All these circumstances, jointly considered, are strong indications of a lunar atmosphere. M. *Schroeter* made several observations in order to determine this point (see *Phil. Trans.* 1792, *Pt.* II.): he began his observation on the moon when two and a half days' old, in the evening soon after sun-set, before the dark part of the disc could be seen, and continued to observe till it was visible. The two cusps appeared tapering in a very sharp, faint prolongation, each exhibiting its farthest extremity faintly illuminated by the solar rays, before any part of the dark segment was visible. Soon after, the whole dark limb appeared illuminated. He concludes that the prolongation of the cusps beyond the semicircle must arise from the refraction of the sun's rays by the moon's atmosphere: but this might, in part, arise from another cause (Art. 469.). But he also observed that, at an occultation of jupiter's satellites, the third disappeared, after having been indistinct about 1″ or 2″ of time; the fourth also became indiscernible near the limb: this was not observed of the other two. Now, at the

height of about forty-five miles our atmosphere is so rare, as to be incapable of refracting the rays of light; this is about the 180th part of the earth's diameter: but since clouds are never observed higher than four miles (generally not one mile), it appears that the vapourous or obscure part cannot be more than the 1980th of the diameter. The mean apparent diameter of the moon is about 1870″; therefore, supposing her to have an atmosphere similar to ours (though it is probably much less dense, and less capable of sustaining vapours), the obscure parts of it, when viewed from the earth, must subtend an angle of less than one second, which space is passed over in less than two seconds of time: this is very conformable to the time which the satellites appeared indistinct, as observed by M. *Schroeter;* so that, considering all the facts here adduced, there is much greater reason to conclude that the moon has really an atmosphere, than the nature of the enquiry would, at first, lead us to expect, or hope for.

479. Prob. X. *To explain the appearance and cause of the harvest moon.*

It is remarkable that the moon, during the week in which she is full about the time of harvest, rises sooner after sun-setting, and with less difference between the times of two successive risings, than she does in any other full-moon week in the year. By these means she affords an almost immediate supply of light after sun-set, which is very beneficial for gathering in the harvest and fruits of the earth: hence this full moon is distinguished from all others in the year by the appellation of *the harvest moon.* Mr. *Ferguson*, in the 16th chapter of his Astronomy, has given a full explanation of these phenomena, as they appear in different climates of the earth: so much of it as suits our present purpose has been abridged, as below.

480. To conceive the reason of the harvest moon,

it may first be considered, that the moon is always opposite to the sun when she is in full; that she is full in the signs pisces and aries in our harvest months, those being the signs opposite to virgo and libra, the signs occupied by the sun in that season; and that those parts of the ecliptic (in which the moon then is) rise from the horizon in northern latitudes in a smaller angle, and, of course, equal spaces, in shorter intervals of time than any other points, as may be easily shewn by the celestial globe; consequently, since the moon's orbit deviates not much from the ecliptic, she rises with less difference of time, or more immediately after sun-set, about harvest, than when she is full at other seasons of the year.

481. The moon rises with nearly the same difference of times in every revolution through her orbit, when she is in the signs pisces and aries; and hence it may be asked, why we never observe this remarkable rising but in harvest? The answer is obvious: for in winter these signs rise at noon, and, being then only a quarter of a circle distant from the sun, the moon in them is in her first quarter, and rising about noon, when the sun is in his meridian, the phenomenon is neither perceived, nor regarded. In spring these signs rise with the sun, because he is then in them; of course the moon, in conjunction, rises nearly with the sun, and is invisible. In summer these signs rise about midnight, and the sun being then three signs, or a quarter of a circle, before them, the moon is in them about her third quarter, when rising so late, and giving but little light, the peculiarity of her rising passes unobserved. But in autumn these signs, being opposite to the sun, rise when he sets, with the moon in opposition, or at the full, which makes her regular rising very conspicuous at that time of the year. The sun enters libra about the 23d of September, and the full moon which is nearest that day, is, properly speaking, the *harvest moon.*

482. These circumstances would regularly take place, if the moon's orbit coincided with the plane of the ecliptic; but, as her orbit makes an angle with the ecliptic, varying from 5° to 5° 18′, and crosses it only in the nodes, her rising when in pisces and aries will sometimes not differ above 1^h^ 40′ through the whole of seven days; and at other times, when in the same two signs, the time of her rising, in the course of a week, will differ full 3½ hours, according to the different positions of the nodes with respect to those signs; which positions are constantly changing, the nodes going backward (Art. 448.) through the whole ecliptic, in about eighteen years, 228 days. This revolution of the nodes will cause the harvest moons to go through a whole course of the most and least beneficial states, with respect to the harvest, in about nineteen years. The following table shews in what years the harvest moons are most or least beneficial, from the year 1800 to 1861: the columns of years under the letter L are those in which the harvest moons are least beneficial; those marked M shew when they are most beneficial; the former falling nearest the descending node, the latter nearest the ascending node.

HARVEST MOONS.						
M	L	M	L	M	L	M
1800	1807	1816	1826	1835	1844	1853
1801	1808	1817	1827	1836	1845	1854
1802	1809	1818	1828	1837	1846	1855
1803	1810	1819	1829	1838	1847	1856
1804	1811	1820	1830	1839	1848	1857
1805	1812	1821	1831	1840	1849	1858
1806	1813	1822	1832	1841	1850	1859
	1814	1823	1833	1842	1851	1860
	1815	1824	1834	1843	1852	1861
		1825				

483. PROB. XI. *To account for the great apparent magnitude of the horizontal moon.*

The great apparent magnitude of the moon, and indeed of the sun, at rising and setting, is a phenomenon which has greatly embarrassed almost all who have endeavoured to account for it. According to the ordinary laws of vision, they should appear to be least when nearest the horizon, being then farthest from the eye; and yet it is found that the contrary is the fact. The apparent diameter, when viewed in the horizon by the naked eye, is two or three times larger than when at the altitude of 30° or 40°; and yet when measured by an instrument her diameter is not increased at all.

484. *Gassendus*, *Hobbs*, *De Veil*, Dr. *Wallis*, *Des Cartes*, and others, have attempted to account for this phenomenon; but as their respective hypotheses are now abandoned, it would be useless to recite them in this place. Dr. *Desaguliers* appears to have been the first who came near the true explication of the appearance: he proceeded on the supposition that the visible heavens seemed to an observer to be only a small portion of a spherical surface, he would, therefore, regulate his judgment of the magnitude of objects agreeably to this notion. Mr. *Rowning*, in his *System of Natural Philosophy*, assigned the same cause of the phenomenon, but did not enter into a full explanation, because he was in expectation of seeing the subject considered at large, in Dr. *Smith's Optics*, which were then preparing for the press. Accordingly, when Dr. *Smith*'s treatise appeared, it was found that he had pursued the idea of Dr. *Desaguliers* with much ingenuity and success: the mode of illustration he adopted is here given, premising his determination of the apparent curvature of the sky.

485. "The concavity of the heavens," says the doctor, "appears to the eye, which is the only

judge of an apparent figure, to be a less portion of a spherical surface than a hemisphere: I mean that the centre of the concavity is much below the eye; and by taking a medium among several observations, I find the apparent distance of its parts at the horizon is generally between three and four times greater than the apparent distance of its parts overhead. For, let the arch A B C D (fig. 7, Pl. VI.) represent the apparent concavity of the sky, O the place of the eye, O A and O C the horizontal and vertical apparent distances, whose proportion is required. First observe when the sun, or the moon, or any cloud or star, is in such a position at B that the apparent arches B A, B C, extended on each side of this object towards the horizon and zenith, seem equal to the eye; then taking the altitude of the object B with a quadrant, or a cross-staff, or finding it by astronomy from the given time of observation, the angle A O B is known. Drawing therefore the line O B in the position thus determined, and taking in it any point B at pleasure, in the vertical line C O produced downwards seek the centre E of a circle A B C, whose arches B A, B C, intercepted between B and the legs of the right angle A O C, shall be equal to each other; then will this arch A B C D represent the apparent figure of the sky. For by the eye we estimate the distance between any two objects in the heavens by the quantity of sky that appears to lie between them; as upon earth we estimate it by the quantity of ground that lies between them. The centre E may be found geometrically by constructing a cubic equation, or as quick and sufficiently exact by trying whether the chords B A, B C, of the arch A B C drawn by conjecture, are equal; and by altering its radius B E till they are so. Now, in making several observations upon the sun, and some others upon the moon and stars, they seemed to me to bisect the vertical arch A B C at B, when their apparent altitudes or

the angle A O B was about 23 degrees; which gives the proportion of O C to O A as 3 to 10, or as 1 to $3\frac{1}{3}$, nearly. When the sun was but 30 degrees high, the upper arch seemed always less than the under, and I think always greater when the sun was about 18 or 20 degrees high.

486. " I have been the more particular in considering the apparent figure of the sky, because I do not find it has ever yet been determined; although it be absolutely necessary to a satisfactory solution of several noted appearances in the heavens. For instance, supposing the arch A B C to represent that apparent concavity (fig. 8, Pl. VI.). I find the diameter of the sun or moon will seem to be greater in the horizon than at any proposed altitude, measured by the angle A O B, in the proportion of its apparent distances O A, O B. The numbers that express these proportions are set down in this table, over against the corresponding altitudes of the sun or moon, and are also exactly represented to the eye in the figure, in which the moons placed in the quadrantal arch F G, described about the centre O, are all equal to each other, and represent the body of the moon at the heights there noted; and the unequal moons in the concavity A B C are terminated by the visual rays that come from the circumference of the real moon, at those heights, to the eye at O. The diameters of these unequal moons at A and B do, therefore, bear the same proportion to each other as their apparent distances O A, O B; and they must appear in the very same proportion that they really have in this concave, because we judge all objects in the heavens to be in this very surface; and so the appearance to the eye is exactly the same as if several moons were painted upon a real surface A B C in the proportions here assigned;

The sun's or moon's altitudes in degrees.	Apparent diameters or distances.
00	100
15	68
30	50
45	40
60	34
75	31
90	30

in which case we should certainly judge the real magnitudes of the larger paintings of the lower moons to be really larger; though the visible magnitudes of them all, answering to their equal images upon the retina, were exactly equal."

487. In confirmation of this theory of the cause of the horizontal moon, Dr. *Smith* observes, that if we take two stars near each other in the horizon, and two others near the zenith, at the same real angular distance from each other, the two former will appear at a much greater distance from each other than the two latter. Upon the same principle also the lower part of a rainbow appears broader than the upper part: and from this there is reason to conclude that the faintness of the object enters not into any part of the cause. The doctor also, in his *Remarks* in vol. II. farther adds the following observations upon this subject. "In these articles I have shewn in general why the moon appears always larger in the horizon than in the meridian, and have confirmed the reason of it by experiments. I say, *in general*, because it is agreed, that at different times the horizontal moon appears of different magnitudes even in the same horizon, and now and then of a size extraordinarily large. This, I am inclined to believe, is chiefly owing to an extraordinary largeness of her picture on the retina at those times; which picture in the present theory was supposed invariable. This might best be examined by taking her diameters with a micrometer; or, because this instrument is seldom at hand, by registering the year and day of the month, together with the heights of the barometer and thermometer. For if it should appear by many such observations that the largest horizontal moons happened generally at her perigee in the warmest summer evenings, the barometer being low and the thermometer high; and, on the contrary, that the

smallest horizontal moons happen generally at her apogee in the coldest winter mornings, the barometer being high and the thermometer low; we might reasonably conclude, that these extraordinary moons are chiefly owing to the concurrence of these three independent circumstances.

488. "Since the picture of the horizontal moon is constantly more or less contracted by refractions, and that of the meridional moon at her greatest height with us, of 60 or 65 degrees, scarce sensibly, it follows that in these two positions, the diameters of her disc considered as portions of the concave sky, must be in a less ratio than that of their apparent distances in the sky. Because, in the present theory, the moon's picture was supposed invariable during her ascent or descent. The ratio of those apparent distances, by the table in Art. 486. is about 1 to 3, or 3 to 9; and this would be the ratio of the diameters of her circular discs in the concave, but for refractions; by which alone I reckon the horizontal disc is generally contracted in the ratio of two circles whose diameters are nearly as 9 to 8. Therefore the highest meridional disc is generally to the horizontal one but as 3 to 8 in diameter."

489. Prob. XII. *To find the moon's age, and time of coming to the south.*

To find the moon's age in days at any given time, requires a knowledge of the *epact*, which is the excess of the solar month above the lunar synodical month; or of the solar year above the lunar year of 12 synodical months; or of several solar months or years, above as many synodical months, or dozens of synodical months. The epact for the beginning of any year may be found very easily, by reckoning from the time of the last *mean* conjunction; but it is generally stated among the chronological notes in the Almanacs. The monthly epacts are the moon's age

at the beginning of each month, when the solar and lunar years begin together; and are thus:

0	2	1	2	3	4	5	6	8	8	10	10
Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.

490. Then knowing the epact at the beginning of the year, to it must be added the number of the month in the above table, and the day of the month; their sum abating thirty, if it exceed that number, is the moon's age; and her age taken from 30, shews the day of the change.

491. To find nearly the time of the moon's southing or coming to the meridian, take $\frac{4}{5}$ of her age, which will be nearly the time; after *noon* if it be less than 12 hours; but if greater, the excess is the time after the last *midnight*.

492. But the above rule being merely a gross approximation, one of the following may be used, as being more accurate.—Calculate the longitude and latitude of the moon, as also the longitude of the sun, and their right ascension, for noon, or any other hour of the given day, in the meridian of the given place; and subtract the sun's right ascension from the moon's, both being expressed in time; the remainder is nearly the time when the moon is on the meridian. Calculate the right ascensions of the sun and moon again for this time, and then subtract as before; the true time when the moon is on the meridian will be expressed by the remainder. If great precision is required, calculate the right ascensions again for the time now found, and subtract as before.

493. Or, the moon's right ascension being calculated from her longitude and latitude, and converted into time, subtract the sun's right ascension in time at noon from it, to find the time of the moon's

passing the meridian nearly, which call T; take the difference of the sun's and moon's daily variations in right ascension in time, which call X; then say, by the rule of proportion, as $24^h - X : T :: X : e$, and $T + e$ will be the correct time of the moon's passing the meridian.

494. But it may, in some instances, be found more readily by continual approximation as follows: Take the proportional part of the difference of the sun's and moon's daily motion in right ascension, answering to the time of the moon's passing the meridian, found nearly in proportion to 24 hours; and take a farther like proportional part of this proportional part; and again of this last, and so on as far as is necessary. The sum of all these proportional parts, added to the approximate time of the moon's passing the meridian, gives the apparent time of her passing the meridian. N. B. The last two rules may, without difficulty, be applied to the determination of the culminating of the planets; observing only, if the planet's motion be retrograde, to take the sum, where the difference is required for the moon, *et contra*; and, if the planet's motion in right ascension be less than that of the sun, the proportion in Article 493. becomes $24^h + X : T :: X : e$, and $T - e$, will be the correct time.

Dr. *Maskelyne* has thus exemplified the last rule: Let it be required to find the time of the moon's passing the meridian July 1, 1767? . . . The sun's R. A. in time, July 1, is $6^h\ 40^m\ 25^s$, and July 2, is $6^h\ 44^m\ 33^s$ by the *Ephemeris*. Therefore his daily motion in R. A. is $4'\ 8''$. The moon's R. A. July 1, at noon, by the *Ephemeris*, is $159°\ 2'$, answering to $10^h\ 36^m\ 8^s$ of time, and July 2, is $169°\ 39'$, answering to $11^h\ 18^m\ 36^s$. The difference is $42^m\ 48^s$ of time, from which $4^m\ 8^s$ being subtracted, leaves $38^m\ 20^s$. Subtract $6^h\ 40^m\ 20s$ the sun's R. A. July 1, at noon, from $10^h\ 36_m\ 8^s$, the moon's R. A. the same noon, the remainder $3^h\ 55^m\ 43^s$ is the approx-

imate time of the moon's passing the meridian. The proportional part of $38^m\ 20^s$ answering to this, is $6^m\ 17^s$, and the proportional part of $6^m\ 17^s$ is 9^s; therefore $6^m\ 17^s$ and 9^s, or $6^m\ 26^s$, added to $3^h\ 55^m\ 43^s$, give $4^h\ 2^m\ 9^s$, the apparent time of the moon's passing the meridian.

CHAPTER XVII.

On the Satellites.

Art. 495. Lord Bacon, in his *advancement of Learning*, when treating on the best way "of drawing down, as it were, from Heaven, a whole shower of new and profitable inventions," lays considerable stress on "*the fortunes or adventures of experiment*," which he observes may frequently discover "some secret of great use; since *the wonders of Nature commonly lie out of the high road, and beaten paths.*" And indeed we shall often find that apparently trivial circumstances have led, by a series of lucky incidents and conjunctures, to very important consequences, with which they did not at first seem to have the most remote connection. Who would have ever conjectured that any thing beneficial to the cause of commerce, or of science, would have resulted from so trifling an occurrence as that of some children amusing themselves with spectacle-glasses? Yet there is reason to believe that the children of one *Zachariah Hansen*, a spectacle-maker of Middleburgh in Holland, by mere accident, while engaged in play, placed a concave and convex glass in such a manner, that in looking through them at the weather-cock of a church, it appeared to be nearer and much larger than usual; and that this circumstance coming to the knowledge of *Galileo*, led him, if not to the first invention of telescopes,

at least to a considerable improvement of them, and to their first application to astronomical purposes. These instruments, in the hands of *Galileo,* and *Thomas Harriot* (the celebrated algebraist), soon enriched astronomy by the discovery of the *satellites of jupiter;* and fortified it with strong confirmations of the hypothesis of Copernicus. (Art. 403.) Observations on the satellites of jupiter have paved the way to the discoveries of the *velocity of light*, and of its *aberration*, which accounts for an apparent motion of the fixed stars (Art. 631.): and by means of these satellites, we are enabled to perform, with facility, that very important problem in navigation, which requires the determination of the longitude of places on the earth. (Art. 667.)

496. Each of the planets, *jupiter, saturn,* and *georgium sidus,* is attended by satellites; of which jupiter has four, saturn seven, and georgium sidus six. In order to arrive at certainty concerning the laws of these bodies, it is necessary to consider facts; we therefore briefly describe the chief phenomena.— 1. These satellites are sometimes to the eastward, sometimes to the westward, of their respective planets, moving successively from one side to the other: each at its greatest excursion, as observed from the earth, is nearly as far distant from the one side as it was from the other; and is found on the same side again in much about the same interval of time. 2. All the satellites (except those of georgium sidus), in going from the western excursion to the eastern, are often hid by the planet's disc, and consequently pass behind it: sometimes one or other of them passes *above* or *below*, but never *on*, the planet's disc. On the contrary, in going from the eastern excursion to the western, those which passed behind, now pass over the disc; and those which passed above, now pass below, and reciprocally. 3. The paths of the satellites being reduced to their respective planet's centre, sometimes appea

rectiline passing through that centre, and inclined in a certain direction to its orbit. Afterwards, they change more and more into ellipses, during one quarter of the planet's annual revolution; and all the superiour conjunctions are then made *above* the planet's centre, and the inferiour conjunctions *below* it: during a second quarter of the planet's revolution, these ellipses become narrower, the satellites are nearer the centre in their conjunctions, and at the end of a second quarter of the revolution, all the ellipses are again become right lines with equal inclination, but in a contrary direction. In the third quarter of the revolution, they are formed anew into ellipses, the superiour conjunctions are made below the centre, and the inferiour ones above; lastly, in the fourth quarter of the revolution, when the planet is returning to the same point of its orbit, these ellipses again decrease in breadth, and all returns to its first state. 4. The times of the superiour and inferiour conjunctions of the satellites being compared, their intervals are nearly equal to their semirevolution.

497. From the first of these phenomena, considered jointly with the fourth, we conclude, that *the orbits of the satellites are curves returning into themselves,* like a circle or ellipsis; that *the primary planet is in one of the diameters of each curve;* and that *the motions of the satellites are nearly uniform.* From the second, it follows, that *all the satellites* (except those of the georgian planet), *go one way;* that *the primary planet is within their orbit,* or, which amounts to the same, that *they move round their primary planet;* and that *the planes of the orbits of the satellites are inclined to the plane of the orbit of their primary.* And, from the third phenomenon, which is a consequence of the obliquity of the satellite's orbit, it appears that the paths of the satellites have similar appearances to the spots of the sun and planets. It follows, therefore, 1. When the paths

of the satellites appear rectilinear, then the earth is in the right line where the plane of their orbit intersects the plane of the planet; (this is called the *line of the nodes;* the *ascending node* is that where the superiour conjunctions commence above the planet's centre; and the *descending node*, that where the superiour conjunctions are formed below the planet's centre): therefore all the conjunctions of the satellite are then made opposite to the centre of the planet. 2. The inclination of these right lines, shews the inclination of the plane of the satellite's orbit, to that of the planet's orbit.

498. Since, then, the satellites uniformly describe orbits nearly circular or elliptical, with their respective primary at their centre or focus, they are probably moved by a force of the same nature with that which moves their planets round the sun; that is, they revolve about their primaries in consequence of a central force, and of a constant impulsive force: if so (which indeed observations render certain), they must follow *Kepler*'s two rules (Art. 209.), namely, 1. *The satellites must describe areas of their orbits proportional to the times;* 2. *Their mean distances from the centres of their respective primaries, must be as the cube roots of the squares of the times of their revolutions.*

499. The time of a *synodic revolution* of a satellite may be found in the following manner: Observe, when the primary planet is in opposition, the passage of a satellite over its body, and mark the time when it is half way between the two opposite edges of the planet's disc, for then it will be nearly in conjunction with the centre of the planet, and also in conjunction with the sun. After a long space of time, observe again when the primary planet is in opposition, and the secondary in conjunction with its centre; and divide the intervening space of time between the two observations, by the number of conjunctions of the sun in that space, the quotient is the time of a syno-

dic revolution. Or, the same thing may be found by means of the eclipses of the satellite: observe when the satellite enters the shadow of its primary, called its *immersion;* or when it comes out of the shadow, called its *emersion;* and after a very long interval of time, when an eclipse happens as nearly as possible in the same situation both with respect to the node and to the place of the primary planet, again mark the time of the *emersion* or *immersion* (whichever was used in the former observation); and from the interval of these times, and the number of eclipses in that interval, the mean time of a synodic revolution will be obtained by division.

500. To determine the time of a *periodic revolution,* it must be considered, that in the return of a satellite to its mean conjunction, it describes a revolution in its orbit, together with the mean angle m, described by the primary planet in that time: hence this analogy, *as* $360° + m : 360 : :$ *time of a synodic revolution : the time of a periodic revolution.*

501. The distance of a satellite from its primary may be easily found by means of its greatest elongation, as seen from the earth, in the following manner: Let S (fig. 1, Pl. VII.) represent the sun, E the earth, P any planet, one of its satellites at A, and the angle of elongation P E A a maximum; then the distances P E and E S are known from the theory of the planet, and the time of observation; and since the angle P E A is known in the right-angled triangle A E P, also the side P E, the side A P is readily obtained. After the same manner, when the earth and planet are in any other situations, e, p, and the satellite at a at its greatest elongation, find $e\,p$ by the theory of the earth and planet, and the given time, whence from the angle of elongation $p\,e\,a$, determine the distance $p\,a$. Make a series of such observations at all suitable opportunities, and calculate $a\,p$ or A P from each of them; for thus we may ascertain the

greatest and least distance of the satellite from its primary; and half the difference of these will give the excentricity of the orbit. These elements, if not determined accurately, may be corrected by subsequent observations.

501. The distances of a satellite from the centre of its primary may also be found, by measuring with a micrometer *, at the time of the satellite's greatest elongation, its distance from the centre of the planet, also the semidiameter of the planet; for then the distance is known in terms of that semidiameter. Or, if the planet and satellite cannot at once be brought into the field of view of a telescope (which may sometimes happen), the distance of the satellite may be measured, at its greatest elongation, by observing the time of the passage of the planet's disc over a wire adjusted as an hour circle in the field of a telescope, and comparing it with the interval between the passage of the planet's centre, and that of the satellite. To give this method the greatest degree of accuracy, the observations should be repeated so long as the interval from the passages of the planet to that of the satellite continues increasing; for when it begins to decrease, the satellite will have passed its greatest elongation. The methods given in this article, may also be applied to the determination of the distances when in the apsides, the excentricity, and the greatest equation, by means of a long series of observations.

502. Or, when the periodic times of all the satellites of a planet are known, and the mean distance of one of them; the mean distances of the others may be found from the proportion between the squares of the periodic times, and the cubes of the distances.

* Mr. *Flamsteed* invented an instrument for finding the distances of jupiter's satellites from his axis, a description of which is given in the *Phil. Trans.* No. 178, or *Abridgement*, vol. I.

And, on the contrary, if the relative distances of all the satellites from their primary be known, and the periodic time of one of them, the periodic times of the others may be found by the same proportion.

503. Attempts have been made to determine *the magnitudes of the satellites*; but they have not been very successful: indeed it is not to be expected that they should. Sometimes the magnitudes have been ascertained by observing the times which they occupy in entering into the shadow of their primary in a central eclipse (Art. 505.): but since we cannot tell the exact instant when the satellite enters or quits the shadow, this method will give the diameters too small. The diameters have also been sometimes measured, by taking the diameters of their shadows upon the disc of their primary; or by noticing the length of time occupied in entering the disc, when they pass centrally over it: but the difference of the conclusions resulting from these methods, shews that we must not place much dependance upon them. M. *Bailly* and *La Grange* have endeavoured to ascertain the quantities of matter in the satellites of jupiter, by considering the effects of their actions upon each other; yet, although by proper assumptions these may be nearly determined, still, as the densities are unknown, we cannot find the diameters by theory alone; nor, indeed, with accuracy, by a union of theory and observation.

504. *Cassini*, *Maraldi*, and *Pound*, conclude from their observations, that the satellites revolve upon their axes; but their very great distances prevent the determination of the periods of their rotation.

505. *The eclipses of the satellites* are of considerable importance to astronomy and geography; it therefore becomes requisite to explain them. To this end, let S (fig. 2, Pl. VII.) be the sun; E *e* the orbit of the earth; P the place of the planet; *v s o* the orbit of a satellite: then, it is evident, that when-

ever the satellite, in its orbit, passes through the shadow of the primary, its light is obscured, and it becomes *eclipsed*; the duration of the eclipse will depend upon the obliquity of the satellite's orbit, to that of the planet, and the distance of the satellite from the node: for (fig. 3, Pl. VII.) a less portion *m n* of the inclined orbit will pass through the shadow O I *m n*, than if the orbit coincided with O N; and in the nodes the satellite is both in its own orbit, and that of the planet, and *n m* becomes equal to O I. Now (fig. 2, Pl. VII.), when the earth is at E before the opposition of the planet, the spectator will see the *immersion* at *i*, and in some particular instances, the *emersion* at *o* also: when the earth is at *e* after opposition, an observer will see the *emersion* at *o*; but whether he sees the *immersion*, will depend upon the position of the earth with respect to the plane of the planet's orbit. When the earth is at E, the conjunction of the satellite happens later at the earth, than at the sun; but when it is at *e*, the conjunction occurs later at the sun than at the earth. If the earth be at *e*, and the satellite at *v*, it cannot be seen by an observer at *e*, because the body of the planet intervenes; this is not an eclipse, but an *occultation*. In order to find the diameter of the planet's shadow at the distance of any of the satellites, let the time of an eclipse (that is, the time occupied from the immersion to the emersion) be observed when the satellite is in one of its nodes, for then the satellite passes through the centre of the shadow, and the eclipse is called *central*: then use this analogy, *as the time of a synodic revolution of the satellite* : *the duration of the eclipse* : : 360° : *the diameter of the shadow, in degrees of the satellite's orbit.* But ſhould it so happen that both the immersion and emersion cannot be seen when the satellites are in the nodes, which is always the case with the first and second of jupiter's satellites, then it is usual to compare the *immersions*

some days before the opposition of the planet, with the *emersions* some days after, and hence, knowing how many synodic revolutions have been made in the intervening time, the time of the transit through the shadow, and consequently the corresponding measure in degrees, become known.

506. Suppose in fig. 15, Pl. V. N $b\,d\,n$ the plane of a planet's orbit, N $a\,c\,n$ the orbit of one of its satellites, N n the line of the nodes; draw P c, P d, perpendicular to N n, and $c\,d$ perpendicular to the plane of the planet's orbit; then, supposing a to be the point in opposition to the sun at S, let $a\,b$ be drawn parallel to $c\,d$. Put s for the sine of c P d, the inclination of the orbit of the satellite to that of the planet (radius being 1), and z for P c; then $1 : s :: z : dc = s\,z$; and if w denote the sine of N a the distance of the node from the opposition, we have $1 : (d\,c)\,s\,z :: w : a\,b = w\,s\,z$, the latitude of the satellite at the time of opposition.

507. Now, to find when an emersion will be visible before opposition, and an immersion after opposition, we have the following method given by M. *de la Lande*. Let s (fig. 4, Pl. VII.) be the centre of the shadow A B of the planet, r the centre of its disc C D, the radius $s\,n$ being expressed in minutes of the orbit of the satellite, and $r\,n$ in the same measure: let $r\,s$ be a portion of the orbit of the satellite equal to the annual parallax *, expressed also in minutes, and let A B and C D be represented as seen from the earth; that is, let the diameter of C D be equal to the minutes in the planet's apparent diameter; and let the diameter of A B be to that of C D, as the time in which the satellite transits the planet's disc when in the node, to the time in which it passes

* The angle S P c subtended by S e the distance of the earth from the sun, at the planet P, is called the *annual parallax* (fig. 2, Pl. VII.): it is manifestly equal to the angle s P v, which is measured by the arch $s\,v$ of the satellite's orbit.

through the shadow. Then if $w c n$ be the path of the satellite, the immersion at w being visible, the emersion at n will also be just visible, for n is the limit between which and m if the emersion happen it will be invisible. In the triangle $r n s$ we know all the sides, and thence find the perpendicular $n m$ let fall on $r s$, which as the orbit $w n$ is very nearly parallel to $r s$, will be very nearly equal to $s c$: but (Art. 506.) $s c = w s z$; or, supposing z to be repreented by 1, $s c = w s$; make therefore $m n = w s$, and from thence deduce w the sine of the distance of the planet from the node. Here the earth is supposed in the plane of the planet's orbit; but as it is not in that plane, the planet will appear higher or lower in the shadow by the latitude of the earth seen from the planet; this varies as the sine of the distance of the earth from the node of the planet. If $r s$ represent the orbit of the planet, the centre of the shadow will, for six months of the year, lie to the south of s, suppose at t; then, knowing $s r$ and $s t$, we can find $r t$ and the angle $s r t$; and therefore we know the three sides of $n r t$, and thence find $n r t$, which lessened by $s r t$ gives $n r s$: therefore, lastly, in the triangle $n r s$, are known $r n$, $r s$, and the included angle $r n s$, whence we find $n m$, which determines whether the emersion is visible or invisible.

508. The place of a satellite's node being determined (Art. 497.), we are enabled, by the eclipses of that satellite, to ascertain the inclination of its orbit: for, let I C O D (fig. 3, Pl. VII.) be a section of the shadow of the planet through which the satellite passes, N I O the plane of the orbit of the planet, N $m n$ the orbit of the satellite; draw Pa perpendicular to Nn, and (Art. 506.) $Pa = w s z$, w denoting the sine of Na the distance of the node, s the sine of the orbit's inclination, and z representing Pa. Let $r = PI$, and $h = m a = a n$; then

$\sqrt{r^2 - h^2} = w s z$: hence $s = \frac{\sqrt{r^2 - h^2}}{w z}$. Here we must convert r, z, and h, into time; that is, r must denote half the greatest duration of an eclipse, h half the real duration of the eclipse at any position out of the nodes, and z the time the satellite would pass over a space equal to P a, which may be found by saying *as* $360^{\circ} : 57^{\circ}\ 17'\ 45\frac{4}{5}''$ *(an arc equal in length to radius)* : : *the time of a synodic revolution* : *the time of describing a space equal to* z. Then s, the sine of the inclination of an orbit, will be $= \frac{\sqrt{r^2 - h^2}}{w z}$. And, on the contrary, if the inclination of the satellite's orbit be known, we shall find the semiduration of the eclipse, by the equation $h = \sqrt{r^2 - w^2 s^2 z^2}$. If we draw P c perpendicular to O I, then in the triangle P $a c$, knowing P a, the right angle P $a c$, and the angle c P $a = c$ NP the inclination of the orbit, we may find $a c$ the distance from the middle of the eclipse to the conjunction of the satellite. These conclusions will not be quite correct, because we take not into account the spheroidical form of the planet, the curvature of the orbits, and the disturbing forces of the planets upon each other. But we are not sufficiently acquainted with all these particulars, to deduce from much more operose calculations very accurate results.

509. The eclipses of the satellites afford us a manifest proof of the *velocity of light*. For, if the motion of light were infinite, or it came to us from immense distances in an instant, it is evident we should see the commencement of an eclipse of a satellite at the same moment, whatever distance we might be from it; but, on the contrary, if light move progressively, then it is equally evident, that the farther we are from a planet, the later it will be before we perceive the beginning of an eclipse of one of its satellites, because the light will occupy a longer

time in travelling to us. M. *Roemer* was the first who deduced, from actual observations, the real velocity of light: he found (and it has since been confirmed by repeated experiments) that when the earth is exactly between jupiter and the sun, his satellites are seen eclipsed about $8\frac{1}{4}$ minutes *sooner* than they would be according to the tables; but that when the earth is at its greatest distance from jupiter, these eclipses happen about $8\frac{1}{4}$ minutes *later* than the tables predict them. Hence it follows that light takes up about $16\frac{1}{2}$ minutes of time in passing over the diameter of the earth's orbit, which is, at a mean, 190 millions of miles; this is very nearly at the astonishing rate of 200,000 miles in a second. Dr. *Bradley* found very nearly the same velocity from observations on the fixed stars. Hence also it appears, that in determining the time of an eclipse of a satellite as seen from the earth, an allowance must be made, corresponding with the different distances of the earth and planet; this allowance is called the *equation of light* *.

510. The satellites of jupiter, saturn, and the georgian planet, are subject to changes in their orbits, with respect to the situation of the apsides and nodes, the inclination of the orbits, their excentricity, &c. similar to those of the moon, and from similar causes. But in many instances they prevail to a greater degree, in consequence of the disturbing forces of the satellites upon each other; hence, therefore, a frequent revision of the tables is necessary.

* The various directions necessary for calculating the eclipses of the satellites, their configurations, &c. are not given here; since they would be of no service without the tables, and the tables are always accompanied by proper precepts. For the satellites of jupiter the best tables are those of M. *Wargentin*, given in the *Nautical Almanac* for 1771 and 1779. Very useful tables for the satellites of both saturn and jupiter, are given in Mr. Vince's *System of Astronomy*.

511. The satellites of jupiter, being situated nearer to the earth than those of any other planet, and being of very considerable importance in astronomy and navigation, have been more regularly and carefully observed than the other satellites; in consequence of which the elements of their orbits are determined with greater precision than those of the other satellites. The chief of these elements, as given by M. *de la Lande*, are exhibited in the table next following.

SATELLITES OF JUPITER.

Elements.	I.	II.	III.	IV.
Periodic revolution . .	$1^d\ 18^h\ 27^m\ 33^s$	$3^d\ 13^h\ 13^m\ 42^s$	$7^d\ 3^h\ 42^m\ 33^s$	$16^d\ 16^h\ 32^m\ 8^s$
Synodic revolution . .	1 18 28 36	3 13 17 54	7 3 59 36	16 18 5 7
Distance in semidiam. ♃ .	5·67	9·00	14·38	25·30
Mean dist. at mean dist. ♃	$1'\ 51''$	$2'\ 57''$	$4'\ 42''$	$8'\ 16''$
Semid. shad. in time .	$1^h\ 7^m\ 55^s$	$1^h\ 25^m\ 40^s$	$1^h\ 47^m\ 0^s$	$2^h\ 23^m$
. . . . that of ♃ = 1	·9941	·9967	·9857	·9913
Semidur. of eclipse 90° from node, when inclin. is least	$1^h\ 3^m\ 45^s$	$1^h\ 16^m\ 5^s$	$1^h\ 3^m\ 40^s$	$0^h\ 0^m\ 0^s$
For circular shadow { greatest inclin.	$3°\ 18'\ 38''$	$3°\ 46'\ 0''$	$3°\ 25'\ 57''$	$2°\ 36'\ 0''$
mean . . .	3 18 38	3 16 0	3 13 58	2 36 0
least . . .	3 18 38	2 46 0	3 2 0	2 36 0
For ellip. shadow { greatest .	3 4 27	3 29 42	3 11 14	2 24 51
least . .	3 4 27	2 34 0	2 49 0	2 24 51
Epoch of conj. 1760, for merid. Greenwich . .	$0^d\ 10^h\ 35^m$	$1^d\ 14^h\ 49^m\ 36^s$	$2^d\ 5^h\ 32^m\ 29^s$	$1^d\ 7^h\ 20^m\ 50^s$
Mean place node 1760 .	$10^s\ 14°\ 30'$	$10^s\ 13°\ 45'$	$10^s\ 14°\ 24'$	$10^s\ 16°\ 39'$
Annual motion node .	$0\ 0''$	$-\ 2\ 3''$	$0\ 0''$	$4\ 19''$
Diurnal motion satel. .	6 23 29 20	3 11 22 29	1 20 19 4	0 21 34 16
Secular motion . .	7 25 31 13	3 23 10 39	1 22 9 19	6 29 50 29

512. Of the satellites of saturn, five were discovered by *Cassini* and *Huygens*, before the year 1685; and it was imagined there were no more, till two were discovered by Dr. *Herschel*, in 1787 and 1788. These are nearer to saturn than any of the other five; but in order to prevent confusion in the numbers, with regard to former observations, they are called the 6th and 7th satellites. The periodical revolutions of these satellites, their distances from the body of saturn expressed in semidiameters of that planet and in miles, and the angles under which their orbits are seen at the earth, at the mean distance from saturn, are expressed below:

SATELLITES OF SATURN.

Satellites.	Periods.				Distances in		Diam. of orbit.	
					Semidiameters.	Miles.		
I.	1d	21h	18m	27s	$4\frac{3}{8}$	170000	1′	27″
II.	2	17	41	22	$5\frac{1}{2}$	217000	1	52
III.	4	12	25	12	8	303000	2	36
IV.	15	22	41	13	18	704000	6	18
V.	79	7	48	0	54	2050000	17	4
VI.	1	8	53	9	$3\frac{5}{9}$	135000	1	14
VII.	0	22	40	46	$2\frac{5}{6}$	107000	0	57

The inclination of the first four satellites to the ecliptic is from 30 to 31 degrees. The fifth describes an orbit inclined in an angle of from 17 to 18 degrees to the orbit of saturn. Dr. *Herschel* observes that this satellite turns once round its axis exactly in the time in which it revolves about saturn; in which respect it resembles our moon.

513. As to the satellites of the georgian, two of them, which were discovered by Dr. *Herschel* in 1787, revolve about that planet in periods of $8^d\ 17^h\ 1^m\ 19^s$ and $13^d\ 11^h\ 5^m\ 1\frac{1}{2}^s$ respectively, the angular distances from the primary being 33″ and $44\frac{2}{9}$″: their orbits are nearly perpendicular to the plane of the

ecliptic. The history of the discovery of the other four, with such elements as could then be ascertained, are given in the *Philosophical Transactions* for 1798, Part I. The precise periods of these additional satellites cannot be ascertained without a greater number of observations than had been made when Dr. *Herschel* sent the account of their discovery to the *Royal Society*; but he gave the following estimates as the most probable which could be formed by means of the *data* then determined. Admitting the distance of the interiour satellite to be $25''.5$, its periodical revolution will be $5^{d}\ 21^{h}\ 25^{m}$. If the intermediate satellite be placed at an equal distance between the two old satellites, or at $38''.57$, its period will be $10^{d}\ 23^{h}\ 4^{m}$. The nearest exteriour satellite is about double the distance of the farthest old one; its periodical time will therefore be about $38^{d}\ 1^{h}\ 49^{m}$. The most distant satellite is full four times as far from the planet as the old second satellite; it will therefore take at least $107^{d}\ 16^{h}\ 40^{m}$ to complete one revolution. All these satellites perform their revolutions in their orbits contrary to the order of the signs; that is, their real motion is retrograde.

CHAPTER XVIII.

On the Ring of Saturn.

ART. 514. WHEN speaking of the appearances of some of the planets when viewed through a telescope (Art. 402.), we mentioned the discovery of a curious ring which encompassed the body of saturn. This ring, according to the observations of *Huygens*, is about 22000 miles broad, and its greatest diameter in proportion to that of the planet, as 9 to 4. Later observations have furnished us with a much better acquaintance with this curious phenomenon. *De la Lande* and *De la Place* inform us that *Cassini* saw the breadth of this ring divided into separate parts, nearly equal in breadth. And Mr. *Hadley*, with an excellent $5\frac{1}{2}$ feet reflector, saw the ring divided into two parts. Mr. *Short*, and some few others, thought they saw several divisions upon the ring: but the long-continued and accurate observations of Dr. *Herschel* seem to confirm the division of the ring into only two concentric parts, almost beyond the possibility of doubt. This gentleman says, there is one single, dark, considerably broad line, belt, or zone, which he has constantly found on the north side of the ring. As this dark belt is subject to no change whatever, it is probably owing to some permanent construction of the surface of the ring: this construction cannot be owing to the sha-

dow of a chain of mountains, since it is visible all round on the ring; for there could be no shade at the ends of the ring: a similar argument will apply against the opinion of very extended caverns. It is pretty evident that this dark zone is contained between two concentric circles; for all the phenomena correspond with the projection of such a zone. The nature of the ring Dr. *Herschel* thinks no less solid than that of saturn itself, and it is observed to cast a strong shadow upon the planet. The light of the ring is also generally brighter than that of the planet; for the ring appears sufficiently bright when the telescope affords scarcely light enough for saturn. The doctor concludes that the edge of the ring is not flat, but spherical or spheroidical. The dimensions of the ring, or of the two rings with the space between them, Dr. *Herschel* gives as below:

	Miles.
Inner diameter of smaller ring . . .	146345
Outside diam. of ditto	184393
Inner diam. of larger ring	190248
Outside diam. of ditto	204883
Breadth of the inner ring	20000
Breadth of the outer ring	7200
Breadth of the vacant space, or dark zone	2839

515. The ring revolves in its own plane in $10^h 32^m 15^s.4$, being nearly the same time (Art. 402.) as that in which the planet performs a rotation upon its axis. The ring, being nearly a circle, appears elliptical from its oblique position; and it appears most open when saturn is 90° from the nodes of the ring upon the orbit of the planet, or when saturn's longitude is about 2^s 17° and 8^s 17°. In such a situation the minor axis is very nearly equal to half the major, when the observations are reduced to the sun; consequently the plane of the ring makes an angle of about 30° with the orbit of saturn. Or, according

to some observations, the inclination of the ring to the ecliptic is about 31° 22′.

516. There have been various conjectures relative to the nature of this ring. Some persons have imagined that the diameter of the planet saturn was once equal to the present diameter of the outer ring, and that it was hollow; the present body being contained within the former surface, in like manner as a kernel is contained within its shell: they suppose that, in consequence of some concussion, or other cause, the outer shell all fell down to the inner body, and left only the ring at the greater distance from the centre, as we now perceive it. This conjecture is in some measure corroborated by the consideration, that both the planet and its ring perform their rotations about the same common axis, and in very nearly the same time. But from the observations of Dr *Herschel*, he thus concludes: "It does not appear to me that "there is sufficient ground for admitting the ring of "saturn to be of a very changeable nature, and I "guess that its phenomena will hereafter be so fully "explained, as to reconcile all observations. In the "meanwhile we must withhold a final judgment of "its construction, till we can have more observa- "tions. Its division, however, into two very une- "qual parts, can admit of no doubt."

517. As saturn moves round the sun in an orbit nearly circular, his obliquely posited ring, like our earth's axis, keeps parallel to itself, and is therefore turned edgewise to the sun twice in a saturnian year, which is nearly as long as thirty of our years. But the ring, though probably considerably broad, is invisible (except when very good telescopes are used) whenever its plane passes through the sun, or the earth, or between them: the chief phenomena of the ring may be illustrated by fig. 7, Pl. VII. where S represents the sun, A B C D E F G H the orbit of saturn, I K L M N O that of the earth; both planets

moving according to the order of the letters. When saturn is at A, his ring is directed towards the sun at S, and he is then seen from the earth with common telescopes, as though he had lost his ring, let the earth be in whatever part of its orbit it may, except between N and O; for whilst it describes that space, saturn is apparently so near the sun as to be hid in his beams. As saturn passes from A to C, his ring appears more and more open to an observer on the earth: at C the ring appears most open; and becomes narrower and narrower, as saturn passes from C to E. When he comes to E, the ring is again directed edgewise both to the sun and earth, and as neither of its sides is illuminated, it is again invisible to us. But as he goes from E to G, his rings open more and more to our view on the under side; and seems just as open at G as it did at C; and may be seen in the night-time from the earth in any part of its orbit, except about M, when the sun obscures the planet from our view. As saturn passes from G to A, his ring turns more and more edgewise to us, and therefore it seems to become more and more narrow; and at A it disappears, as before. Also, while saturn goes from A to E, the sun shines on the upper part of his ring, and the underside is dark; and whilst he passes from E to A, the sun shines on the under side of his ring, and the upper side is dark. According to the computation of Dr. *Maskelyne*, the plane of the ring passed through the earth on January 29, 1790; the earth passing from the northern, or dark, to the southern or enlightened side of the ring; the ring, therefore, then became visible, and will continue so till 1803.

518. The mean distances of the earth and saturn from the sun, are respectively as 1 : 9·54072 (Art. 360.); therefore the length of the semidiameter of the earth's orbit will be the sine of 6° 1′ 2″ in saturn's orbit. Saturn, then, describes an arch of 6° 1′ 2″ from the instant the plane of his ring produced touches the earth's orbit, to the instant it passes

through the sun: and as saturn's periodic time in his orbit is about 10761·615 days, he will be 179·83 days in running 6° 1′ 2″. The plane of his ring then takes nearly 360 days to run over the diameter of the earth's orbit; but as this is nearly the $\frac{113}{355}$ part of it, it is therefore $\frac{355}{113} \times 360$ days in running through a space equal to the whole orbit: but the earth takes 365·25 days nearly to run the same distance; hence, the velocity of the earth in its orbit, is to the simultaneous velocity of the ring of saturn, as $\frac{355}{113} \times 360 : 365{\cdot}25$, or nearly as 3·098 to 1. Hence, during the 179·83 days the ring is passing over a part of its orbit equal and nearly parallel to I M, the plane of the ring may pass through the earth three times, though sometimes not so many. This might be pursued much farther; but it is unnecessary here. Those who are desirous of seeing more on this head, are referred to a treatise entitled, *Essai sur les Phénomènes relatifs aux Disparitions périodiques de l'Anneau de Saturne*, par M. *Dionis du Séjour* *, in which they will find a curious analytical solution of the problem respecting the times of the appearance and disappearance of this ring, wherein is also exhibited a new species of curves.

* I cannot recommend this work more highly than by extracting the encomium upon it (signed by *D'Alembert, Borda, Bezout, Vandermonde, La Place,* and *Fouchi*) from the Register of the Royal Academy of Sciences at Paris, for 1775:—*Tels sont les objets que* M. du Séjour *a traités dans son ouvrage; et l'on voit qu'il n a rien laissé à désirer sur la théorie des phases de l'anneau de saturne. L'élégance, la finesse, et la simplicité des méthodes dont il a fait usage, rendent cet ouvrage très intéressant pour les géomètres; et la discussion des phénomènes depuis* 1600 *jusqu'en* 1900, *le rend nécessaire aux astronomes qui voudront dans la suite observer avec précision ces apparences: ainsi nous croyons qu'il mérite d'être imprimé avec l'approbation et le privilège de l'Academie.*

CHAPTER XIX.

On Eclipses of the Sun and Moon, and Occultations of the fixed Stars.

ART. 519. AN *eclipse* is a privation of the light of one of the luminaries, occasioned by the interposition of some opaque body, either between it and the eye, or between it and the sun. Eclipses are divided, with respect to the objects eclipsed, into *eclipses of the sun*, *of the moon*, and of *the satellites*. (Art. 505.) And with respect to circumstances, into *total*, *partial*, *annular*, and *central*. A *total* eclipse, is one in which the whole disc of the luminary is darkened; a *partial* one, is when only a part of the disc is darkened; in an *annular* eclipse, the whole is darkened, except a ring or annulus, which appears round the dark part, like an illuminated border; and in a *central* eclipse, the centres of the two luminaries, and that of the earth, are in one and the same right line.

On Eclipses of the Moon.

520. AN eclipse of the moon is occasioned by an interposition of the body of the earth directly between the sun and the moon, and so intercepting the sun's rays that they cannot illuminate her; or, the moon is eclipsed when she passes through part of the shadow of the earth as projected from the sun. Hence it is obvious that lunar eclipses can only happen at the time of full moon, for it is then only

that the earth can be between the moon and the sun. Were the orbits of the earth and moon in the same plane, there would be a lunar eclipse at every full moon; but as they are not coincident, an eclipse can only happen, either when the full moon takes place in one of the nodes of the moon's orbit, or so near it, that the moon's latitude does not exceed the sum of the apparent semidiameters of the moon and the earth's shadow a the moon's orbit.

521. To determine the distance from the node, beyond which there can be no eclipse; we must ascertain the apparent semidiameter of the earth's shadow at the moon: but it may be proper, first, to shew that this shadow extends farther than the moon's orbit. Let S (fig. 6, Pl. VII.) represent the sun; E, the earth; M, the moon: the respective diameters and distances of all which are known. Then A D being the difference between the semidiameters of the sun and the earth, we have, by similar triangles, A D : B E : : D B or S E : E C. Here the mean distance of the earth and sun being about 23840 semidiameters of the earth, and the excentricity of the earth's orbit about 354 semidiameters, we may take 24194 for the greatest distance of the earth and sun; and the diameters being very nearly as 1 to 112, we have 111 : 1 : : 24194 : 217·99 semidiameters of the earth = E C the length of the earth's shadow. Or, E C may be ascertained by a rather different process, thus: The angle B A E, or the sun's parallax, is 8″·7; and the angle AES, or the sun's apparent semidiameter, is about 15′ 56″: the former of these taken from the latter, leaves 15′ 47″·3 for the angle A C E: then, tang. 15′ 47″·3 : radius : : BE = 1 : CE = 218 nearly, as before. Hence, since the moon's distance from the earth varies from about 56 to 64 semidiameters, the shadow will be extended considerably farther than the moon's orbit. But the extreme end of the shadow falls short of the orbit of mars, there-

fore none of the heavenly bodies can be involved in the shadow besides the moon.

522. The apparent semidiameter of the earth's shadow at the moon, may be determined from these considerations: the angle $r\,E\,o = E\,r\,A - E\,C\,A$; but $E\,C\,A = S\,B\,A - B\,S\,E$, therefore $r\,E\,o = E\,r\,A + B\,S\,E - S\,B\,A$; that is, *the semidiameter of the earth's shadow at the moon's orbit, appears under an angle equal to the sum of the horizontal parallaxes of the sun and moon, diminished by the sun's apparent semidiameter.* Thus, if the moon's parallax be $61'\ 12''$, the sun's parallax $8''\!\cdot\!6$, and his apparent semidiameter $15'\ 56''$; then $61'\ 12'' + 8''\!\cdot\!6 - 15'\ 56'' = 45'\ 24''\!\cdot\!6$ the apparent semidiameter of the shadow. Some astronomers remark that the apparent dimensions of the shadow are larger than when found by this rule, owing to the earth's atmosphere; they therefore augment the semidiameter of the shadow by about $\frac{1}{60}$ of itself.

523. The sum of the apparent semidiameters of the shadow and of the moon never exceeds $1°\ 4'$, which is equal to that latitude of the moon between which and the node all lunar eclipses must happen: then, taking the inclination of the moon's orbit at about $5°\ 16'$, from which it will not vary considerably, we have $1°\ 4'$ the perpendicular of a right-angled spherical triangle, and $5°\ 6'$ the angle opposite to it; from which we find the side opposite the right angle by this analogy, $5°\ 16'$: radius : : sine $1°\ 4'$: sine $11°\ 42'$. Hence, if the moon be at a less distance from either node than $11°\ 42'$, there will be an eclipse of some duration; otherwise none can happen. This $11°\ 42'$ is called the *lunar ecliptic limit*: or, more accurately, the *limit* is from $10\frac{1}{2}°$ to $12°$, according to the different distances of the sun, earth, and moon.

524. To compute a lunar eclipse, there must be had the true distance of the moon from the node, at

the mean opposition; also, the true time of the opposition, with the true places of the sun and moon reduced to the ecliptic; also, the moon's true latitude at the time of the true opposition, the angle of the moon's way with the ecliptic, and the true horary motions of the sun and moon.

525. The nature of the computations may be illustrated by taking for an example the lunar eclipse which happened on the 25th of February, 1793. The necessary elements deduced from *Mayer's Tables improved*, are as follow:

The true time of opposition, at Greenwich . . .		10^h	49^m	28^s
The apparent time . .		10	36	18
The sun's place . . .	11^s	7°	$51'$	$43''$
The moon's place . . .	5	7	51	43
The place of the moon's ascending node	5	15	28	37
The moon's latitude, south .			40	12
The sun's horary motion .			2	31
The moon's horary motion .			30	11

526. Let N B be a part of the ecliptic (fig. 5, Pl. VII.), S the centre of the earth's shadow; through which, perpendicular to N B, draw S D equal to the moon's latitude, either above or below, according as the moon's latitude is north or south; therefore, in the present case, below N B: let N D be the moon's orbit, N the place of the ascending node, and D the centre of the moon when in opposition; and suppose the small circles described about G, M, and K, to be the moon's disc in its proper situations at the beginning, middle, and end of the eclipse. Now, the moon is approaching her ascending node, and N S $= 5^s\ 15^\circ\ 28'\ 37'' - 5^s\ 7^\circ\ 51'\ 43'' = 7^\circ\ 36'\ 54''$: if N A be to N S, as the sun's horary motion to the moon's, in which case A S $= 6^\circ\ 58'\ 49''$, A D will be the moon's *relative orbit*, or her orbit with respect to the eye of the observer, which is in motion; then

in the right-angled spherical triangle S A D, are given A S (as above), and S D the moon's latitude, whence we find A D S = 84° 32′ 41″, the complement of the angle which the relative orbit makes with the ecliptic. M being the relative place of the moon's centre at the middle of the eclipse, S M must be perpendicular to A D; and the triangle S D M being small, may be conceived rectilinear; wherefore, knowing S D, and S D M, we find M S = 40′ 1″, and M D = 3′ 50″, which is called the *reduction*. The horary motion upon the relative orbit will be found by saying, *as the co-sine of its inclination* : *radius* : : *the difference of the sun's and moon's horary motion* : *the horary motion upon the relative orbit*; and as this relative horary motion : D M : : one hour : $8^m\ 18^s$, time of describing D M, which added to the apparent time of opposition, gives $10^h\ 44^m\ 36^s$ for the time of the middle of the eclipse.

527. Now, at the time of the opposition, the sun's apparent semidiameter was 16′ 9″, his horizontal parallax 9″; the moon's apparent semidiameter 14′ 50″, and her horizontal parallax 54′: hence M H = 14′ 50″, and S B or S *n* = 54′ + 9″ — 14′ 50″ = 39′ 19″ apparent semidiameter of the earth's shadow (Art. 522.). Then, in the plane triangle S G M, equal and similar to S K M, we know S M, and S G, the sum of the semidiameters of the earth's shadow, and of the moon's disc, whence we find M G = M K = 35′ 12″; the time of describing which, is $1^h\ 18^m\ 20^s$, and the whole time of moving from G to K, $2^h\ 36^m\ 40^s$; whence we readily find the time of the beginning and end of the eclipse. The magnitude of the eclipse at the middle, is represented by H *n*, being the greatest distance of the moon within the earth's shadow, which is measured in twelfth parts of the diameter of the moon's disc, called *digits*: to find which, we know S M = 40′ 1″, and S *n* = S B = 39′ 19″; hence M *n* = 0′ 42″; which taken from

HM = 14′ 50″, leaves 1′ 8″ = H*n*; therefore 14′ 50″ : 14′ 8″ : : 6 digits or 360′ : 5^d 43′. Consequently, on the night of the 25th February, 1793,

The eclipse began at . .	9^h	26^m	16^s
Middle . . .	10	44	36
End . . .	12	2	56
Duration . . .	2	36	40
Digits eclipsed . .	5^d	43′	

And since the moon did not set till nearly four hours after midnight, it is evident the whole of the eclipse was visible, unless the moon was hidden from view by clouds.

528. The chief circumstances in lunar eclipses are here mentioned: 1. All lunar eclipses are universal, or visible in all parts of the earth which have the moon above their horizon, and are every-where of the same magnitude and duration. 2. In all lunar eclipses, the eastern side (or the left-hand side, as we look towards her from the north) is what first immerges into the shadow, and emerges again; for the proper motion of the moon being swifter than that of the earth's shadow, the moon approaches it from the west, overtakes and passes through it with the moon's east side foremost, leaving the shadow behind, or to the westward. 3. Although total eclipses of the longest duration happen in the node, yet there may be total eclipses within a small distance of the nodes, namely, within that distance where the moon's latitude is equal *to the apparent semidiameter of the earth's shadow, minus the semidiameter of the moon's disc*, but in these situations the duration of total darkness will be short; whereas in *central* eclipses it will continue nearly two hours. 4. If the earth had no atmosphere, the moon, when she was totally eclipsed, would be invisible; but as the earth has an atmosphere, some of the light from the sun will be refracted thereby, and transmitted to the moon, on which account the moon will be visible at that time,

and appear of a dull red colour. 5. The moon becomes sensibly more and more dim previous to her entering the real shadow of the earth; and, after she has passed through the shadow, becomes more and more bright till she acquires her native lustre: this is occasioned by a part of the sun's rays being obstructed while she passes over the portions of her orbit *r n*, and *s m*, contained between the lines B *n*, F *m*, which bound the part *e* B C F *i* (fig. 6, Pl. VII.) called the *Penumbra.*

On Solar Eclipses.

529. An eclipse of the sun is an occultation of his body, occasioned by an interposition of the moon between the earth and sun: on which account it is by some considered as an eclipse of the earth, since the light of the sun is prevented from coming to the earth by the moon, whose shadow involves a part of the earth. Let S A R (fig. 8, Pl. VII.) be a section of the sun, T G H that of the earth, and B D C that of the moon interposed directly between them; then the part B K C bounded by the lines B K, C K, drawn from A and R to touch B and C respectively, will be the section of the moon's shadow, or *umbra*, and the parts B G *e*, *n* C H, bounded by R B and A C produced, will represent the *penumbra*. Now, if the shadow of the moon extend so far as to cover a portion of the earth's surface *e a n*, an observer any-where in that portion will have the sun *totally* eclipsed or hidden from his sight; but if the vertex of the conical shadow reach not so far as *a*, a spectator there will have the sun *centrally*, but not totally, eclipsed; for his disc will appear like a bright *annulus*, or luminous ring, the width of which will be equal to half the difference of the apparent diameters of the two luminaries: and an observer situ-

ated in any part *e* G or *n* H within the penumbra, will have the sun *partially* eclipsed, some part of his disc being visible, and the remainder hidden by the intervening body of the moon. That the moon's shadow does sometimes extend sufficiently far to cover a part of the earth's surface, and at other times does not reach so far, may be thus shewn: the angle S D R = D R K + D K R, hence D K R = S D R — D R K; but S D R : S T R : : S T : S D : : 390 : 389 (Art. 457.), therefore S D R = $\frac{390}{389}$ S T R; and D C being to T L, as 2·180 : 7960 (Art. 461.), or nearly as ·274 : 1, D R C will be to T R L, the sun's horizontal parallax, in a ratio compounded of the direct ratio of D C : T L, and of the inverse ratio of S D : S T; whence, T R L being nearly 9″, D R C will be 9″ × $\frac{274}{1000}$ × $\frac{390}{389}$, or nearly 3″: consequently D K R = $\frac{390}{389}$ S T R — 3″. Now, the angle S T R, when least, is about 15′ 47″ (Art. 304.); and in this case D K R = $\overline{15' \, 47''} \times \frac{390}{389}$ — 3″ = 15′ 41″: hence, sine 15′ 41″ : radius : : B D : D K : : ·274 : 60·06 semidiameters of the earth, length of the moon's shadow, when the earth is in its aphelion; and when it is in perihelion, S T R will be greater, and, of consequence, D K will be less. But the moon's distance from the earth varies from about 56 to 64 semidiameters of the earth, therefore in some cases, the shadow of the moon may fall upon a part of the earth, and in others, the vertex of the shadow may not extend so far by two or three semidiameters.

530. Let us next determine how much of the earth's surface may be involved in the moon's shadow; in order to which, we must find the angle *e* T *a*: here we will suppose the sun to be in apogee and the moon in perigee, and in that case, the conical shadow will be extended about 4½ semidiameters beyond T; then, as T *e* (1) : T K (4½) : : sine T K *e* = 15′ 41″ : sine K *e* T = 1° 7′ 16″; wherefore K *e* T + T K *e*

$= 1° \ 22' \ 57'' = a$ T e or the arc $a\,e$, and double this, or 2° 45′ 54″, is the measure of $n\,a\,e$, supposing the earth to be a sphere; allowing $69\frac{1}{2}$ miles to a degree, we have 179·98, or very nearly 180 English miles, for the breadth of that spot on the earth's surface which is covered by the moon's dark shadow, when the sun's distance is greatest, and the moon's distance least.

531. After a nearly similar manner, the greatest portion of the earth ever covered by the penumbra may be ascertained: in this case, the moon will be in apogee, and the sun in perigee. The angle B I D = B R D + S D R = S D R + D R K = 16′ 21″ + 3″ = 16′ 24″; for here S D R = $\frac{390}{380}$ × 16′ 19″, and B G D = 14′ 45″ (Art. 461.). Then, as sine B I D : sine B G D : : G D : I D : : 64 : 57·561 semidiameters of the earth; hence, D T being 64, we have I T = 121·56 semidiameters, and T G : I T : : sine T I G (16′ 24″) : sine T G I = 144° 33′ 36″; therefore I G N = 35° 26′ 24″. But I G N = T I G + I T G, of course I T G = I G N − I T G = 35° 10′; the double of which is 70° 20′ for the arch H a G: the English miles answering to which are 4888, or nearly 4900; the breadth of the portion of the earth's surface under the penumbral shadow *. It is manifest from this article, and the preceding, that when $n\,e$, under the shadow, is a maximum, H G, under the penumbra, is a minimum, and *vicè versa*.

532. The moon's mean motion about the centre of the earth is nearly 33′ in an hour, and 33′ of the moon's orbit is about 2280 miles; which, therefore, we may consider as the horary velocity with which

* In the case here considered, the moon is supposed in the node, and the axis of the shadow and penumbra pass through the centre of the earth; but in every other case, both the shadow and penumbra will be cut obliquely by the earth's surface, and the bounds of the sections will be ovals, and very nearly true ellipses.

the moon's shadow passes over the earth, or, at least, over that part on which the shadow falls perpendicularly; in every other place the velocity will be increased in the proportion of the sine of the angle which D K makes with the surface, in the direction of its motion, to radius. But it must also be recollected, that the earth's rotation upon its axis will affect the apparent velocity of the shadow, and consequently the duration of the eclipse: if the point of the earth's surface be moving in the direction of the shadow, its velocity in respect to that point will be diminished, and consequently the time in which the shadow passes over that point will be increased; but if the point be moving in a direction contrary to that of the shadow, as is the case when the shadow falls on the other side of the pole, the relative velocity of the shadow will be increased, and the time diminished.

533. If the moon's parallax were insensible, or very small, the limits of a solar eclipse would be determined after the same manner as those of a lunar; but because the parallax is considerable, the method is rather altered. 1. Add together the apparent semidiameters of the luminaries, both in apogee and perigee; this gives 33′ 16″ for the greatest sum, and 30′ 31″ for the least. 2. Since the parallax diminishes the northern latitude, and augments the southern, let the greatest parallax in latitude be added to the former sums, and also subtracted from them; the sum or difference will in each case give the latitude beyond which there can be no eclipse; this latitude being given, the distance from the node is readily determined, and is found to be from $16\frac{1}{2}°$ to $18\frac{1}{3}°$, according to circumstances, for the *limit*, beyond which there can be no solar eclipse. Or, for this purpose M. *Cassini* gives the following directions: "Find the time of the mean conjunction, and at that time find the sun's mean longitude, also the longitude of the moon's node; and if the difference of

these be less than 21°, there *may* be an eclipse; but if the difference be less than 15°, there *must* be one." But perhaps it may be better, to explain more fully on what the solar ecliptic limit depends; to this purpose let us enquire what would be the apparent semidiameters of the earth's disc, and of the moon's shadow and penumbra thereon, if viewed from the moon: the earth's apparent semidiameter is manifestly equal to the moon's horizontal parallax; the apparent semidiameter of the moon's dark shadow is nearly equal to the moon's apparent semidiameter *minus* the sun's apparent semidiameter (unless the sun's apparent semidiameter be greatest, at which time the shadow does not reach the earth); for in the triangle B *a* K, K B *a* = B *a* S — B K S = B *a* D — A K S = B *a* D *minus* sun's apparent semidiameter nearly; and the apparent semidiameter of the moon's penumbra upon the earth's surface is nearly equal to the apparent semidiameter of the sun, *plus* the horizontal semidiameter of the moon augmented according to its altitude; for G D *a* = G I D + I G D = B I D + B G D = (by reason of the great distance of S T with respect to T D) A T S + B G D nearly. Therefore, let A E (fig. 9, Pl. VII.) represent a portion of the annual orbit, and F H the visible path of the centre of the lunar shadows, which will exactly correspond to the position of the moon's orbit with respect to the ecliptic in the heavens, and therefore the point of intersection N will be the node, and the angle H N E the inclination of the lunar orbit to the plane of the ecliptic. Also, let Æ P Q S represent the disc of the earth (according to the *orthographic projection*) in the several places N, B, C, D, its semidiameter being made equal to the minutes in the moon's horizontal parallax at the time of the eclipse; let the small dark circles described about the centres N, R, L, G, be the apparent sections of the shadow in those positions, and the larger dotted

circles about the same centres, the respective sections of the penumbra. Then it is evident, if the moon, when near conjunction, be at the distance N G from the node, the penumbra will not fall near the earth's disc, wherefore there can be no eclipse in that situation. If the moon's distance from the node be equal to N L, the penumbra will just touch the disc, and consequently N C will be the earth's greatest distance from the node at which the sun can be at all eclipsed: here L C is perpendicular to N L, and is nearly equal to the moon's horizontal parallax, added to the sum of the sun's and moon's apparent semidiameters, which varies from about 1° 35′ to 1° 32′, and gives the solar ecliptic limit as before stated. When the moon's distance from the node is N R, the ecliptic conjunction being at *n*, the dark circle touches the earth's disc, and N B is the limit for total eclipses; therefore between B and C, or R and L, the eclipse will be partial. If the moon be new at N the node, there will be a central eclipse, which will be either total or annular, according to the respective distances of the sun, moon, and earth, as has been previously observed in Art. 529.

534. Solar eclipses, or, as they may be called, eclipses of the earth, may be calculated in the same manner as eclipses of the moon, if we have not respect to a particular place; but when we wish to know the times of beginning and ending, the digits eclipsed, &c. as seen from any particular point on the earth's surface, the calculations become much more complicated and tedious; we will, however, endeavour to exemplify them by taking a particular instance, as was done in lunar eclipses. At the new moon which happened on September 5, 1793, the place of the moon was within the limits for solar eclipses; let us, therefore, ascertain the duration and magnitude of the eclipse, as it was then observed at

Greenwich. Here the elements, as deduced from *Mayer's Tables improved*, are as below:

The true time of conjunction, Sept.	4^d 23^h 52^m 32^s
Longitude of the sun and moon	5^s 13° $16'$ $34''$
Latitude of the moon N, ascending	40 21
Moon's horizontal parallax .	54 3
Her apparent diameter . .	29 28
The sun's apparent diameter .	31 45
The moon's horary motion .	29 48
The sun's ditto . .	2 26
Horary motion of the moon from the sun . . .	27 22

525. In the first place we must find the way of the moon from the sun; in order to which, let H Z O (fig. 1, Pl. VIII.) be the meridian of the place, H O the horizon, G B the equinoctial, G E B the ecliptic, Z the zenith, P the elevated pole, S and M the places of the sun and moon in conjunction; P S F the sun's meridian. Having found n S $= 7^\circ\ 37'\ 39''$, the sun's distance from the node, take n A to n S as the sun's horary motion to that of the moon; then S A $= 6^\circ\ 59'\ 52''$. Draw M A: then in the spherical triangle A S M, right-angled at S, are given A S and S M, to find the angle A M S $= 84^\circ\ 32'\ 18''$, M A being the moon's way from the sun. But because the eye of the observer is in motion, in consequence of the rotation of the earth, which communicates an apparent motion to the moon, contrary to that of the observer, the quantity and direction of that motion must be found. Here, since the observer is carried eastward, that is, towards the point C, the apparent motion in the moon caused thereby will be in the line C S; to determine the position of which in respect of A M or S M, it is requisite to proceed as follows: in the right-angled triangle B F S, there are given S B and the angle B, to find F S $= 6^\circ\ 35'\ 3''$, and B S F $=$ P S A $= 67^\circ\ 24'\ 58''$. In the triangle

ZPS are given PZ the co-latitude, PS the complement of FS, and the hour angle ZPS, to find ZS = 44° 58′, PZS = 177° 22′ 32″, and ZSP = 1° 38′ 34″. Then ZSP+ASP—ASM (90°) = ZSM = 20° 56′ 28″. In the right-angled triangle CSD are given CD and DS, for CD is the measure of the angle CZD = PZS—CSP (90°), and DS = the comp. ZS; whence CS = 88° 8′ 45″, and CSD = NSZ = 88° 8′ 42″. Then NSZ —ZSM = NSM = 67° 12′ 14″, and 180° — AMS = NMS = 95° 27′ 42″; consequently, SM being known, the angle MNS may be found = 17° 20′ 46″.

536. The quantity of the apparent motion along SN must then be found, and compounded with that along AN, which is already known. The sine of 15°, the horary motion of a point in the equator, is ·258819 to the radius 1: and if p be the moon's horizontal parallax, the radius of the earth appears at the moon under the angle p, and 15° of the equator under the angle ·2588 p, which is the horary motion of a point in the equator, viewed directly from the moon; and the moon's apparent motion, as seen from that point, is manifestly the same. But this motion must be diminished, first, in proportion to the co-sine of the latitude of Greenwich (or any other place given out of the equator), because its motion is in a parallel circle; and, secondly, in proportion to the sine of CS, the sun's distance from the east (or west) point of the horizon (Art. 532.); therefore, to the log. of ·2588 p, add the log. co-sine latitude, and the log. sine of CS, the sum, abating twice log. radius, is log. of 8′ 41″, the apparent horary motion along SN. To compound this with the known motion along AMN, let AS (fig. 2, Pl. VIII.) be a portion of the ecliptic, MA the moon's way from the sun, SN the direction of the apparent motion: draw MR parallel to SN, and let MR be the horary

motion along S N, and M K the horary motion of the moon from the sun; then complete the parallelogram R M K W, draw the diagonal M W Q, which is the direction of the motion compounded of that of the observer and that of the moon, and M W is the total apparent horary motion, supposing the observer at rest. Then, in the plane triangle M W K are given M K = 27′ 22″, and K W = 8′ 41″, and the angle M K W = M N S = 17° 20′ 46″; whence we find W M K = 7° 45′ 43″, and M W = 19′ 13″ the absolute horary motion; and the angles W M K and K M S being known, we know S N W = 92° 18′ 1″. Here, the sun being in the eastern hemisphere, the concave side of that hemisphere is projected in fig. 1, Pl. VIII. the moon's motion from the sun is from M towards A, and the other apparent motion from S towards N in either figure. But when the sun is in the western hemisphere, the convex side of the sphere is represented in the projection; and then the moon moves from the sun in the direction A M; the other apparent motion is also contrary.

537. Next find the moon's parallax of altitude, by saying, as radius : co-sine D S, the sun's altitude, : : 54′ 3″ the moon's horizontal parallax : her parallax M *m* = V *t* (fig. 3, Pl. VIII.) = 38′ 11″. Then, in the right-angled spherical triangle M *n m*, are given M *m*, and the angle *m* M *n*, to find M *n* = 35′ 38″ her parallax in latitude, whence S *n* = 40′ 21″ — 35′ 38″ = 4′ 43″ her remaining latitude, and *m n* = S *s* = 13′ 39″ her parallax in longitude, which must be *added*, because the moon is to the *east* of the nonagesimal degree; but it must be *subtracted* when the moon is to the *west*.

538. Draw G S (fig. 4, Pl. VIII.) for the ecliptic, and at any point S erect the perpendicular M S = 4′ 43″ the moon's latitude corrected for parallax; through M draw the line N M Q, making the angle S M Q = 92° 18′ 1″ (found Art. 536.); this line will be the moon's apparent path. Draw S P perpendicular to

M Q, then S P will be the least distance of the centres of the luminaries, and S the point of greatest obscuration; or, on the supposition that L Q is a straight line (which is nearly, though not exactly, true), P will be the place of the moon's centre in the middle of the eclipse. From the centre S, with the radius S B = 15′ 52½″, describe the circle B D C for the sun's disc; and from the centre P, with the radius P A = 14′ 44″, describe the circle D A C for the moon's disc; also make S H and S L each equal to the sum of the radii of the discs of the sun and moon; and the moon's centre will appear at L when the moon's disc first touches the sun's, or at the beginning of the eclipse, and H will be the apparent place of the moon's centre at the end of the eclipse. Then, in the triangle P S L are given S L and S P (found from the triangle S P M), to find $PL = \sqrt{SL^2 - SP^2}$ = 30′ 14″ = B H: hence, 19′ 13″ (Art. 536.): 1 hour : : 30′ 14″ : $1^h\ 34^m\ 23^s$ half the duration of the eclipse. Now the moon at the middle of the eclipse is past P, or (since P and M are exceedingly near) past the apparent conjunction by the distance of 13′ 39″ her parallax in longitude, which, at the rate of 19′ 13″ per hour, gives $42^m\ 37^s$ to be subtracted from the time of apparent conjunction, for the middle of the eclipse.

539. To find the digits eclipsed, we have $no = Sn + Po - PS$, or the sum of the apparent semidiameters of the luminaries *minus* the moon's apparent latitude, which gives 25′ 53″ for the scruples immersed; then 15′ 52″ : 6^d : : 25′ 53″ : $9^d\ 47_m$ the digits eclipsed, on the sun's north or upper limb.

Hence, on Sept. 5, 1793, the apparent time at Greenwich, of the

Beginning, was . . .	9^h	35^m	32^s	A. M.
Middle	11	9	55	
End	0	44	48	P. M.
Duration	3	8	46	
Digits eclipsed . .	9^d	47′		

In latitudes north of Greenwich the eclipse was greater; in the northern parts of Scotland it was annular, and the digits eclipsed were about $11\frac{1}{2}$. But this eclipse was not any-where *total*, because the dark conical shade of the moon did not quite extend to the earth. This was the greatest solar eclipse which has happened in England since that on April 1, 1764. There will be one nearly as great on Nov. 19, 1816; when about $9\frac{1}{2}$ digits will be eclipsed: and on Sept. 7, 1820, and May 15, 1836, there will be solar eclipses which will be nearly total, in the middle parts of England *.

540. In the above calculation no allowance has been made for the variation of the moon's apparent velocity during the time of the eclipse; this will induce some errour, which, as well as other errours which may have crept into the process, may be thus corrected. At the estimated time of the beginning of the eclipse, find, from the horary motions and computed parallaxes, the apparent latitude L r of the moon, and its apparent longitude S r from the sun; then $SL = \sqrt{Sr^2 + rL^2}$: if this be equal to the sum of the apparent semidiameters of the sun and moon, and the horary motions be properly taken, the estimated time requires no correction; but if S L be not equal to this sum, assume another time, either a small interval *before* or *after* the estimated time of beginning, according as S L, thus found, is *less* or *greater* than the sum of the apparent semidiameters; to that time compute the moon's apparent latitude and longitude, and thence find S L, by taking the square root of the sums of the squares of this com-

* Other great solar eclipses will happen in England, on Oct. 9, 1847, A.M. . . March 15, 1858, A.M. . . Aug. 19, 1887, A.M. . . April 17, 1912, A.M. . . April 8, 1921, A.M. . . Feb. 15, 1961, A.M. . . August 11, 1999, A.M. The duration and magnitude of either of which, the student may calculate, as an agreeable exercise for his leisure hours.

puted latitude and longitude; from these two values of S L the correction in time may be readily obtained by the method of *trial and errour:* and in a similar way we may correct the estimate time of the end of an eclipse. But even after all this additional labour, the result can only be considered as accurate in proportion to the reliance which may be placed upon the tables; and we have no *lunar tables* but what are subject to an errour of about half a minute in longitude, which may occasion an errour of about a minute and a half in the calculation of a solar eclipse.

541. Precepts for the computation of solar eclipses might have been delivered in smaller compass; but then what was gained in point of room would not compensate for the want of conviction which would attend a mere delivery of such rules as do not carry their rationale along with them; on this account we have combined the precepts with an example, and shall now mention a few other circumstances, most of which naturally follow from what has been previously shewn. 1. An eclipse of the sun does not appear the same in all parts of the earth where it is seen, but is in some, *total* or *annular*, while in others it is *partial*. 2. A solar eclipse does not happen at the same time in all places where it is seen; but appears more *early* to the *western* parts, and more *late* to the *eastern*; as the motion of the moon beyond the sun, and consequently of her shadow, is from west to east. 3. An eclipse of the sun begins on the *western* side, and ends on the *eastern*. 4. No eclipse of the sun happens to all places where the sun is visible; for the penumbra does at no time cover a hemisphere of the earth. 5. The position of the cusps of the horns of the unobscured part of the sun's disc may be easily found in the middle of the eclipse; for the line which joins them is parallel to the moon's apparent way. 6. The middle of a solar eclipse will not be at the same time in all places

on the same meridian; for the parallax of longitude will be different in different latitudes. 7. The excess of the apparent semidiameter of the moon above that of the sun in a total eclipse is so small, that total darkness seldom continues more than four minutes in the latitude of London. 8. In most solar eclipses the moon's disc is covered with a faint light, which is attributed to the reflection of the light from the illuminated part of the earth. 9. In total eclipses of the sun, the darkness is sometimes so great as to render visible the planets above the horizon, and stars of the second magnitude. In such eclipses the moon's limb is seen surrounded with a ring which appears much brighter and whiter near the moon's body than at a distance from it; this ring in all respects resembles the appearance of an enlightened atmosphere viewed from a distance; but whether it belongs to the moon or the sun is not entirely decided, though it is generally supposed that it belongs to the former *.

542. With respect to the *number of eclipses* of both luminaries, it may be observed that there cannot be fewer than two, nor more than seven, in one year; the most usual number is four, and it is rare to have more than six. The reason is obvious; for the sun passes by both the nodes of the moon's orbit but once in a year, unless he pass by one of them in the beginning of the year, in which case he will pass by the same again a little before the end of the year; because (Art. 448.) the nodes retrograde about $19\frac{1}{3}$ degrees every year, and therefore the sun will come to either of them about 173 days after the other, for he will have to move $180^\circ - \frac{19\frac{1}{3}^\circ}{2}$, which

* A very ingenious method of constructing and calculating solar eclipses was first given by Mr. *Flamstead* in vol. I. of *Sir Jonas Moore's System of Mathematics*: the same method in substance is given by Dr. *Keill*, M. *de la Caille*, Mr. *Ferguson*, and Mr. *Vince*, in their respective treatises on Astronomy.

will occupy nearly 173 days. And if neither node be within 17° of the sun at the time of new moon, he will be eclipsed; and at the subsequent opposition the moon will be eclipsed near the other node, and come round to the next conjunction before the former node be 17° from the sun, when he will be again eclipsed. When three eclipses happen about either node, the like number commonly happens about the opposite one; as the sun comes to it in 173 days afterward, and six lunations contain only four days more. Thus there may be two eclipses of the sun, and one of the moon, about each of the nodes; and the twelfth lunation from the eclipse in the beginning of the year may give a new moon before the year is ended, which, in consequence of the retrogradation of the nodes, may be within the solar ecliptic limit; and hence there may be seven eclipses in a year, *five* of the sun and *two* of the moon. But when the moon changes in either of the nodes, she cannot be near enough the other node at the next full moon to be eclipsed; and in six lunar months afterwards she will change near the other node; in which case there cannot be more than two eclipses in a year, both of the sun. The ecliptic limits of the sun (Art. 533.) are greater than those of the moon (Art. 523.), and hence there will be more solar than lunar eclipses, as they will nearly bear the proportion of their limits, viz. about four to three. But more lunar than solar eclipses are seen at any given place; because a lunar eclipse is visible on a whole terrestrial hemisphere at once, whereas a solar eclipse is visible only on a portion of a hemisphere.

543. Since eclipses depend upon the position of the sun and moon, with respect to the nodes, it is manifest, that when they are in the syzygies, and have the same situation with respect to the nodes, the eclipses will return and appear as before: the space

of time after which the same eclipses return again is called *the period of eclipses.* Now, as the nodes move backwards $19\frac{1}{3}°$ every year, they would shift through all the points of the ecliptic in eighteen years and 225 days; and this would be the regular period of the return of eclipses, if any complete number of lunations were performed in it, without a fraction; but this is not the case. However, in 223 mean lunations, after the sun, moon, and nodes, have been once in a line of conjunction, they return so nearly to the same state again, that the same node which was in conjunction with the sun and moon at the beginning of these lunations will be within 28′ 12″ of the line of conjunction, when the last of these lunations is completed; and in this period there will be a regular return of eclipses, till it be repeated about forty times, or in about 720 years, when the line of the nodes will be $28' \times 40$ from the conjunction, and will, consequently, be beyond the ecliptic limits: this is called the *Plinian period*, or *Chaldean saros;* it contains, according to Dr. *Halley*, 18 Julian years 11 days 7 hours 43 minutes 20 seconds; or, according to Mr. *Ferguson*, $18^y\ 11^d\ 7^h\ 42^m\ 44^s$. In an interval of $557^y\ 21^d\ 18^h\ 11^m\ 51^s$, in which there are exactly 6890 mean lunations, the conjunction or opposition coincides so nearly with the node, as not to be distant more than 11″. If, therefore, to the mean time of any solar or lunar eclipse, we add this period, and make the proper allowance for the intercalary days, we shall have the mean time of the return of the same eclipse. This period is so very near, that in 6000 years it will vary no more from the truth, as to the restitution of eclipses, than $8\frac{1}{4}$ minutes of a degree.

544. The doctrine of eclipses is of considerable advantage and utility. From hence we deduce a strong confirmation of the conclusions arrived at in the first chapter, respecting the figure of the earth;

for, in all lunar eclipses, the shadow of the earth upon the moon's disc is always bounded by an arc very nearly circular; which could not happen, unless the shadow of the earth in all situations was nearly conical, and, consequently, the earth itself nearly spherical. Lunar eclipses also prove that the sun is larger than the earth, and the earth larger than the moon; that the sun is larger than the earth, because the shadow of the earth ends in a point, which it could not do if the sun were smallest; and that the earth is larger than the moon, because the whole of the moon's body is sometimes involved in the earth's shadow, and a section of this shadow at the moon's orbit is much less than the earth itself. Eclipses also, that are similar in all circumstances, and that happen at considerable intervals of time, serve to ascertain the period of the moon's motion (Art. 440.). In geography, the advantages resulting from eclipses are very great; for by them we can discover the longitudes of different places, especially by those of the moon, which are most often visible, and the same lunar eclipse being of equal magnitude and duration, and beginning at the same *absolute* moment, at all places where it is seen (Art. 666.). Eclipses also lend their assistance in removing doubts respecting the particular times of ancient historical events, and thus render essential service to *chronology*; for we find that, even in the histories of *Herodotus* and *Thucydides*, there are no regular dates for the events recorded, and that no attempts were made to establish a fixed æra until the time of *Ptolemy Philadelphus*, who endeavoured to effect it, by comparing and correcting the dates of the Olympiads, the kings of Sparta, and the succession of the priestesses of Juno at Argos. Now if, near the time of any memorable event recorded in history, a remarkable solar or lunar eclipse be also recorded, we may know whether the real time of this event is rightly determined, by a proper

application of the doctrine of eclipses. Thus we learn from the testimony of the ancients, that *Thales* predicted a total solar eclipse, which was to happen either at *Sardis*, or *Miletus*, in the 4th year of the 48th Olympiad; which happening agreeably to the prediction, brought about a peace between the *Medes* and *Lydians*. Modern chronologers say this peace took place in the 585th year before Christ; and we find, by means of the periods of eclipses, that a very signal solar eclipse happened on the 28th of May in that year, corresponding to the present 10th of that month, which confirms the accuracy of the date. Again, *Thucydides* relates, that a solar eclipse happened on a summer's day in the afternoon, in the first year of the *Peloponnesian* war, which eclipse was so great that the stars appeared. This is stated by modern authors to have been in the year 431 before Christ; and, by computation, it appears, that on the 3d of August that year there was a great solar eclipse, which passed over Athens about six o'clock in the afternoon; this, therefore, corroborates the decision of the chronologers. And the same might be shewn in many other instances, where the appearances resulting from the computations are found to correspond as nearly with those recorded as can reasonably be expected, considering the unavoidable errours which must attend the best constructed tables. In one material instance, the doctrine of eclipses furnishes us with an important correction to the vulgar mode of reckoning: "All the best masters of chronology," as Mr. *Whiston* observes in his *Astronomical Lectures*, "are bold to assert that the christian æra, which hath been used for so many ages past, is less than what it ought to be, by three whole years at least. For whereas it is most certain, that the christian æra, which is taken from the birth of our Saviour, ought to begin in the reign of Herod the Great; and by the testimony of *Josephus*, an

eclipse of the moon happened in Judea a few months before the death of that Herod; which same eclipse the astronomical tables do most certainly shew to have been the *fourth* year before the christian æra, and in the month of March; it appears plain that there is a great mistake in the christian accounts, and that there is more credit to be given to this one astronomical criterion, than to the longest tradition."*

On Occultations of the fixed Stars by the Moon.

545. There is no method of *determining* whether there will be an occultation of a fixed star by the moon, previous to the calculation of the apparent place of the moon at the ecliptic conjunction; but we may, from the course of the moon, conjecture when there will probably be one. According to M. *Cassini*, those stars whose north and south latitudes do not exceed 6° 36′ may suffer an occultation somewhere upon earth; and, if the latitudes are not greater than 4° 32′, they may be eclipsed on any part of the earth. Compute the time of the mean conjunction,

* It appears worth while to observe here, that the time of a very celebrated event in the early history of our own country has been ascertained with great exactness, and that in a considerable degree, though not entirely, by means of an eclipse. The invasion of *Julius Cæsar* is stated by some historians to have taken place in the year 52 before Jesus Christ, by others in the year 54, and by others in the year 56. But Dr. *Halley*, in No. 193 of the *Phil. Trans.* determines the time of this invasion *to a day*, and says the landing was effected on the 26th of August, in the year of the world 3950, the year of Rome 699, or the 55th before the usual christian æra. He was led to this decision, partly by calculating the time of that *lunar eclipse*, of which *Drusus* made such successful use to quiet the *Pannonian* army upon the death of *Augustus*, and partly by computing the time of the full moon, previous to which Cæsar landed his legions, and considering the phenomena of the tides on the Kentish coast at that particular period.

and at that time the moon's true latitude; then, if the difference of the latitudes of the moon and star exceed 1° 37′, there can be no occultation; if the difference be less than 51′, there *must* be one somewhere on the earth; between these limits it is doubtful whether there will be one or not.

546. When it is determined that there may be an occultation, deduce from astronomical tables the *apparent time* of the ecliptical conjunction of the moon and star at the place for which the calculation is made, and for the same time their longitudes and latitudes, parallax and true diameter of the moon, the horary motion of the moon in longitude and latitude, as also the star's right ascension, declination, and the obliquity of the ecliptic; then take the moon's sidereal horary motion, which is her horary motion diminished by $\frac{1}{365}$ of itself; for the latitude of the moon at her conjunction with the star, take the *excess* or *defect* of the moon's latitude, compared with that of the star either *north* or *south*; assume the *apparent time* of the star's transit over the meridian, which is (Art. 171.) the sun's right ascension in time subtracted from that of the star; and to calculate the time of occultation proceed agreeably to the following rules:—

547. *Rule* I. To the *arithmetical complement* of the *co-sine* of the star's latitude, *add* the *co-sine* of its right ascension, and the *sine* of the obliquity, the *sum* is the *sine* of the angle of inclination; and is to the *right* hand of the *northern* axis of the ecliptic, if the star's longitude be between *three* and *nine* signs, otherwise to the left.

548. II. To the *arithmetical complement* of the logarithm of the *moon's sidereal horary motion* in seconds, add the logarithm of the *moon's horary motion in latitude* in seconds, the *sum* is the *tangent* of the angle of inclination of the moon's way with the

northern axis of the ecliptic; which is to the *right* hand, if the moon has *north latitude ascending*, or *south latitude descending*; otherwise to the *left*.

549. III. In case the axis of the moon's way, and the earth's axis, are both on the *same* side of the axis of the ecliptic, the complement of their *difference*; but when on *contrary* sides, the complement of their *sum* to 90° is an angle, which call E.

550. IV. The time that the moon's centre is at E is *before* the true conjunction, if the earth's axis is to the *right* hand of the *northern* axis of the ecliptic, and the moon's latitude at the conjunction *north*, otherwise *after*; but if the moon's latitude at conjunction is *south*, the contrary.

551. V. When the *northern* axis of the moon's way is to the *right* hand of the earth's *northern* axis, the motion in the direction of the earth's axis is from *north* to *south* for any time *before* E, but from *south* to *north* for any time *after* E; otherwise, the contrary.

552. VI. In order to reduce the latitude of the place, and the horizontal parallax of the moon, to the spheroidal figure of the earth, *subtract* the logarithm of the *less* from that of the *greater* axis of the earth (or of any numbers expressing their ratio), the remainder *added* to the *co-tangent* of the latitude in the *sphere*, gives the *tangent* of an arc; to which *add* again the remainder, the *sum* is the *co-tangent* of the latitude in the *spheroid*; or, to twice the remainder *add* the *co-tangent* of the latitude in the *sphere*, the *sum* is the *co-tangent* of the latitude in the *spheroid*.

553. From the *sine* of the said arc subtract the *co-sine* of the latitude in the *spheroid*, and to the remainder *add* the logarithm of the seconds contained in the moon's horizontal parallax in the *sphere* (being the equatoreal horizontal parallax), the *sum* will be

the logarithm of the moon's horizontal parallax in the *spheroid* in seconds, or radius of the disc.

554. Take out from the tables the following sixteen logarithms, and place them one under another, as below:

Call the co-sine complement arithmetical of the angle of the moon's way . . . . *e*
The logarithm of the moon's sidereal horary motion . . . . *f*
Add 6·44370 to *e*, the sum call . . . . . . *g*
The co-sine of the angle of the moon's way . *h*
The logarithm of the seconds in the moon's latitude, at the conjunction . . . . . . *i*
The arithmet. comp. sine of the angle E . . *k*
To the sine of the angle of the earth's axis add 3·55630, call the sum . . . . *l*
The arith. comp. of the sum of *e* and *f*, call . *m*
The sine of the angle E . . . . *n*
The sum of *g* and *f* . . . . *o*
The co-sine of the angle E . . . . *p*
The sine of the star's declination . . . . . *q*
The co-sine of the latitude of the place in the spheroid . . . . *r*
The logarithm of the seconds in the radius of the disc . . . . *s*
The sine of the latitude . . . . *t*
The co-sine of the declination . . . . . . *u*

Collect the following sums; the sum of *i*, *k*, *l*, *m*, is the logarithm of a number of seconds of time before or after the conjunction that the moon's centre is at E, by Rule IV. from which you have the apparent time E.

The sum of *s*, *t*, *u*, is a constant logarithm F, the seconds answering thereto G; the sum of *h*, *i*, *k*, is a constant logarithm H, the seconds corresponding thereto I.

The sum of *n* and *o* is a constant logarithm . A
The sum of *o* and *p* . . . . B

The sum of r and s C
The sum of q, r, and s . . . . D

Now, to find the apparent distance of the centres of the moon and star, reduced to the plane of projection, at any interval of time from E, proceed thus:

555. VII. To the constant logarithms A and B add the logarithm of the assumed interval of time in seconds, the number of seconds answering to the sums call a and b, and to the constant logarithm C *add* the *sine*, and to D the *co-sine*, of the hour from the star's culminating, the number of seconds answering to the sums call c and d; then, if the time is *before* E, and the hour *before* the star's culminating, or the time *after* E, and the hour *after* culminating, the *difference* of a and c, otherwise their *sum*, is a number of seconds, which call K.

556. VIII. Take the *sum* of G and d, when the star's declination and the latitude of the place are of the *same* denominations; but if of *different* denominations, the *difference*; which *sum* or *difference* call L.

557. IX. Take the $\left\{\begin{matrix}\text{sum}\\ \text{difference}\end{matrix}\right\}$ of I and b as the motion in the direction of the earth's axis is from $\left\{\begin{matrix}\text{south to north}\\ \text{north to south}\end{matrix}\right\}$, by Rule V. and the moon's latitude at the conjunction *north*; but if *south*, the contrary; which *sum* or *difference* call M: then take the *difference* of L and M, except when the *difference* of G and d is to be taken for L*, and d is greater than g; in which case take the sum of L and M, unless it should happen at the same time that b is greater than I; and the *difference* of them is to be taken for

* Which is, when the assumed time is nearer twelve hours from culminating than it is to the time of culminating: for then, to C and D, in Rule VII. the *sine* and *co-sine* of the hour from the star's being on the meridian *below the horizon*, is added to obtain c and d; and then, in Rule VIII. the *difference* of G and d is taken instead of the *sum*.

M, then the *difference* of L and M must still be assumed; this *sum* or *difference* will be one leg, and K the other, of a right-angled triangle, by which may be found the hypothenuse, which is the *apparent distance* of the centres at the time required.

N.B For the distance of the centres at the time E, *c* is equal to K, and I is equal to M.

558. In case equal intervals of time are taken from the time E, *a* and *b* will be doubled, tripled, &c. and, therefore, need only be computed once for any number of instants: and the time between the instants converted into degrees, being *added* to, or *subtracted* from, the time in degrees of the first instant, reckoned from culminating, or twelve hours from it, as the times *recede* from, or *approach* to, the one or the other, will produce the hour angles at their respective instants.

559. When the apparent distance of the centres of the moon and star, *approaching* to, or *receding* from, each other, is equal to the semidiameter of the moon, the beginning or *immersion*, and end or *emersion*, of the occultation takes place. . . . In any of these operations, when the indices of the logarithms exceed ten, &c. reject the tens.

560. The chief advantage of this method is, that after the apparent distance of the centres is obtained for any instant of time, as many more as is thought proper may be derived from it, with the greatest facility, by four analogies only; whereas, in the common way, by parallaxes, *three* times the labour, at least, is requisite to arrive at the same accuracy*.

* This method is capable (with slight alterations) of being applied with great benefit to the computation of *solar eclipses*, as may be seen in Dr. *Hutton*'s *Miscellanea Mathematica*, where the rules were first given by Mr. J. *Keech*, and accompanied by examples, the results of which agreed exceedingly nearly with the actual observations of Professor *Hornsby* and Mr. *Reeves*.

CHAPTER XX.

On the Transits of Mercury and Venus over the Sun's Disc.

Art. 561. IF the orbits of the inferiour planets, *mercury* and *venus*, were in the same plane with the orbit of the earth, it is obvious that every time either of these planets was in conjunction with the sun, it would seem in actual contact with the body of the luminary; or, since the unillumined or darkened side of the planet would be then towards the earth, it would appear like a small dark spot upon the sun's disc. But since the orbit of mercury makes an angle of 7°, and that of venus an angle of $3^\circ\ 23\frac{1}{2}'$, with the orbit of the earth (Art. 356.), neither of these planets can pass over, or *transit*, the sun's disc, unless at the time of conjunction the planet be so near its node, that its geocentric latitude is less than the apparent semidiameter of the sun. Now if we know the time in which a transit happened at either node, we may determine when there will be another transit at the same node, by considering that, before another transit can so happen, the earth must perform a certain number of complete revolutions through its orbit, in very nearly the same period that the planet performs a greater certain number of complete revolutions, which will bring both the earth and inferiour planet in the same situation relatively to the sun. The respective numbers of the revolutions of the earth and

planet will manifestly be in the inverse ratio of the times of revolution; and as the periodic times of the earth, mercury, and venus, are 365·256, 87·968, and 224·7, respectively, we must find whole numbers in the ratios of 87·968 : 365·256, and 224·7 : 365·256. Here 7 : 29, 13 : 54, 33 : 137, and 46 : 191, are ratios which approximate more and more nearly to that of 87·968 : 365·256; therefore, in seven years, mercury will have nearly performed twenty-nine revolutions, in thirteen years, fifty-four revolutions, and so on, approaching nearer the truth as the numbers expressing the ratio increase. Hence, if to the time of a transit of mercury, which happened at either its ascending or descending node, thirteen years, thirty-three, or forty-six, be continually added, the times when other transits, at the ascending or descending node, may be expected, will be nearly known; and if for each of these times the shortest geocentric distance of mercury from the sun's centre at the time of conjunction be computed, we may be assured there will be a transit, if that distance is less than the sun's apparent diameter. Now, on May 7, 1799, there was a transit of mercury at its descending node, and by adding thirteen, thirty-three, &c. it appears that others may be expected at the descending node in May, 1832, 1845, 1871, and 1891. Also, on November 5, 1789, there was a transit of mercury at the ascending node, whence others may be expected at that node in 1802, 1815, 1822, 1835, 1848, 1861, 1868, and 1894. Again, with relation to venus, the ratios are 8 : 13, 235 : 382, 243 : 395, 713 : 1159, approximating more and more nearly to equality with the ratio of 224·7 : 365·256; consequently, the periods are 8, 235, 243, and 713 years, at the ends of which transits may be expected to happen. Now these *transits of venus* are of considerable advantage in astronomy, on which account the times of their taking place have been carefully computed,

particularly by Dr. *Halley**, who, after exhibiting the principles on which these calculations depend, proceeds as follows:

562. "After eight years venus returns to the sun, taking away $2^d\ 10^h\ 52\frac{1}{2}^m$ from the moment of the foregoing transit, and the planet proceeds in a path which is 24′ 41″ more to the south than the former.

563. "After 235 years, adding $2^d\ 10^h\ 9^m$, venus may again enter the sun, but in a more northern path by 11′ 33″; but, if the foregoing year is bissextile, $3^d\ 10^h\ 9^m$ must be added.

564. "After 243 years venus may also pass the sun, only taking away $0^h\ 43^m$ from the time of the former, but proceeds more southerly by 13′ 8″; but, if the foregoing year be bissextile, add $23^h\ 17^m$.

565. "And in all these appulses of venus to the sun in the month of November, the angle of her path with the ecliptic is 9° 5′, and her horary motion within the sun is 4′ 7″; and, since the semidiameter of the sun is 16′ 21″, the greatest duration of the transit of the centre of venus comes out $7^h\ 56^m$.

566. "Then let the sun and venus be in conjunction at the descending node in the month of May, and by the same numbers the same intervals may be computed. After eight years, let there be taken away $2^d\ 6^h\ 55^m$, and venus will make her transit in a more northern path by 19′ 58″.

567. "After 235 years, add $2^d\ 8^h\ 18^m$; or, if the foregoing year be bissextile $3^d\ 8^h\ 18^m$, and you will have venus more to the south by 9′ 21″.

568. "Lastly, after 243 years, add $0^d\ 1^h\ 23^m$, or, if the foregoing year be bissextile, $1^d\ 1^h\ 23^m$, and venus will be found in conjunction with the sun, but in a more northerly path by 10′ 37″.

569. "In every transit within the sun at this node, the angle of venus's path with the ecliptic is 8° 28′,

* See *Philosophical Transactions*, No. 193, or *Abridgement* of ditto, vol. I. p. 427.

and her horary motion is $4' \; 0''$; and the semidiameter of the sun subtending $15' \; 51''$, the greatest duration of the central transit comes out also $7^h \; 56^m$, exactly the same as the other node.

570. "As to the epochs, from that only ingress which *Horrox* observed, the sun being then just ready to set, it is concluded that venus was in conjunction with the sun at London in the year 1639, Nov. $24^d \; 6^h \; 37^m$, and that she declined towards the south $8' \; 30''$. But in the month of May, no mortal has seen her, as yet, within the sun. But from my numbers, which I judge to be not very different from the heavens, it appears that venus for the next time will enter the sun in 1761 *, May $25^d \; 17^h \; 55^m$, that being the middle of the eclipse, and then will be distant from his centre $4' \; 15''$ towards the south. Hence, and from the foregoing revolutions, all the phenomena of this kind will be easily exhibited for a whole millennium, as I have computed them in the following table.

TABLE.

In November.							In May.						
Years.	Times of Conjunction.			Distance from Sun's Centre.			Years	Time of Conjunction.			Distance from Sun's Centre.		
918	20d	21h	53m	6′	12″	N.	1048	24d	13h	45m	3′	50″	N.
1161	20	21	10	6	55	S.	1283	23	8	14	5	31	S.
1396	23	7	20	4	38	N.	1291	25	15	9	14	27	N.
1631	26	17	29	16	11	N	1518	25	16	32	14	52	S.
1639	24	6	37	8	30	S.	1526	23	9	37	5	6	N.
1874	26	16	46	3	3	N.	1761	25	17	55	4	15	S
2109	29	2	57	14	36	N	1769	23	11	0	15	43	N.
2117	26	16	3	10	5	S.	1996	28	2	13	13	36	S.
							2004	25	19	18	6	22	N.

* The dates which Dr. Halley here gives, are conformable to the *old style* they therefore require an addition of eleven days for the last century, twelve days for the present, thirteen for the next, and so on, in order to reduce them to the current dates; so that these transits, instead of falling in May and November, will hap-

571. "As for the durations of these eclipses of venus, they may be computed after the same manner as those of mercury in respect of the centre*. But since venus's diameter is pretty large, and since the parallaxes also may bring a very notable difference as to time, a particular calculation must necessarily be made for every place.

572. "Now the diameter of venus is so great, that while she adheres to the sun's limb, almost twenty minutes of time will be elapsed, that is, when she applies directly to the sun. But when she is incident obliquely, she continues longer in the limb. Now that diameter, according to Horrox's observation, takes up 1′ 18″, when she is in conjunction with the sun at the ascending node, and 1′ 12″ at the other node.

573. "Now the *chief use* of these conjunctions, is, *accurately to determine the sun's distance from the earth, or his parallax,* which astronomers have in vain attempted to find by various other methods; for the minuteness of the angles required, easily eludes the nicest instruments. But in observing the ingress of venus into the sun, and her egress from the same, the space of time between the moments of the *internal contacts*†, observed to a second of time, that

pen in June and December; that in 1761, for instance, happening on the 6th of June.

* Thus, if CODI (fig. 3, Pl. VII.) represent the disc of the sun, N*mn* the orbit of venus, then the duration of the transit when venus enters the disc at *m*, and quits it at *n*, will be to the duration of the central transit (Art. 565. 569.) as *mn* to the diameter. And P*a* being known, we shall have, $OP \times PI : OP^2 :: \overline{OP + Pa} \times \overline{OP - Pa} : an^2$; whence *na* becomes known, and consequently *nm*.

† What Dr. *Halley* means by the *internal contacts*, will appear from the following passage extracted from Dr. *Maskelyne*'s instructions relative to the observation of the transit of venus, on June 3, 1769: "Venus will begin to enter the sun's disc to the north-east part of the same, reckoning the point of the sun's circumference or limb nearest the north pole of the world, to the north point. Her diameter will subtend near a minute; therefore, as that of the sun is about $31\frac{1}{2}$ minutes, her diameter will be about $\frac{1}{32}$ of that

is, $\frac{1}{15}$ of a second, or 4‴ of an arch, may be obtained by the assistance of a moderate telescope, and a pendulum clock, that is consistent with itself exactly, for the space of six or eight hours. Now, from two such observations rightly made in proper places, the distance of the sun within a 500th part may be certainly concluded." These remarks of Dr. *Halley* are, in the main, consistent with truth, his deductions deviating from accuracy no farther than his tables stood in need of correction; and, with respect to the chief purpose to which he recommends these transits to be applied, it has since been done with very singular success, in the transits which took

of the sun. She will move a space of about four minutes, or four times her own diameter, in an hour, along a line or chord, which at its nearest approach is ten minutes distance from the sun's centre. Thus she will take about six hours and twenty minutes (exclusive of the effect of parallax) to move over the sun's disc, from her first making a small impression in the limb on the NE. point, to her going out of the sun's disc on the NW. point, and leaving the sun's limb there entire again. The first least visible impression made by venus's inner limb upon the sun's limb, is commonly called the sun's *first external contact*; though, properly speaking, the true external contact of venus, and the sun's limb, happens a few seconds of time sooner, before the least particle of venus has entered upon the sun. . . . After this, venus gradually makes a deeper and deeper impression, or notch, in the sun's limb; and in about nine minutes half her opaque body will be seen upon the sun's body, eclipsing an equal part of the same, the other half of her body being without the sun's disc, and therefore invisible: in nine minutes more, the whole body of venus will be introduced within the sun's disc, and her outer limb will touch the sun's limb internally, or form what is called the *first internal contact*. . . . She will now recede from the sun's limb, by approaching nearer to his centre, and in about 2^h 52^m more, will be found at the nearest distance of ten minutes from the centre; she will then recede from the centre again, and in 2^h 52^m more, her outer limb will touch the sun's limb internally on the north-west part of the sun's circumference, and form the *second internal contact*: she will now be leaving the sun, and in nine minutes her centre will appear upon the sun's limb, and about nine minutes after that, her inner limb will touch his limb externally, or make the *second external contact*; and the impression made by her body in the sun's limb being gradually imperceptible, she will go entirely out of the sun." See *The Nautical Almanac*, for 1769.

place in 1761 and 1769, by means of which, the sun's parallax and distance have been ascertained with as much precision as the nature of the enquiry will permit us reasonably to look for.

574. A general idea of this method of determining the parallax of venus, and thence that of the sun, may be obtained by referring to fig. 11, Pl. VIII. where D B A represents the earth, V venus, and T S R the eastern limb of the sun. To an observer at B, the point *t* of that limb will be on the meridian, its place referred to the heaven, will be at E; and venus will appear just within it at S. But, at the same instant, to an observer at A, venus is east of the sun, in the right line A V F, the point *t* of the sun's limb appears at *e* in the heavens, and if venus were then visible, she would appear at F. The angle C V A is the horizontal parallax of venus, and is manifestly equal to the angle F V E, whose measure is the arc F E. A S C is the sun's horizontal parallax, equal to the opposite angle *e* S E, whose measure is the arc *e* E: and F A E = V A *v* is venus's horizontal parallax from the sun, which may be found by observing how much later in absolute time (allowing for the difference of the meridians of A and B) her total ingress on the sun is as seen from A, than as seen from B, which is the time she takes to move from V to *v* in her orbit. Now V A *v* = A V C − A S C, the difference of the horizontal parallaxes of venus and the sun; and from the nature of parallax (Art 82.) A V C : A S C : : S C : V C, consequently V A *v* varies as S C — V C or S V: but, from the respective theories of venus and the earth, the proportions of S V, S C, or of S V, VC, may be known at the time of the transit; therefore we have S V : S C : : V A *v* : A V C, and SV : VC : : V A *v* : A S C, whence the horizontal parallaxes of venus and the sun become known.

575. Or, it is possible to effect the same thing by

a single observer, as may be shewn in the following manner: Let B M A (fig. 5, Pl. VIII.) be the earth, V venus, and S the sun. The earth's motion on its axis from west to east, or in the direction A M B, carries an observer on that side contrary to the motion of venus in her orbit, which is in the direction U V W, and will, therefore, cause her motion to appear quicker on the sun's disc, than it would appear to an observer placed at the earth's centre C, or at either of its poles; for, if venus were to stand still in her orbit at V for twelve hours, and the earth had no motion but that of rotation, the observer on the earth's surface would, in that time, be carried from A to B, through the arc A M B. When he was at A, he would see venus on the sun at R; when at M, he would see her at S; and when he was at B, she would appear at T; so that his own motion would cause the planet to appear in motion on the sun, through the line R S T, which being in the direction of her apparent motion on the sun, as she moves in her orbit U W, and the earth in its orbit, that apparent motion will be accelerated on the sun to this observer, just as much as his own motion would shift her apparent place on the sun, if neither she nor the earth moved in their respective orbit. But as the whole duration of the transit, from the first to the last internal contact, seldom exceeds six hours, an observer, who has the sun on his meridian at the middle of the transit, will be carried only from *a* to *b* during the whole time thereof; and, therefore, the duration will be much less contracted by his own motion, than if the planet were to be twelve hours in passing over the sun, as seen from the earth's centre. Now it is manifest, that the nearer venus is to the earth, the greater is her parallax, and the more will the true duration of her transit be contracted; the farther she is from the earth, the contrary; so that the contraction will be in direct proportion to the parallax. Therefore, by

observing, at proper places, how much the duration of the transit is less than its true duration at the earth's centre, as given by the Astronomical Tables, the parallax of venus may be ascertained, and thence that of the sun.

576. Dr. *Maskelyne*, the astronomer royal, whose labours have, for a series of years, been so successfully directed for the promotion of science, and the honour of his country, has paid a peculiar regard to the doctrine of transits: he has given a method of computing the effect of parallax in accelerating or retarding the time of the beginning or end of a transit of venus over the sun's disc, which, as we think it preferable to any other method that has yet been given, is here inserted. The diagram (fig. 6, Pl. VIII.) refers particularly to the transit of venus, which took place in May, 1769; but the method is equally applicable to any other transit of venus, or mercury. C represents the centre of the sun's disc L Q, P the celestial north pole of the equator, S the south pole, P C S a meridian passing through the sun, Z the zenith of the place, A D B ☋ the relative orbit of venus, ☋ being the relative place of the descending node, A the geocentric place of venus at the ingress, B at the egress, and D at the nearest approach to the sun's centre, as seen from the earth's centre, and *o* the apparent place of venus at the egress to an observer whose zenith is Z: draw *o u* Z, and *u* is the *true* place of venus when the *apparent* place is at *o*, and *u o* is the parallax in altitude of venus from the sun; the time of contact will manifestly be diminished by the time which venus takes to describe *u* B: draw *n' o' h o n* E parallel to A B, meeting Z B produced in E, and B *n*, A *n'*, tangents to the circle, and let C *h* D be perpendicular to A B. Now, the trapezium *u o* E B, on account of the minuteness of its sides, may be considered as rectilinear, and as Z B is of immense magnitude in comparison with B *u*, B E may be considered as parallel to *u o*; consequently,

$u\,o$ E B may be looked upon as a parallelogram, and E o as equal to u B. But E o = E $n \pm n\,o$, the upper or lower sign obtaining, according as E falls without or within the circle representing the sun's disc; and, by trigonometry, E n : E B : : sine E B n = co-sine C B Z : sine B n E = sine B C D = co-sine C B D; hence, E $n = \frac{\text{E B} \times \text{co-sine C B Z}}{\text{C B D}}$; also, by the nature of the circle $n\,o = \frac{\text{B}n^2}{n\,o'} = \frac{\text{B}n^2}{\text{AB}}$ very nearly; but B n : B E : : sine B E n = sine Z B D : sine B n E = co-sine C B D; therefore, $\text{B}n^2 = \frac{\text{B E}^2 \times \text{sine Z B D}^2}{\text{co-sine C B D}^2}$, and $n\,o = \frac{\text{B E}^2 \times \text{sine Z B D}^2}{\text{A B} \times \text{co-sine C B D}^2}$. Put h = horizontal parallax of venus from the sun, and (Art. 83.) B E = $h \times$ sine Z o = $h \times$ sine Z B; hence u B = o E = E $n \pm n\,o = \frac{h \times \text{sine Z B} \times \text{co-sine C B Z}}{\text{co-sine C B D}} \pm h^2 \times$ sine Z B $\times \frac{\text{sine Z B D}^2}{\text{B A} \times \text{co-sine C B D}^2} = h \times$ sine Z B $\times$ co-sine CBZ $\times$ sec. CBD $\pm \frac{h^2 \times \text{sine ZB}^2 \times \text{sine ZBD}^2 \times \text{sec. CBD}^2}{\text{A B}}$. Put t = the time which venus takes by her geocentric relative motion, to describe the space h; to find which, let m be the relative horary motion of venus; then $m : h$: : 1 hour = 3600″ : $t = \frac{h \times 3600''}{m}$; hence, to find the time of describing u B, we have h : to the last value of u B above expressed : : t : $t \times$ sine Z B $\times$ co-sine C B Z $\times$ sec. C B D $\pm \frac{t \times h \times \text{sine Z B}^2 \times \text{sine Z B D}^2 \times \text{sec. C B D}^2}{\text{A B}}$, the time of describing u B, or the effect of parallax in accelerating or retarding the time of contact; the upper sign is to be used when C B Z is acute, and the lower when it is obtuse. If C B Z be very near a right angle, but obtuse, it may happen that n E may be less than $n\,o$,

in which case n E is to be taken from $n\,o$, according to the rule The principal part n E of the effect of parallax will increase or diminish the planet's distance from the sun's centre, according as the angle Z B C is acute or obtuse; but the small part $n\,o$ of parallax will always increase the planet's distance from the centre; take, therefore, the sum or difference of the effects, with the sign of the greater, as to increasing or decreasing the planet's distance from the centre of the sun. Or the rule may be otherwise stated thus: take the sum or difference of n E and $n\,o$, according as Z B C is acute or obtuse, and the distance of the planet from the sun's centre will always be increased in the first case, and diminished in the second, except when Z B C is obtuse and near 90°, for then n E may be less than n , and the distance from the sun's centre will be increased by the difference. If Z B C be acute, the part n E will retard the ingress, and accelerate the egress; but if Z B C be obtuse, the part n E will accelerate the ingress, and retard the regress. In like manner, the parallax affects the time of the planet's coming to any given distance from the sun's centre, before or after the middle of the transit. The second part of the correction did not exceed 9^s or 10^s of time in the transits of venus, in 1761 and 1769, where the nearest approach to the sun's centre was about 10′. In the transits of mercury, the first part of the correction will generally be sufficient.

577. The transit of 1769 was observed at *Wardhuys*, a seaport of Danish Lapland, standing on a small island of the same name, near the continent; also at *Otaheite*, an island in the South Pacific Ocean; and it appeared, from a comparison of the observations, that the duration at the former place exceeded that at the latter by 23^m 10^s. But, from the known situation of the two places, Dr. *Maskelyne*, assuming the sun's mean horizontal parallax 8·83 (agreeably to the result of observations on the transit in 1761),

and calculating from the expression in Art. 576. found that the ingress at *Wardhuys* was accelerated by the parallax $6^m\ 38^s{\cdot}74$, and the egress retarded $4^m\ 38^s{\cdot}14$; so that the whole duration was *lengthened* $11^m\ 16^s{\cdot}88$. At *Otaheite*, according to this gentleman's calculation, the ingress was retarded $5^m\ 43^s{\cdot}11$, and the egress accelerated $6^m\ 26^s{\cdot}96$; so that the whole duration was *diminished* $12^m\ 10^s{\cdot}07$: consequently, the computed difference of the times is $23^m\ 26^s{\cdot}95$.

578. Now, since the observed difference of the total durations at *Wardhuys* and *Otaheite* is $23^m\ 10^s$, and the computed difference, from the assumed sun's mean horizontal parallax $8''{\cdot}83$, is $23^m\ 26^s{\cdot}95$, the true parallax of the sun is less than that assumed. Let the true parallax be to that assumed as $1 - e$ to 1, and (Art. 576.) the first parts of the computed parallax will be lessened in the ratio of $1 - e : 1$, and the second parts in the ratio of $\overline{1 - e}|^2 : 1$, or of $1 - 2e : 1$ nearly. All the first parts, according to Dr. *Maskelyne*'s calculation, viz. $406^s{\cdot}05$, $287^s{\cdot}05$, $341^s{\cdot}48$, $382^s{\cdot}47$, in all $= 1417^s{\cdot}05$, combine the same way to make the total duration at *Wardhuys* longer than that at *Otaheite*. As to the second parts, the effects at *Wardhuys* are $-7^s{\cdot}31$ and $-8^s{\cdot}91$, and at *Otaheite* are $+1^s{\cdot}63$ and $+4^s{\cdot}49$; these four incorporated give $-10^s{\cdot}10$. Therefore, $1417^s{\cdot}05 \times \overline{1 - e} - 10^s{\cdot}10 \times \overline{1 - 2e} = 1390^s$, the excess of the observed total duration at *Wardhuys* above that at *Otaheite*; or $1417^s{\cdot}05 - 10^s{\cdot}10 - 1390^s = \overline{1417^s{\cdot}05 - 20^s{\cdot}20} \times e$, and $e = \frac{16{\cdot}95}{1396{\cdot}85} = 0{\cdot}0121$: hence the mean horizontal parallax of the sun $= 8''{\cdot}83 \times \overline{1 - 0{\cdot}0121} = 8''{\cdot}72316$. And by computing the effects of this parallax at both places, and comparing them with the observations, the differences were so trifling, that the above conclusion, or $8\frac{3}{4}''$, or $8\frac{13}{18}$, which is very near it, must be reckoned within

a very small degree of the true horizontal parallax at the sun's mean distance from the earth.

579. Mr. *Short* states the mean horizontal parallax, as a medium resulting from many observations, at about 8″ 47; *Euler* makes it 8″·68; Dr. *Hornsby*, 8″·69 and 8″·92; M. *Pingre*, 8″·8; M. *du Sejour*, 8″·81; M. *de la Lande*, 8″·6. The mean of all these determinations agrees very nearly with the parallax, as deduced from the calculations of Dr. *Maskelyne*. Hence, then, knowing the radius of the earth, we may find the mean distance of the sun, according to the method given in Chap. XI. Prob. 7.

580. The transits of mercury and venus are not only useful in determining the sun's parallax, but they may be applied to the ascertaining of the longitudes of different places; they occur so seldom, however, that they cannot be of great utility in this respect. They are of considerable benefit in correcting the elements of the respective planets, particularly the places of the aphelia and nodes, and the inclination of the orbit. The transits of venus in 1761 and 1769 contributed much to the correction of the theory of that planet; as to the transit of mercury in 1799, the weather in England was so unfavourable, that the sun could not be viewed at the time of the transit; it was, however, observed in France, by *La Lande*, who, in his *History of Astronomy* for 1800, says, "The transit of mercury over the sun enabled me to verify the place of the aphelion, and by my result there appears no necessity for changing the Tables of mercury." Dr. *Herschel*, in consequence of the effect of the solar rays upon the glasses of his seven-feet telescope, when prepared for viewing this transit, was led to an enquiry into the method of viewing the sun advantageously, with telescopes of large apertures and high magnifying powers; in pursuit of this enquiry he instituted a series of very curious experiments, an account of which is inserted in the *Philosophical Transactions* for 1800, Part II.

The results he obtained from them furnished him with various means of adapting his telescopes more successfully to solar observations; and, as the two methods which he found most effectual may be highly useful to the practical astronomer, they are here described: he placed a very dark green glass behind the second eye-glass, that it might be sheltered by both glasses, which, in his double eye-piece, are close together, and of an equal focal length. Here, as the rays were not much concentrated, the coloured glass received them on a large surface, and stopped light and heat, in the proportion of the square of its diameter then used, to that on which the rays would have fallen had it been placed in the focus of pencils. For the same reason he also placed a dark green smoked glass close upon the former, with the smoked side towards the eye, that the smoke might likewise be protected against heat, by a passage of the rays through two surfaces of coloured glass. This position had the advantage of leaving the telescope, with its mirrours and glasses, completely to perform its operation, before the application of the darkening apparatus; and thus to prevent the injury which must be occasioned by the interposition of the heterogeneous colouring matter of the glasses, and of the smoke. . . . He also placed a deep blue glass, with a blueish-green smoked one upon it, as in the former method, and found the sun of a whiter colour than before; there was no disagreeable sensation of heat, though a little warmth might be felt. . . . The doctor adds, that with either of these "he has seen uncommonly well; and that, in a long series of very interesting observations upon the sun, the glasses have met with no accident."*

* In the *Memoirs of the Academy of Sciences at Berlin*, from August, 1786, to the end of 1787, there is inserted an account of "*Observations on the Passage of Mercury over the Sun's Disc, on May 4th*, 1786," by Professor *Beitler*, of Mitaw, in Courland. From these observations the diameter of mercury, seen from the earth, was found to be 10″·37; as seen from the sun, 12″·836; and as seen at the mean distance of the sun from the earth, 5″·79.

CHAPTER XXI.

On Comets.

Art. 581. BESIDES the ordinary planets, which are always in our neighbourhood (if we may so speak), and within our view, or, at least, within the reach of our telescopes, there is another sort of planets, which may, perhaps, be called temporary; for, after any one of them has been conspicuous for a season, it withdraws, and is no longer visible: these bodies are called *comets*, or, vulgarly, *blazing stars*, having this to distinguish them from other stars, that they are usually attended with a long train of light, tending always opposite to the sun, and being of a fainter lustre the farther it is from the body of its respective comet. There is a popular division of comets into three kinds, *bearded*, *tailed*, and *hairy*; but, in general, this division rather relates to the several circumstances of the same comet than to the phenomena of several. Thus, when the comet is eastward of the sun, and moves from him, it is said to be *bearded*, because the light precedes it in the manner of a beard; when the comet is westward of the sun, and sets after him, it is said to be *tailed*, because the train of light follows it; and lastly, when the sun and comet are diametrically opposite, the earth being between them, all the train, except the extremities, is hid behind the body of the comet; this, therefore, appears round it like a border of hair or *coma*, whence it is called *hairy*,

and whence the name of comet is derived. The magnitude of comets has been observed to be very different; many of them without their hair have appeared no larger than stars of the first magnitude; but some authours have given us accounts of others, which appeared much greater; such was that which appeared in the time of the emperor *Nero*, which, as *Seneca* relates, was not inferiour, in apparent magnitude, to the sun himself. The comet which *Hevelius* observed, in the year 1652, did not seem to be less than the moon, though it was deficient in splendour, for it had a pale, dim light, and appeared with a dismal aspect. Most comets have dense and dark atmospheres surrounding their bodies, which weakens the sun's rays that fall upon them; but within these appears the *nucleus*, or solid body of the comet, which, when the sky is clear, will often give a more splendid light.

582. The ancient philosophers allowed comets a place among the heavenly bodies, and ranked them in stations far above the moon; for *Aristotle* (in his first book of *Meteors*, Chap. VI.*), *Plutarch* (in his third book of *Philosophical Opinions*, Chap. II. †), and *Seneca* (in his seventh book of *Natural Questions*, Chap. III.), testify, that the Pythagoreans, and the whole Italian sect, maintained, that a comet was a kind of *planet*, or *wandering star*, which did not constantly appear in the heavens, but in stated times, after having gone through a determinate circuit, arose or appeared again. *Hippocrates*, *Chios*, and *Democritus*, were of the same opinion, as we are informed by *Aristotle* and *Seneca*. *Seneca* also assures us, that *Apollonius Myndius*, one of the most skilful in the

* Των δε Ἰταλικων τινες και καλεμενων Πυθαγορειων ενα λεγεσιν αυτον ειναι των πλανητων αςερων, ἀλλα δια πολλε τε χρονε τη φαντασιαν ἀυτε ειναι και την υπερβολην επι μικρον ὁπερ συμβαινει και περι τον τε Ερμε αςερα.

† Των ἀπο Πυθαγρε τινες μεν ἀςερα φασιν ειναι τον κομητην των ἐκ ἀει φαινομενων, δια τινος δε ὡρισμενε χρονε περιοδικως ἀνατελλοντων.

search of natural causes, asserted, "that the *Chaldeans* reckoned comets among the other *wandering stars*, and that they knew their courses." *Apollonius* himself maintained, that a comet was a star of its own kind, as the sun and moon are, but that its course was not yet known; that by its motion it mounts very high in the heavens, and only appears when it descends into the lower part of its orbit. *Seneca* himself embraces this opinion (see the 22d chapter of the book before referred to): "I cannot believe," says he, "that a comet is a fire suddenly kindled, but that it ought to be ranked among the eternal works of nature. A comet has its proper place, and is not easily moved from thence; it goes its course, and is not extinguished, but runs off from us. But you will say, if it were a wandering star, it would keep in the zodiac; but who can set one boundary to all the stars? who can restrain the works of the divinity to a narrow compass? for each of those bodies, which you imagine to be the only ones that have motion, have very different circles; why, therefore, may there not be some that have peculiar ways of their own, wherein they recede far from the rest?"

583. Yet, for all this, the whole sect of *Peripatetics*, fearing that generations and corruptions should be introduced into the heavens by placing the comets in them, maintained that they were nothing more than a peculiar kind of meteors within our atmosphere; and some later philosophers have held nearly the same opinion. But that comets are far beyond the limits of our atmosphere, and, indeed, at a greater distance than the moon, is manifest, because, when seen from different places, they appear at the same distance from fixed stars which are near them, which could not be, unless their parallax was very small, and, consequently, their distance very great. We may easily, by the help merely of a thread, find if a comet have any sensible parallax; for a comet, just

before it disappears, goes so slowly, that for several days it appears to have scarcely any motion among the fixed stars: let it, then, be twice observed; first, when it is very high above the horizon, take any two stars between which the comet lies in a right line parallel to the horizon, which may be easily found by extending the thread directly between the stars; afterwards, when the comet approaches near the horizon, it must again be tried, whether it continue in the right line between the same two fixed stars. Now, if there be any sensible parallax which depresses the comet, it cannot be seen on the same right line in both situations; therefore, since this is found to be the case with all comets at their first appearance, or just before they disappear, it is a convincing proof that they have, in these positions, no sensible parallax, and must, therefore, be at a prodigious distance from us. Since refraction will equally affect the comet and the stars, it need not be regarded in these observations.

584. Comets, since they are at such distances from the earth, must, like all other stars, have their apparent motion round it from east to west, which arises only from the rotation of the earth upon its axis. But, besides this, they have a real and proper motion of their own, by which they are continually shifting their place in the heavens, and have their proper courses in the celestial regions: but the determination of their orbits is a very troublesome and difficult business, chiefly because none of them is visible through the whole of its revolution. A comet is very seldom seen in conjunction or opposition to the sun, and seldom at its node in the plane of the ecliptic; and as it can be observed but in a very small part of its orbit, we cannot apply those methods which enable us to ascertain the orbit of a planet. Our mode of procedure must, however, in many respects be similar to them; and all possible diligence must be used in making the observations. It will be

advisable to take the apparent diameter of the comet very frequently; as by these means a judgment may be formed of its relative distance at different times: its degree of motion, its brightness, &c. must also be regarded; for when it moves with greatest velocity, or appears most bright, it may be fairly concluded that it is near its perihelion. If the place of the comet can be observed when it has no latitude, the place and time of its being in one of its nodes will then be exactly known; but as this can seldom be actually observed, these elements are generally approximated to, from other observations. The proper course of a comet may be found by observing every night its distance from two fixed stars, whose longitudes and latitudes are known; or by finding its altitude when in the same azimuth with two known fixed stars; or, by noting four fixed stars, in the point of intersection of the two lines connecting which the comet is found; for by either of these means we can easily compute (Art. 190. 191. 192.) the place of the comet for each night, and thence its course. If the places of the comet, as thus observed every night, be marked on the celestial globe, a line drawn through them will represent the comet's path among the stars: a great circle drawn through three distant places, will nearly shew the way it has to go; if it be continued till it intersect the ecliptic, it will shew nearly the place of the node, and the inclination of the orbit to the ecliptic: the place of the node and inclination of the orbit being thus found from several triplets of places independent of each other, a medium of the results may be looked upon as tolerably accurate.

585. Now, from observations thus conducted, with proper calculations, it appears that comets describe curve lines about the sun; they must therefore, by the first law of motion (Art. 262.), be acted upon by some force which draws them from their rectilinear

course. Since this force, in all the planets, tends to the sun, as being by far the largest body in the system; we conclude, for the same reason, that in the comets it respects the sun also; and, as this force in the planets is inversely in the duplicate ratio of the distance from the sun (Art. 281.), the same law, we conclude, is observed by the comets; which, therefore, move in conic sections about the sun, having one of their foci in the centre of that luminary. Hence, if comets return in their orbits (as we know some of them do, and as we conclude all, or nearly all, do), those orbits must be *ellipses*, or nearly so; and their periodical times will be to the periodical times of the planets in the sesquiplicate ratio of their principal axes. (Art. 284.) Comets, therefore, being for the most part beyond the planetary regions, and, on that account, describing orbits with much larger major-axes than the planets, revolve more slowly. Thus, if the major-axis of a comet's orbit be four times as long as that of the orbit of *georgium sidus*, the time of the comet's period, would be to that of the planet as $4^{\frac{3}{2}} : 1$, or, as $8 : 1$; its periodic time would therefore be nearly 666 years.

586. The orbits in which comets move, are extremely excentric; therefore, since astronomical calculations of very excentric ellipses are complex and tedious, and as comets are visible during only a very short part of their revolution, during which the portions of their elliptic orbits are nearly of the same curvature as *parabolas*, it follows, that *the arc of a comet's visible orbit may, without any sensible errour, be taken for a portion of a parabola*. We therefore, for the ease of computation, imagine the orbits of comets, or at least those parts of them which chiefly come under our consideration, to be *parabolic*; and on this hypothesis, the motions of these bodies when near the vertices or perihelia of their orbits are ascertained. Let P*p* (fig. 9, Pl. VIII.) be an indefinitely small

arc described by the body, S the place of the sun, S N a line drawn from the focus S, perpendicular to a tangent to the parabola A P D at the point P; then, 1ſt, *The velocity* u *in any point* P *of the parabola, is as the square root of the parameter to the axis, divided by* S N: for, the velocity is as the arc P p or $u = p$ P: now, p M being perpendicular to P S in the similar right-angled triangles p P M, P S N, S N : S P : : p M : p P $= \frac{p\,M \times S\,P}{S\,N}$. But the parameter is as the square of the described sectors; therefore R (the parameter) $= p\,M^2 \times S\,P^2$, and $\surd R = p\,M \times S\,P$; and by substitution, p P or $u = \frac{\surd R}{S\,N}$, or $u = \frac{\surd 4\,A\,S}{S\,N}$, from the nature of the parabola. 2dly. *The velocity* u *in any point* P *of the parabola, is to the velocity* V *of a body running through the circumference of a circle with a central force tending to its centre, the radius being equal to* S P, *as* $\surd 2 : 1$. For, since $u = \frac{\surd 4\,A\,S}{S\,N}$, $u^2 = \frac{4\,A\,S}{S\,N^2}$; or, because $S\,N^2 = S\,P \times S\,A$ (*Hutton on Parabola*, Prop. VI. Cor. 3.), $u^2 = \frac{4\,A\,S}{S\,P \times A\,S} = \frac{4}{S\,P}$. But the circle whose radius is S P, being taken as an ellipsis, its parameter is $= 2\,S\,P$; and the velocity V being uniform, it is every-where (Art. 283.) as $\frac{\surd 2\,S\,P}{S\,P}$; therefore V V $= \frac{2\,S\,P}{S\,P^2} = \frac{2}{S\,P}$: consequently $u^2 : V^2 : : \frac{4}{S\,P} : \frac{2}{S\,P} : : 2 : 1$; and therefore $u : V : : \surd 2 : \surd 1 : : \surd 2 : 1$.

587. Since the comets appear, as far as we are acquainted with their motions, to follow the same laws as the planets, we must therefore with these, on a like account as we did with them, distribute the inequalities of their motions in the different parts of their orbits: and in order to do this the more readily, it will be neceſſary to premise a few properties of

parabolic areas, which are either well known, or manifestly deducible from such as are.

588. The parabolic area A P Q (fig. 8, Pl. VIII.) comprehended between the curve A P, its ordinate P Q, and absciss A Q, is equal to two-thirds of the rectangle contained by P Q and A Q.

589. If A P and the tangent P T be drawn, then the area of the triangle A P Q is to the parabolic area A P Q, as $\frac{1}{2}$ is to $\frac{2}{3}$, or as 3 to 4: and the parabolic segment included between the arc A P and the right line A P, is one-third of the triangle A P Q.

590. The area of the triangle P Q T, is to the parabolic area A P Q, as 1 to $\frac{2}{3}$, or as 3 to 2: for A Q = A T, whence P Q T is double the triangle A P Q.

591. If the ordinate S M pass through the focus S, the parabolic area A S M is equal to $\frac{1}{12}$ of the square of the parameter: for putting A S = *b*, the parameter will be denoted by 4 *b*, and the ordinate S M by 2 *b*; the parabolic area will therefore be $\frac{2}{3}$ of 2 *b b*, and the square of the parameter will be 16 *b b*; the former of which is evidently $\frac{1}{12}$ of the latter.

592. When there are several parabolas, their areas included between the part A S of the axis, from the vertex to the focus, and the ordinate S M passing through the focus, are to one another as the areas of circles, whose radius is A S: for the relation of those parabolic areas being expressed by $\frac{2}{3}$ of 2 *b b*, by substituting unity for the constant quantities, they become as *b b*, that is, as the squares of A S: but the areas of circles whose radii are A S, are to each other as the squares of A S; therefore the parabolic areas A S M are to one another as the areas of circles, having A S for radius.

593. Since the times are always as the areas, *the time a comet requires to pass from the perihelion to* 90°, *is always as the time it would take to describe a*

circle, whose radius is equal to the comet's perihelion distance. . . . Thus much being premised, we may lay down the chief rules respecting the motion of comets in the three following problems.

594. PROB. I. *To find an equation expressing the relation between the true anomalies, or a comet's true motions reckoned from its perihelion; the time it requires to pass from its perihelion to 90°; and also the time required to arrive at those true anomalies.*

Put t for the tangent of half any true anomaly A S P (fig. 8, Pl. VIII.); a for the time taken to go from the perihelion to 90°; b for the time in passing from the perihelion to the point P; and make the perihelion distance S A = 1. From P draw P D perpendicular to the tangent, meeting the axis in D: draw the ordinate P Q, and join P A. Then, 1st, the angle P D A = $\frac{1}{2}$ A S P: for, let S N be drawn perpendicular to P T; then the triangle P S T is isosceles, the angle P S N = $\frac{1}{2}$ A S P; and, because of the similar right-angled triangles P S N, T P D, the angle P D A = N S P = $\frac{1}{2}$ A S P. 2dly. $\frac{1}{2}$ P Q = t: for, taking D Q for radius, P Q is the tangent of the angle P D Q. Now (*Hutton. Parab.* Pr. VI. Cor. 2.), D Q = 2 A S; therefore, taking A S for radius, P Q is double the tangent of P D Q, or P Q = 2 t, consequently $\frac{1}{2}$ P Q = t. 3dly. A Q = $t\,t$: for in the right-angled triangle T P D, Q D : P Q : : P Q : Q T; or 2 : 2 t : : 2 t : 2 $t\,t$ = Q T = 2 A Q (by Conic Sections); therefore A Q = $t\,t$. 4thly. The area of the parabolic sector A S P, is $t + \frac{1}{3} t^3$: for, the area of the triangle Q A P = $\frac{1}{2}$ Q P × Q A = $t \times t^2 = t^3$, and the area of the segment A O P A = $\frac{1}{3} t^3$ (Art. 589.); also the area of the triangle A P S is $\frac{1}{2}$ A S × P Q = $\frac{1}{2} \times 2\,t = t$: therefore the area of the sector A S P is $\frac{1}{3} t^3 + t$. 5thly. The area of the sector A S M O A is $\frac{4}{3}$: for it is (Art. 528.) $\frac{2}{3}$ of S M × S A, or $\frac{2}{3}$ of

$\frac{4}{3}\times 1=\frac{4}{3}$. Lastly, since the time a is as the area A S P O A, and the time b as the area A S M O A, we have this proportion, $a:\frac{4}{3}::b:\frac{1}{3}t^3+t$; from this analogy, we deduce $3at+at^3=4b$, for the equation sought.

595. COROL. 1. The times a and b being given, the tangent of half the true anomaly P S A is manifestly found by resolving the cubic equation $3at+at^3=4b$, or $t^3+3t=\frac{b}{\frac{1}{4}a}$. There are various methods of doing this; but it may in many cases be easily effected, as follows: Let the side A B of the triangle A B C, right-angled at A, be $=1$, A C $=\frac{b}{\frac{1}{2}a}$ (fig. 7, Pl. VIII.); compute B C the hypothenuse; and find two mean proportionals between B C + A C, and B C — A C, the difference of these mean proportionals is the value of t. This process may be readily performed by means of logarithms.

596. COROL. 2. The equation $3at+at^3=4b$, being reducible to $\frac{1}{4}t^3+\frac{3}{4}t=\frac{b}{a}$, it follows, that if *the same true anomaly be found in different parabolas, the times required to arrive at those anomalies from the perihelion of each orbit, are to each other as the time of passing from the perihelion to* 90°; *and the contrary:* for if those anomalies are equal, $\frac{1}{4}t^3+\frac{3}{4}t$ is a constant quantity; therefore $\frac{b}{a}$ is in a constant proportion, of the times occupied in running through those anomalies, to the times of passing from the perihelion of each parabola to 90°.

597. COROL. 3. A true anomaly, and the time a comet takes to pass from its perihelion to 90°, being given; the time corresponding to that true anomaly is found by the equation $b=\frac{3}{4}at+\frac{1}{4}at^3$, or $4b=3at+at^3$.

598. COROL. 4. A true anomaly, and the corresponding time, being given; the time of a comet's

passage from its perihelion to 90°, is determined by the equation $a = \frac{4b}{3t + t^3}$. For instance, suppose a comet's perihelion distance is equal to the earth's mean distance from the sun, the time required by the comet to pass from its perihelion to 90°, may be thus computed:

The time of the earth's sidereal revolution is $365^d\ 6^h\ 9^m\ 12^s$ (Art. 56.); whence we get $2'\ 27''\ 50'''\frac{1}{2}$ for the horary motion at the mean distance. But at equal distances from the centre of force, the velocity in a circle, is to that in a parabola, as 1 to $\sqrt{2}$ (Art. 586.); therefore the horary velocity of the comet in its perihelion is $2'\ 27''\ 50'''\frac{1}{2} \times \sqrt{2} = 3'\ 29''\ 4'''\frac{3}{4} = 2t$. Now $4b = 4$ hours $= \frac{1}{6}$ of a day, the logarithm of which is 9·2218488; the logarithm of the tangent of t, or of $1'\ 44''\ 32'''\frac{3}{8}$, is 6·7048558, to which the logarithm of 3 being added, the sum is 7·1819771, the log. of $3t$; $6{\cdot}7048558 \times 3 = 0{\cdot}1145674$ the log. of t^3, the number expressing the value of this logarithm, is so exceedingly minute (having nine cyphers preceding the first significant figure), that it may be safely neglected. Therefore, lessening 9·2218488 by 7·1819771, the remainder 2·0398717 is the log. of a, the value of which is 109·615429, or $109^d\ 14^h\ 46^m\ 13^s$, for the time occupied by a comet, whose perihelion distance is equal to the earth's mean distance from the sun in going from the perihelion to 90°.

599. Prob. II. *To find an equation expressing the relation between a comet's perihelion distance* S A (fig. 8, Pl. VIII.), *any one of its true anomalies* A S P, *and the distance* S P *from the sun.*

Let p denote the perihelion distance, c the co-sine of half the true anomaly, r the tabular radius, and d the distance from the sun: then, because S N is perpendicular to T P, the angle N S P is half A S P; therefore S P N is the complement of half the true

anomaly. But, taking S P for radius, we have, r : sine S P N $= c$: : S P : S N ; consequently, by the nature of proportions, $r\,r : c\,c :: d : p$, and therefore $c\,c\,d = p\,r\,r$, is the equation sought.

CoroL. If in different parabolas, the same true anomaly be found, the corresponding distances from the sun are as the perihelion distances ; for the constant relation of $c\,c$ to $r\,r$, gives also that of d to p.

600. Prob. III. *To reduce all the perihelion distances of different comets to a common measure, viz. to the earth's mean distance from the sun, supposed unity.*

The time of a comet's passage from its perihelion to 90°, is (Art. 593.) as the time it would take to describe, about the sun as a centre, a circle whose radius r is equal to its perihelion distance p. But it has been shewn (Art. 285.) that the time of a revolution in a circle, is as $\sqrt{r^3}$: the times therefore of passing from the perihelia to 90°, are as $\sqrt{p^3}$. Hence, when the measure of a comet's perihelion distance is to be determined in parts, such as those of which the earth's mean distance from the sun is 1, say, as 109·615429, or $109^d\ 14^h\ 46^m\ 13^s$ (Art. 598.), to the time of a comet's passage from perihelion to 90° ; so is 1 (for $\sqrt{1^3} = 1$), to $\sqrt{p^3}$, or to the square root of the cube of the perihelion distance required.

601. Corol. From hence and the preceding demonstrations, it follows, that *the true anomalies in any parabola being computed, they may be used for all comets whose perihelion distances are known* : for when true anomalies are the same in different parabolas, the distances d from the centre of force, are as the perihelion distances p (Art. 599.), or $\sqrt{d^3}$ is as $\sqrt{p^3}$. Now (Art. 600.) the $\sqrt{p^3}$ are as the times required to pass from the perihelion to those anomalies ; therefore, when the true anomalies are the same in different parabolas, the $\sqrt{p^3}$ are as the times of passing from the perihelion to those true anomalies.

602. Having thus explained the nature of the computations for the motions of comets, we proceed to the calculus for the geocentric longitudes and latitudes of these bodies, which is, in almost every respect, similar to that which was pursued in determining these particulars in the planets; the necessary elements are:

1. The epoch and place of the comet's perihelion.
2. The distance of the perihelion from the sun, in terms of the radius of the earth's orbit.
3. The position of the ascending node.
4. The inclination of the comet's orbit to the ecliptic.

These being known, proceed in conformity to the following precepts:

603. I. Subtract two-thirds of the logarithm of the perihelion distance from the logarithm of the time (in days and decimals) between the comet's passing the perihelion and the given time, the remainder will be the logarithm of that interval, reduced to the distance, in days, from the perihelion of the orbit.

604. II. Find the true anomaly answering the time found, and, when the comet is direct, add that anomaly to the perihelion's place, if the given time follow that of its passage through the perihelion; or subtract it, if the given time precede the passage. But when the comet's motion in its orbit is retrograde, add the true anomaly to the place of the perihelion, if the given time precede that of the passage; or subtract it, if the time be after the passage of the perihelion; the result will be the true heliocentric place of the comet in its orbit.

605. III. The place of the ascending node, taken from the comet's true heliocentric place, will leave the argument of the comet's latitude (Art. 370).

606. IV. As radius, to the co-sine of the orbit's inclination, so is the tangent of the argument of latitude, to the tangent of the comet's distance from the

ascending node, measured in the ecliptic, *i. e.* the tangent of the reduced distance (Art. 372.).

607. V. Add the reduced distance just found to the true place of the node, the sum will be the comet's true heliocentric place reduced to the ecliptic, or (Art. 373.) the comet's true heliocentric longitude.

608. VI. As radius, to the sine of the inclination of the comet's orbit; so is the sine of the argument of latitude, to the sine of the heliocentric latitude (Art. 371.).

609. VII. Calculate the sun's distance from the earth, and his true place, when the earth's true heliocentric place is known, being six signs distant; then the difference between the comet's true heliocentric longitude, and the heliocentric longitude of the earth, will give the angle at the sun between the earth and the comet, that is (Art. 292.), the angle of commutation.

610. VIII. As the square of the co-sine of half the anomaly, to the square of radius; so is the comet's perihelion distance, to its true distance from the sun (Art. 599.).

611. IX. As radius, to the co-sine of the comet's heliocentric latitude; so is the comet's real distance from the sun, to its distance measured upon the plane of the ecliptic, that is, to its curtate distance (Art. 374.).

612. X. As the sum of the comet's curtate distance from the sun, and of the earth's real distance from the sun, to their difference; so is the tangent of the complement of half the angle of commutation (Art. 609.), to the tangent of an arc; which added to this complement, if the comet's curtate distance from the sun exceed the earth's real distance from him; or subtracted, if the comet's curtate distance from the sun be less than the earth's distance, the sum or difference will give the angle at the earth comprehended between the sun's place and the comet's geocentric

place, that is, the angle of the comet's elongation; whence may easily be deduced the comet's geocentric longitude (Art. 375.).

613. XI. Lastly (Art. 376.), as the sine of the angle of commutation (Art. 609.), to the sine of the angle of elongation, just found; so is the tangent of the comet's heliocentric latitude (Art. 608.), to the tangent of its geocentric latitude.

614. Prob. IV. *To determine that part of a comet's orbit through which it moves when subject to observations from the earth.*

This is a problem which is universally acknowledged to be of considerable difficulty; such, indeed, that as the data are not sufficiently accurate for a direct geometrical method, the solution is generally effected by repeated approximations. Several of the approximating methods which have been adopted are exceedingly tedious; and as it would not be consistent with the plan of this work to enter at length into an account of these, we shall merely explain the nature of one, namely, that given by *Boscovich*, which is in substance as follows: Having collected the greatest possible number of observations, choose three of them which were taken when the comet was not too near its perihelion (because near the perihelion the orbit does not differ sensibly from a circle), and make these the basis of the operations: let S (fig. 10, Pl. VIII.) be the sun, U W the orbit of the earth, supposed here to be a circle, E the place of the earth at the first observation, *e* at the third; draw E C, *e c*, to represent the observed directions of the comet, and let L, *l*, *w*, be the longitudes of the first, second, and third observations, *m* and *n* the geocentric latitudes of the comet at the first and third observations, and *t*, T, the intervals of time between the first and second, second and third observations. Assume C for the place of the comet at the first observation, reduced to the ecliptic; then, to determine the place

at the third observation, say $T \times \text{sine}\ \overline{w - l} : t \times \text{sine}\ \overline{l - L} :: EC : ec$, and c will be nearly the place required; join Cc, and it will represent the path of the comet on the ecliptic, according to this assumption. Draw CK, ck, perpendicular to the ecliptic, taking $CK : EC ::$ tang. m : radius, and $ck : ec ::$ tang. n : radius; join Kk, and it will represent the orbit of the comet, if the first assumption be true. Bisect Cc in x, and draw xy parallel to CK, and Kk will be bisected in y; join yS. Let $SE = 1$; then if v be the mean velocity of the earth in its orbit, the velocity of the comet at $y = \frac{\sqrt{2} \times v}{\sqrt{Sy}}$ (Art. 586.); taking, therefore, $v = Ee$, compute $\frac{\sqrt{2} \times v}{\sqrt{Sy}}$, and if this be equal to Kk, the assumed point C was the true point. But if these quantities be not equal, a new point must be assumed for C, in choosing which we must be directed by the nature and quantity of the errour arising from the first assumption; thus, if the computed value of Kk be greater than its value measured in the figure, and the lines CK, ck, diverge from each other as they recede from the sun, the point C must be taken farther from E, by how great a quantity we must conjecture from the magnitude of the errour, and from the consideration that the comet's velocity diminishes, as it recedes from the sun. Find CK, ck, as before, and compare the measured and computed value of Kk; and, if a fresh assumption be necessary, make it in conformity to the considerations above suggested. Having thus ascertained the position of the points C, c, very nearly, produce cC, kK, to meet at N; join NS, and it will be the line of the nodes; and if Cr, cz, be drawn perpendicular to NS, either of the angles KrC, kzc, will measure the inclination of the orbit. Also, from the two distances SC, Sc, and the included angle CSc, the parabola may be easily constructed; thus, having set

off S C, S c (fig. 1, Pl. IX.), in their proper relative position, with the centres C, c, and radii equal to S C, S c, describe the arcs a R o, e r i, and draw the line R r d to touch those arcs, this line will manifestly be the directrix of the parabola; which being known, together with the focus S, and the determining ratio (that of equality), the parabola may be constructed by Prop. 4. *Newton's Conics.* Or, letting fall S D perpendicularly upon R D, and bisecting it in A, the vertex and focus of the parabola A c C will be known; whence it may be drawn by well known methods. From either of these constructions, the ratio of the comet's perihelion distance S A (fig. 1, Pl. IX.) to the earth's mean distance S E (fig. 10, Pl. VIII.) will be known, and consequently (Art. 586.) the comet's velocity in perihelion: the velocity in either of the points C or c will be determined by the observations; and since (Art. 273.) the angular velocities are reciprocally as the squares of the distances from the centre of force, the distances S C, S c, are hence found in terms of S E: if these agree nearly with the construction, the assumptions have been properly made; if not, some farther corrections are necessary. The angles A S C, A S c, may either be measured or calculated from the known distances by means of Art. 599.; then, having the perihelion distance and the true anomaly, the time from the perihelion may be determined by Art. 593. 597.; whence, as the observations will shew whether the comet be approaching to, or receding from, the perihelion, an epoch of the perihelion will readily be ascertained. When a parabola is found to agree tolerably nearly with the positions given, it is useless to proceed farther in the approximation, for either the observations fixed upon are accurate, or nearly so; if they are accurate, we cannot expect that a parabola should agree perfectly with them, because the points they determine are actually (or at least probably) in

an ellipsis; if they are only nearly accurate, if a parabola could be found to agree with *them*, it might not agree with *all* the observations made upon the comet.

615. Prob. V. *To ascertain the periodical time of a comet, and the axes of its orbit.*

If comets, after having receded from the lower regions of the solar system to vast distances beyond the orbits of the most distant planets, return again to the neighbourhood of the sun, the paths they describe must be nearly elliptical; and then, if observations have been made sufficiently exact to be a basis of the operations, the requisites of the problem may be determined in the following manner: Let A K B I (fig. 2, Pl. IX.) be the trajectory of a comet, A B its major axis, I K the minor, S, F, the two foci, the former of which being the place of the sun, C the place of the comet, C S its distance from the sun, C c the space it passes over in a very small portion of time, D C E a tangent to the curve in the point C, S D, F E, perpendiculars demitted thereon from the foci: draw S G parallel to the tangent, and join F C. Also, let A L B be a circle, described on the major axis A B; A P T B a rectangle about the ellipsis A I B, and A Q R B a square about the circle A L B. Lastly, let A N O be the elliptic orbit of any planet, S, f, its foci; put S C $= a$, S D $= b$, C $c = e$, the time in which it is described f, the major axis of the cometary orbit A B $= x$, of the planetary orbit A O $= q$, the circumference of the circle A V O described on the same axis $= p$, the periodical time of the comet $= t$, and that of the planet $= n$.

616. The space C c described, the distance S C, and the angle S C D, being all determinable by observation, are given quantities. The mean distance of the comet is A H $=$ S K $= \frac{1}{2} x$, and of the planet is A g, $=$ S N $= \frac{1}{2} q$; and, because the squares of the periodical times are as the cubes of the mean

distances (Art. 209.), we have $\frac{1}{8}q^3 : \frac{1}{8}x^3 :: n^2 : t^2$; therefore $t^2 = \frac{\frac{1}{8}x^3 n^2}{\frac{1}{8}q^3}$, and $t = \frac{nx}{q}\sqrt{\frac{x}{q}}$.

617. But it is necessary to find another expression for the periodical time t, which may be done thus: Because C c is a very minute portion of the orbit, it may be esteemed a right line, and the sector C S c as a rectiline triangle, whose area $\frac{1}{2}$ S D $\times$ C $c = \frac{1}{2}be$ is given: then, as the area $\frac{1}{2}be$, is to the whole area of the ellipsis A K B I = A; so is the time f, to the whole periodical time t; wherefore $t = \frac{f}{\frac{1}{2}be} \times$ A.

618. Now, in order to determine the area A, we must find the semiconjugate H K; and here, because A B = S C + F C, we have F C $= x - a$; and by the similar triangles S D C, F E C, we have S C : S D :: F C : F E, that is, $a : b :: x - a : \frac{bx - ab}{a}$ = F E; consequently F G = F E — G E $= \frac{bx - 2ab}{a}$. Again, S C : C D :: F C : C E; or $a : \sqrt{a^2 - b^2} :: x - a : \frac{x-a}{a}\sqrt{a^2 - b^2}$: hence D E or S G = C E + C D $= \frac{x-a}{a}\sqrt{a^2 - b^2} + \sqrt{a^2 - b^2} = \frac{x}{a}\sqrt{a^2 - b^2}$. But F G $= \frac{bx - 2ab}{a}$; therefore F S $= \sqrt{FG^2 + SG^2} = \sqrt{\frac{b^2x^2 - 4ab^2x + 4a^2b^2 + a^2x^2 - b^2x^2}{a^2}}$ $= \sqrt{\frac{a^2x^2 - 4ab^2x + 4a^2b^2}{a^2}}$; and, of course, S H = $\frac{1}{2}$ F S $= \sqrt{\frac{a^2x^2 - 4ab^2x + 4a^2b^2}{4a^2}}$.

619. Moreover, since S K = A H $= \frac{1}{2}x$, H K = $\sqrt{SK^2 - SH^2} = \sqrt{\frac{1}{4}x^2 - \frac{a^2x^2 - 4ab^2x + 4a^2b}{4a^2}}$ $= \frac{b}{a}\sqrt{ax - a^2}$; therefore I K = 2 H K $= \frac{2b}{a}\sqrt{ax - a^2}$, and $\frac{xb}{a}\sqrt{ax - a^2}$ = area of the

rectangle APTB. Let P be the periphery of the circle ALB, whose diameter is x, then its area will be $\frac{1}{2}$ LH $\times$ P $= \frac{1}{4} x$ P, and we shall have $x^2 : \frac{1}{4} x P :: \frac{1}{2} x^2 : \frac{1}{8} x P ::$ AQRB : ALB :: APTB : AIB :: $q^2 : \frac{1}{4} q p$; that is $q^2 : \frac{1}{4} q p :: \frac{xb}{a} \sqrt{ax - a^2} : \frac{bpx}{4aq} \sqrt{ax - a^2} =$ AIB. But 2 AIB = AIKB $= A = \frac{bpx}{2aq} \sqrt{ax - a^2}$: therefore, substituting this value of A, in the expression at the end of Art. 617., we have $t = \frac{fpx}{aeq} \sqrt{ax - a^2}$. Equate this value of t, with that given in Art. 616., then $\frac{nx}{q} \sqrt{\frac{x}{q}} = \frac{fpx}{aeq} \sqrt{ax - a^2}$; which equation reduced, gives $x = \frac{af^2p^2q}{f^2p^2q - ae^2n^2} =$ AB, the major axis of the comet's elliptical trajectory.

620. If we substitute this value of x, in the above equation, for t, we shall have $t = \frac{p^3 f^3 n^2 a^{\frac{3}{2}}}{q\overline{f^2p^2 - ae^2n^2}^{\frac{3}{2}}} =$ the periodical time. Also, because the conjugate IK $= \frac{2b}{a} \sqrt{ax - a^2} = c$, we have $x = \frac{c^2a^2 + 4b^2a^2}{4b^2a} = \frac{af^2p^2q}{f^2p^2q - ae^2n^2}$, whence, by reduction, we find $c = 2ben \sqrt{\frac{a}{f^2p^2q - ae^2n^2}}$, the minor axis of the orbit.

621. From these equations, it obviously appears, that when the velocity of the comet is such that $f^2p^2q = ae^2n^2$, the axis x will be *infinite*, and consequently the trajectory will be a *parabola*; if ae^2n^2 be greater than f^2p^2q, the direction of the axis will be on the other side of the curve, which will be an *hyperbola*; in either of which cases, the comet can never return: but in every instance where f^2p^2q is greater than ae^2n^2, the comet will describe an ellipsis; among these we may comprise the

circle, where $x = 2a = \frac{af^2p^2q}{f^2p^2q - ae^2n}$ nd $f^2p^2q = 2ae^2n^2$, whence $e = Cc = \frac{fp}{n}\sqrt{\frac{q}{2a}}$, the arch of the circle described in one day, or one hour, according as the value of n is given in days, or hours.

622. Let the earth be the planet which we supposed to describe the ellipsis A N O; then its mean distance $\frac{1}{2}q = 100000$, or $q = 200000$, and $p = 628318$; also the periodical time $n =$ one year: then if C c be the portion of the comet's orbit described in one day, we have $f = \frac{1}{365\cdot2565} = 0\cdot0027378$. The other expressions will become as follows: for the principal axis, $x = \frac{591826599235 \times a}{591826599235 - ae^2}$, and for the periodical time $t = \frac{4750560000 \times a^{\frac{3}{2}}}{\overline{591826599235 - ae^2}|^{\frac{3}{2}}}$.

623. Hence, if the data deduced from observations are ascertained with precision, that is, if the distance of a comet, and the space it passes over in one day, be accurately known, the axis of its orbit, and the time of its revolution, may be easily found by the theorems given in the preceding articles: which theorems may also be applied with great facility to the determination of the orbits and periods of planets. But with respect to comets, it must be confessed that in very excentric orbits, even a minute errour in one of the observations may have considerable effect upon the result of the computation; therefore, as on account of the irregularity of a comet's appearance it is almoſt impossible to determine with the desirable accuracy the apparent position of either the limb or the centre, errours of consequence must inevitably follow. Hence the value of ae^2 will most frequently come out either equal to or greater than the number 591826599235, and so give the axis x infinite or negative: and even in those cases where

$a e^2$ is less than the said number, if a and e be not defined to the last degree of exactness, the major axis x, and periodical time t, will differ widely from the truth. Thus it happened that M. *Bouguer*, who first made use of this method, determined the orbit of the comet which was observed in 1729 to be an hyperbola. M. *Euler* also, in his first calculation of the comet's orbit which appeared in 1744, made it an hyperbola; but afterward, from more accurate observations, he found the orbit an ellipsis. Since, therefore, much dependence cannot be placed upon the results of these computations, it is thought by many, that the safest way to determine the period of comets, is to compare the elements of all those which have been computed, and (as it is extremely improbable that the orbits of two different comets should have the same inclination, perihelion distance, places of the perihelion, and node, &c.) when the elements of two agree pretty nearly, to conclude that they are in fact one and the same comet, whose period will be known by the interval of time between the two passages of the perihelion; and from this the axes of the orbit may readily be computed. But even this method has failed, particularly in the instance of the comet which was expected to return in 1789 or 1790: perhaps, then, the hypothesis advanced by Mr. *Cole* in his *Theory of Comets*, may in some cases be accurate. He supposes that the orbit of a comet is not an ellipse; but that, when it passes its perihelion, it has acquired so great a velocity, that its centripetal force is overcome by its centrifugal, and that consequently the comet continues to fly off in a parabola or hyperbola, till it come within the attraction of some fixed star; that this attraction may give it a new direction, and increase its velocity till it come to an apsis below that star, when it may again fly off either in a parabola or hyperbola, and proceed till it fall

within the attraction of another star; and thus visit many different systems *.

624. Prob. VI. *To find the length of a comet's tail.*

This has usually been done in the following manner: Let S (fig. 3, Pl. IX.) be the sun, E the earth, C the comet, CF the tail when directed from the sun; then, knowing the angles SEC and CSC from observation, their sum is the angle ECF; the angle CEF under which the tail appears is also known from observation; of course all the angles of the triangle ECF are known, and since the side CE may be found when the comet's orbit is determined, we readily find CF. If the direction of the tail deviate from CF, the angle of deviation FC*e* may be found from observation; this taken from FCE, and added to CFE, will give *e*CE and C*e*E

* M. Lambert has started the same idea in his *Cosmogony*, or *System of the World*, and pursued it with that enthusiastic ardour for which he was so celebrated; though he has, probably through the luxuriance of his imagination, pushed the opinion into consequences which the dictates of a well-regulated judgment cannot entirely warrant. In another part of his disquisition on comets he seems to be more successful: he takes for granted the principle that the universe ought to be as populous as possible, and must therefore contain as many moving bodies as it can, without confusion and disorder. "Hence," says he, "the most perfect plan of our system will be that into which enters the greatest number of orbits, all separated from one another, and which in no point intersect each other. If, then, we should be able to prove that the orbits of comets correspond to this end better than those of the planets, the reason of their superiority in point of number must be seen and admitted by minds of the most ordinary capacity." He therefore discusses the question, "which of these two species of orbits, the elliptic or circular, can be conveniently introduced in the greatest number into the solar system?" and furnishes such an answer as strongly confirms the *Newtonian Philosophy*, and at the same time shews that the excentric orbits of comets, instead of being irreconcileable with the wisdom which is otherwise manifest in the works of creation, are additional proofs that the highest possible perfection obtains in the universe at large.

respectively; whence C E *e* and C E being as before, C *e* soon becomes known. The tails of comets, according to the observations of *Sir Isaac Newton*, *Pingre*, and others, have sometimes appeared under angles of more than 90°; and have thence been computed to be the astonishing length of 40, 60, or 80 millions of miles!

625. SCHOLIUM. The tail of a comet, at its first appearance, is very short, and increases as the comet approaches towards the sun; immediately after its perihelion the tail is longest, and most luminous, and is then generally observed to be somewhat bent, and to be convex towards those parts to which the comet is moving; the convex side being rather brighter and better defined than the concave side. When the tail arrives at its greatest length, which in some comets has been stated as in the last article, it quickly decreases, and soon vanishes entirely, about the same time that the comet itself ceases to be seen. The matter of which the tail is formed is exceedingly rare, and so very pellucid that the light of the smallest stars suffers no diminution in passing through it, as is remarked by *Newton*, in the *Principia*, lib. iii. prop. 41.

626. To account for these phenomena, various conjectures have been advanced: *Newton* mentions three opinions which prevailed about his time.—"Some," says he, "maintain that they are nothing else than the beams of the sun's light transmitted through the comet's head, which they suppose to be transparent: others, that they arise from the refraction which light suffers in passing from the head of the comet to the earth: and others, that they are a sort of clouds or vapour continually rising from the head of the comet, and tending towards the parts opposite to the sun." The first and second of these opinions he refutes, but adopts the third, and adduces several arguments to prove

that the tail must consist of some kind of vapour arising constantly from the head of the comet. The ascent of vapours into the tail of the comet, he supposes occasioned by the rarefaction of the atmosphere when the comet is in perihelion. Smoke, it is observed, ascends the chimney by the impulse of the air in which it floats; and air rarefied by heat, ascends by the diminution of the specific gravity, carrying up the smoke along with it: in the same manner, then, it may be supposed that the tail of a comet is raised by the sun. The tails thus produced, when the comets are in perihelio, will go off along with their heads into remote regions, and be there lost, vanishing by little and little, till the comets are, as it were, left bare; till at the return, descending towards the sun, some short tails are again gradually produced, which afterwards in the perihelion descending down into the sun's atmosphere, will be immensely increased. *Newton* farther observes, that the vapours when thus dilated, rarefied, and diffused through the celestial regions, may probably be gradually attracted down to the planets and intermingled with their atmosphere, so as to be useful for the conservation of the water and moisture of the planets. He also conjectures that comets may be designed to recruit the sun with fresh fuel, and repair the consumption of his light by the streams which he continually sends forth in all directions. All this, it will be seen, is in consistency with the opinion then received, that the sun was a body of fire: but as this opinion is now doubted by most philosophers, and abandoned by many, the hypothesis of *Newton* respecting comets does not now meet with many advocates.

627. M. *Euler* (*Mem. Berlin*, tom. II. p. 117.) thinks there is a great affinity between the tails of the comets, the zodiacal light, and the aurora borea-

lis, and that the common cause of all of them is the action of the sun's light on the atmospheres of the comets, of the sun, and of the earth. He supposes that the impulse of the rays of light on the atmosphere of comets, may drive some of the finer particles of that atmosphere far beyond its limits; and that this impulsive force, combined with that of gravity towards the comet, would produce a tail which would always be in opposition to the sun, if the earth did not move. But the motion of the comet in its orbit, and about an axis, must vary the position and figure of the tail, giving it a curvature and deviation from a line joining the centres of the sun and comet; and that this deviation will be greater as the comet's orbit has the greater curvature, and as its motion is the more rapid. It may even happen, according to this hypothesis, that the velocity of the comet in the perihelion may be so great, that the force of the sun's rays may produce a new tail before the old one can follow; in which case the comet might have two or more tails, as was observed respecting the comet of 1744, while in its perihelion.

628. Dr. *Hugh Hamilton*, in the second of his *Philosophical Essays*, urges several objections against the Newtonian hypothesis: he remarks, that, since the tail of a comet, though exceedingly rare, meets with no resistance in its rapid motion round the sun (except so slight a one as can only cause a very small condensation on that side of it which moves foremost, and thereby may make it a little brighter than the other side), it cannot possibly move in a *medium* denser and heavier than itself, and therefore cannot be raised up from the sun by the superior gravity of such a medium. And since the stars seen through all parts of a comet's tail appear in their proper places, and with their usual colours, he infers that

the rays of light suffer no refraction in passing through the tail; therefore, since bodies reflect and refract light by one and the same power, he concludes that the matter of a comet's tail has not the power of refracting or reflecting light, and is, of consequence, a lucid or self shining substance. Also, from what astronomers say of the splendour of comets' tails, it is manifest they do not shine with such a dull light as would be reflected to us by the clouds or vapours at so great a distance, but with a brisker though a glimmering light, such as would arise from a very thin, volatile, burning matter. Dr. *Halley*, speaking of the great streams of light in the remarkable *aurora borealis* seen in 1716, says, "they so much "resembled the long tails of comets, that at first "sight they might be taken for such:" and afterwards, "this light seems to have a great affinity to "that which the effluvia of electric bodies emit in the "dark." Dr. *Hamilton* improves upon these hints; and since, as he shews, the tails of comets, the aurora borealis, and the electric fluid, agree remarkably, not only in their appearance, but also in such properties as we can observe of each of them, he concludes that they are substances of the same nature. And, because the electric matter, from its vast subtilty and velocity, seems capable of making great excursions from the planetary system, he imagines that the several comets, in their long excursions from the sun in all directions, may overtake this matter; and by attracting it to themselves may come back replete with it, and being again heated by the sun, may disperse it among the planets, and so keep up a circulation of this matter, which there is reason to think is necessary in our system.

629. After all that has been said and done by different persons to determine the nature of comets and their tails, our acquaintance with them is still very

limited. Their real magnitudes have been supposed to vary from about the size of the moon to three times that of the earth, beyond which limits very few have been observed till lately; but Dr. *Herschel* has observed several which he could not find had any solid nucleus: we shall conclude the present chapter with his reflections on this subject.—" Many of the operations of Nature are carried on in her great laboratory which we cannot comprehend; but now and then we see some of the tools with which she is at work. We need not wonder that their construction should be so singular as to induce us to confess our ignorance of the method of employing them, but we may rest assured that they are not mere *lusus naturæ*. I allude to the great number of small telescope comets that have been observed; and to the far greater number still, that are probably much too small for being noticed by our moſt diligent searchers after them. Those six, for instance, which my sister has discovered, I can, from examination, affirm, had not the least appearance of any solid nucleus, and seemed to be mere collections of vapours condensed about a centre. Five more that I have also observed, were nearly of the same nature. This throws a mystery over their destination, which seems to place them in the allegorical view of tools, probably designed for some salutary purposes to be wrought by them; and, whether the restoration of what is lost to the sun by the emission of light, may not be one of these purposes, I shall not presume to determine. The motion of the comet discovered by M. *Messier*, in June, 1770, plainly indicated how much its orbit was liable to be changed, by the perturbation of the planets; from which and the little agreement that can be found between the elements of the orbits of all the comets that have been observed, it appears clearly

that they may be directed to carry their salutary influence to any part of the heavens *."

* Many other interesting particulars respecting the nature of comets, the calculation of their orbits, &c. may be learnt by referring to *Newton's Principia*, lib. iii.; Dr. *Halley's Synopsis of Comets*, in *Phil. Trans.* No. 218, or at the end of Dr *Gregory's Astronomy*; M. *Sejour*, *Essai sur les Comètes*; M. *Pingré's Cométographie*; Sir *Henry Englefield*'s ingenious work *On the Determination of the Orbits of Comets*; Dr. *Hutton's Mathematical and Philosophical Dictionary*, art. *Comet*; and M *Bode's General Considerations on the Situation of the Orbits of all the Planets and Comets which have hitherto been calculated*, inserted in the *Memoirs of the Academy of Sciences at Berlin*, from Aug. 1786 to the end of 1787. The most extensive table for calculating the motions of comets was computed by M. *de Lambre*; it is inserted in Mr. *Vince's System of Astronomy*. A smaller table, on the same principles, may be met with in M. *de la Caille's Astronomy*.

CHAPTER XXII.

On the Aberration of Light.

ART. 630. THE term *aberration* is applied in astronomy to a certain apparent motion of the celestial bodies, occasioned by the progressive motion of light, and the earth's annual motion in her orbit. This apparent motion is so minute, that it could never have been discovered by observations, unless they had been made with extreme care and accuracy; and although it naturally arises from the combination of the two causes just mentioned, yet as it was never even suggested by theorists, until it was discovered by observation, it furnishes us with one of the strongest proofs of the truth of the Copernican system. The discovery is owing to the accuracy and ingenuity of the late Dr. *Bradley*, astronomer royal *; he was led to it accidentally by the result of some careful observations, which he had made with a view of determining the *annual parallax* of the fixed stars, or that which arises from the motion of the earth in its annual orbit about the sun.

631. The nature of this apparent motion may be explained by the motion of a line parallel to itself, in a manner bearing a near affinity to the explana-

* The history of this discovery is related by the doctor himself in No. 406 of the *Philos. Transact.*; or *Abridgment*, vol. IV. p. 150. It may also be seen in Dr. *Hutton's Dictionary*, and Mr. *Vince's Astronomy*.

tions of the composition and resolution of forces: thus, since light has a progressive motion, let the proportion of its velocity to that of the earth in her orbit, be as the line BC to the line AC (fig. 4, Pl. IX.); then, by the composition of these two motions, the particle of light will seem to describe the line BA or DC, instead of its real course BC; and will appear in the direction AB or CD, instead of its true direction CB; so that if AB represent a tube carried with a parallel motion by an observer along the line AC, in the time that a particle of light would move over the space BD, the different places of the tube being AB, *a b*, *c d*, *c* D; when the eye, or end of the tube is at A, let a particle of light enter the other end at B; then when the tube is at *a b*, that particle of light will be at *e*, exactly in the axis of the tube; when the tube is in the position *c d*, the particle will be at *f*, still in the axis of the tube; and when the tube arrives at CD, the particle of light will arrive at the point C the place of the eye, and will consequently appear as though it came in the direction DC of the tube, instead of the true direction BC: and so on, one particle of light succeeding another, and forming one continued ray or stream of light in the apparent direction DC. So that the apparent angle made by the ray of light with the line AE, is DCE, instead of the true angle BCE: the difference of these angles, viz. BCD, or ABC, is the quantity of the aberration.

632. Dr. *Bradley* himself considered the matter in the following manner: He imagined CA (fig. 5, Pl. IX.) to be a ray of light falling perpendicularly upon the line BD: then, if the eye were at rest at A, the object whence the ray proceeded must appear in the direction AC, whether light be propagated in time or in an instant. But if the eye be moving from B towards A, and light be propagated in time, with a velocity that is to the velocity of the eye, as

A C to A B; then, light moving from C to A, whilst the eye moves from B to A, that particle of it by which the object will be discerned, when the eye in its motion comes to A, is at C when the eye is at B. Joining the points B, C, he supposed the line B C to be a tube, inclined to the line B D in the angle D B C, and of such a diameter as to admit of but one particle of light: then it was easy to conceive, that the particle of light at C, by which the object must be seen when the eye arrives at A, would pass through the tube B C, so inclined to the line B D, accompanying the eye in its motion from B to A; and that it would not come to the eye, placed behind such a tube, if it had any other inclination to the line B D. If, instead of supposing B C so small a tube, we conceive it to be the axis of a larger; then, for the same reason, the particle of light at C cannot pass through that axis, unless it be inclined to B D in the same angle D B C. . . . In like manner, if the eye move the contrary way, from D towards A, with the same velocity; then the tube must be inclined in the angle B D C. Although, therefore, the true or real place of an object be perpendicular to the line in which the eye is moving, yet the visible place will not be so; since that must doubtless be in the direction of the tube. But the difference between the true and apparent place will be, *cæteris paribus*, greater or less, according to the different proportions between the velocity of light and that of the eye: so that, if we could suppose light to be propagated in an instant, then there would be no difference between the real and visible place of an object, although the eye were in motion; for in that case, A C being infinite with respect to A B, the angle A C B, which is the difference between the true and visible place, vanishes. But as light is propagated in time, it must be manifest, from the foregoing considerations, that there will always be a difference between the true and visible

place of an object, except when the eye is moving either directly towards or from it. And in all cases, *the sine of the difference between the true and visible place of the object, will be to the sine of the visible inclination of the object to the line in which the eye is moving; as the velocity of the eye, is to the velocity of light.*

633. If light moved only 1000 times faster than the eye, and an object, supposed to be at an infinite distance, were really placed perpendicularly over the plane in which the eye is moving, it follows, from what has been said, that the apparent place of such object would always be inclined to that plane in an angle of 89° 56½′; so that it would constantly appear 3½′ from its true place, and would seem so much *less* inclined to the plane *that* way towards which the eye tended: that is, if A C be to A B or A D as 1000 to 1, the angle A B C will be 89° 56½′, the angle A C B 3½′, and B C D, or 2 A C B, will be 7′, if the direction of the motion of the eye be contrary at one time to what it is at another.... If the earth revolve about the sun annually, and the velocity of light were to that of the earth as above stated, then it is easy to conceive that a star, really placed in the pole of the ecliptic, would, to an eye carried along with the earth, seem to change its place continually; and, neglecting the small difference on account of the earth's diurnal revolution on its axis, it would seem to describe a circle about that pole, every-where distant from it by 3½′. So that its longitude would be varied through all the points of the ecliptic every year, but its latitude would always remain the same. Its right ascension would also change, and its declination, according to the different situation of the sun in respect of the equinoctial points; and its apparent distance from the north pole of the equator, would be 7′ less at the autumnal than at the vernal equinox. The greatest alteration of the place of a star in the pole of the

ecliptic, or, which in effect amounts to the same, the proportion between the velocity of light and the earth's motion in its orbit, being known, it will not be difficult to find what would be the difference, on this account, between the true and apparent place of any other star, at any time; and, on the contrary, the difference between the true and apparent place being given, the proportion between the velocity of light, and the earth's motion in her orbit, may be found.

634. In nearly the above terms, the aberration is accounted for by *Dr. Bradley.* He also relates the result of a number of observations, from which he found, that every star appeared to describe a small ellipsis in the heavens, the transvere axis of which is nearly equal to 40″; and from this greatest variation in the place of the stars, he deduces the ratio of the velocity of light to that of the earth in her orbit, supposing both to be uniform, thus: in the figure last referred to, A C is to A B, as the velocity of light, to that of the earth in her orbit, and the angle A C B is 20″; so that the ratio of those velocities, is that of radius to the tangent of 20″, or (since the tangent has no sensible difference from so small an arc), as radius to 20″: but the radius of a circle is equal to an arc of $57\frac{3}{10}$° nearly, or equal to 206260″; therefore the velocity of light is to that of the earth, as 206260 to 20, or as 10313 to 1. Hence the time in which light will pass from the sun to the earth was easily deduced: for this time is to one year, as A B or 20″ to 360°, or the whole circle; that is, 360° : 20″ : : $365\frac{1}{4}^{d}$: 8^{m} 7^{s}; therefore it appears, from this discovery of *Dr. Bradley*'s, that light passes from the sun to the earth in 8 minutes 7 seconds: thus confirming, in a very satisfactory manner, the conclusion of M. *Roemer* (Art. 509.), deduced from observations of a totally different kind.

635. The effects of aberration in different circum-

stances may be shewn, and the nature of the calculations explained, as in the subsequent articles. Let T L Q I (fig. 6. Pl. IX.) be an indefinitely great circle, representing the ecliptic, with the sun at its centre S; P its pole; C B F D the earth's orbit; Q P E T a circle of latitude, passing through any star E, determining the longitude in T, and the latitude T E. Let T Q be the intersection of the plane of this circle of latitude with the plane of the ecliptic. Suppose the earth's place to be first in C, and consequently when the star is in conjunction with the sun; now having joined C E, and drawn the tangent C *c*, which is perpendicular to the plane of the circle of latitude T P Q, say, as the velocity of light, to that of the earth (or, as radius, to 20″, or to the tangent of 20″); so is C E, to C *c*. The length of C *c* being thus determined, and the parallelogram E C *c k* constructed, the point *k* is the place in the heavens where the impression of the light will occasion the star to appear, and the celestial arc E *k* is the *star's aberration.* From a like construction, made for every point of the earth's orbit, by applying a mode of reasoning nearly similar to what was adopted in the chapter on apparent motions (Chap. IX.), we may make the following deductions:

636. I. Supposing the earth's orbit circular, and her motion uniform, as well as that of light, then all the apparent places of the same star must be in a circle, whose centre is the star's true place, and whose plane is parallel to the ecliptic: consequently the projection of this circle in the heavens is an ellipsis whose *greater axis is parallel to the plane of the ecliptic* (Art. 235.), *its less axis perpendicular to that plane*, and the former to the latter in proportion *as radius to the sine of the star's latitude.*

637. II. The plane of a star's parallelogram of aberration, being determined by the star and by the

position of a tangent to every successive place of the earth in her orbit, changes its situation every instant, and completes a revolution in a year: and because of the almost infinite distance of the stars (Art. 683.) from the sun and earth, the point S may be taken as the earth's orbit, and the plane of the angle of aberration may be supposed to turn on the right line E S, drawn from the star to the sun, in the same manner as the earth moves about the sun.

638. III. When the plane of the parallelogram of aberration is become perpendicular to the plane T P Q of the circle of latitude, which happens *in the syzigy*, because the tangent C *c* is then perpendicular to that plane, the angle of aberration is not in the plane T P Q; nor is there then any aberration in latitude: for this angle being measured by the line E *k* parallel to the ecliptic, and perpendicular to the plane T P Q (and is half the greater axis of the elipsis), it is therefore the arc of a small circle parallel to the ecliptic, and passing through the star's true place: then the aberration is *all in longitude*, and is a *maximum*. But when the plane of the angle of aberration coincides with the plane T P Q (which happens when the earth has run through 90° from the syzygy, and consequently where *the star is in quadrature* with the sun), the angle of aberration then is *wholly in latitude*; and the star being at the extremity of the minor axis of its ellipse, the aberration in latitude is a *maximum*, but it is evidently nothing in longitude. In other positions of the plane of this angle, the aberration is divided, partly in longitude and partly in latitude, much in the same way as a force oblique to a plane. When the star is in conjunction, its apparent place, reduced to the ecliptic, will diminish the longitude; but when it is in opposition, its apparent place by aberration increases the longitude: hence the *longitude* is the *greatest* when the star is in *opposition*, and *least* when in *conjunction*. As to the

latitude, it is *least* in quadrature *before* opposition, and *greatest* in the quadrature *after*.

639. If a circle of declination R V X be conceived to pass through the star E, consequently crossing its ellipsis of aberration by passing through the centre; it is manifest that when a star appears at the points where that circle intersects the ellipsis, it will have *no aberration in right ascension*, since its true and apparent place will be in the same circle of declination: and when the star is in the points where its ellipsis of aberration is intersected by a diameter perpendicular to the circle R V X, it will have *no aberration in declination*, because its true and apparent place will be in the same parallel to the equator. But all circles of declination being oblique to the ecliptic, the star does not pass from the term of no aberration in right ascension, to that of no aberration in declination, in the time the earth describes 90° of her orbit; consequently the aberration is not absolutely nothing in declination when it is greatest in right ascension; and reciprocally.

640. The aberrations of a star for a given instant may be readily determined thus: In a circle *d k b f* (fig. 7, Pl. IX.) let an horizontal diameter *d b* represent that portion of a parallel to the ecliptic, in the midst of which at E is the star's true place; and let the vertical diameter *k f* represent a portion of its circle of latitude. Now, beginning at the point E, divide the radii E *d*, E *b*, into as many equal parts, or seconds, as are found by this analogy; *as co-sine of the star's latitude, to radius; so are* 20″, *to the number sought*: for the arcs E *d*, E *b*, of the small circle, will thereby be divided into the number of seconds they contain. Divide the radii E *k*, E *f*, into parts of 20″ each, because they are arcs of a great circle: also divide the circumference of the circle into signs and degrees, making the point *d* answer to the star's longitude: write the signs in their order,

going from right to left in the upper part of the circle, and making *d* mark the west, *b* the east, *k* the north, *f* the south. In E *k*, take E C to E *k*, as the sine of the star's latitude is to radius; and on the axes *b d*, C D, describe the ellipsis *b* C *d* D, which will evidently be the projection of the star's circle of aberration. Suppose, now, that this ellipsis is made for a star whose longitude is 12° in ♉, and latitude 36° north, the sun being at the given instant in 14° ♍. From the point *m*, answering to the sun's place on the circle, draw *m* N perpendicular to *b d*, and the point M of the ellipsis is (upon the known principles of projection) the star's apparent place: then M P, or N E its equal, perpendicular to C D, is the aberration in longitude, or the quantity of seconds the star appears more to the east than it really is; and M N = E P is the star's aberration in latitude. From this construction we may easily deduce the following rules: *the continual product of* 20″, *the sine of the star's latitude (to radius* 1*), and the sine of the earth's distance from the syzygy, is the aberration in latitude*; and, *the product of* 20″, *into the co-sine of the earth's distance from syzygy, divided by the co-sine of the star's latitude, is the aberration in longitude.*

641. To find the aberration in *right ascension* and *declination*, we must first determine the situation of the star's circle of declination with respect to its circle of latitude: to accomplish which, let *m* (fig. 4, Pl. II.) be the star's true place in the heavens, E Q the ecliptic, N and S its poles, H R the equator, Z and A its poles, H Z R A the solstitial colure; then, taking the instance mentioned in the last article, *p m* will be the star's latitude, and *m* N = 54° the complement of the latitude, the arc E *p* or angle Z N *m* is 48°, being the nearest distance of 12° in ♉ to the solstitial colure, and Z N = 23° 28′, the obliquity of the ecliptic to the equator, or the distance of the

poles of those two circles; hence, knowing the two sides m N, N Z, of the oblique spherical triangle m N Z, and their included angle, we find m Z $= 41°$, the complement of the star's declination, and Z m N $= 26° 50'$, the quantity which the north part m Z of the star's circle of declination is to the east of its circle of latitude p N. Therefore, draw the diameter F G (fig. 7, Pl. IX.), making to the east of E k the angle k E F $= 26° 50'$, this will represent the star's circle of declination; let its radii E F, E G, be divided into parts of $20''$ each; draw perpendicularly to F G the diameter I H, representing a portion of the parallel to the equator passing through the star; the radii E I, E H, of which must be divided into as many seconds as are found by this analogy, *as co-sine of the star's declination, to radius; so are* $20''$, *to the number sought*; this will, in the present case, be $30\frac{1}{2}''$. Then, drawing M L, M Q, respectively perpendicular to F G, I H, we have E Q = M L, for the measure of the *aberration in right ascension*; and E L = M Q, for that of the *aberration in declination.*

642. As it is frequently necessary in practical astronomy to *calculate* the aberrations in right ascension and declination, we add the following rules for this purpose*:

For the right ascension. Seek, in the astronomical tables, that point of the ecliptic which corresponds to the point of the equator marked by the right ascension of the star; this point (which call N) is always that wherein the sun being found, the aberration makes the apparent right ascension the least; take, in the same tables, the angle made by the ecliptic and a meridian passing through that point N; and

* The demonstration of these rules may be seen in Mr. *Simpson's Essays*; also in the *Memoirs of the Royal Academy of Sciences*, for 1737, and 1746, where the subject of aberration is treated upon in a very able manner by M. *Clairaut*. The subject is also very ingeniously discussed in *La Lande's Astronomy*, vol. III. p. 173—210.

say, *as the product of the radius into the co-sine of the star's declination, is to the product of the co-sine of the sun's distance from the point* N, *into the sine of the angle between the ecliptic and meridian of* N; *so are* 20″, *to the aberration in right ascension*, which is additive or subtractive, according to the position of the sun with respect to the point N.

643. *For the declination.* Take from the tables the declination of the point N, and say, *as the tangent of the sum of the declinations of the star and the point* N (if of different names), *or of their difference* (if of the same name), *is to the co-sine of the star's right ascension; so is the sine of the obliquity of the ecliptic, to the co-tangent of an arc, which is the distance between the point* N *in the ecliptic, where the aberration in right ascension makes this right ascension the least, and that point* (T) *of the ecliptic in which the sun is found when the aberration in declination causes the declination to be the least.*

The point T is obtained, by *subtracting* the arc found from the place of N, if the right ascension of the star is proceeding 0° to 90°, with south latitude and north declination; or from 180° to 270° with north latitude and south declination: but, if the declination and latitude be both north, and the right ascension is between 270° and 90°; or between 90° and 270°, having latitude and declination both south; then T is found by *adding* this arc to the place of N. On the contrary, the *supplement* of the said arc from the place N must be *subtracted*, if the right ascension is between 90° and 180° with south latitude and north declination; or between 270° and 360° with north latitude and south declination: or, the *supplement* of this arc from the place N, must be *added*, if the right ascension is between 270° and 90°, with latitude and declination both south; or between 90° and 270°, with latitude and declination both north.

Then say, *as the product of the radius into the*

sine of the distance N T, *is to the product of the co-sine of the sun's distance from the point* T, *into the sine of the sum, or difference, of the declinations of the star and point* N; *so are* 20″, *to the aberration in declination:* which is easily applied to the true declination of the star, according to its situation in regard to the sun.

644. It will be manifest, from the preceding account of aberration, that even with respect to the planets, it may produce effects which are not entirely to be disregarded*. A planet, considered as affected by aberration, appears in the place where it should have appeared at that instant which precedes the time of observation, by the interval of time occupied by light in passing from the planet to the earth. In the sun, the aberration in longitude is constantly 20″, that being the space moved by the sun, or rather by the earth, in the space of $8^m\ 7^s$, which is the time employed by light in passing from the sun to the earth. And, knowing pretty nearly the distance of a planet from the earth at any time, we shall have, *as the distance of the sun, to that of the planet; so are* $8^m\ 7^s$ *to the time of light passing from the sun to the earth:* then, computing the planet's *geocentric motion* in this time, in *longitude, latitude, right ascension,* or *declination,* it will be the *planet's aberration,* for whichever of these the geocentric motion was calculated; and it will be *subtractive* or *additive,* according as the planet's motion is *direct* or *retrograde.*

645. It is evident that the aberration will be

* A quantity of aberration is occasioned by the diurnal rotation of the earth, but whether we consider it with respect to the sun, planets, or fixed stars, it is too small to be perceptible; for, in the space of eight minutes, a point on the earth's surface moves through 32′ of a degree; and since small optic angles are nearly as the diameters they subtend, it is, as radius : sine 32′ : : 8·″75 (sun's parallax) : 4·″88, the *maximum* of aberration from this cause.

greatest in the longitude, and very small in latitude, because the planets deviate in a very small degree from the plane of the ecliptic, or path of the earth; on this account, the aberration in the latitudes of the planets is commonly neglected as nearly insensible; the greatest in mercury being only $4\frac{1}{3}''$, and it is considerably less than this in the other planets. As to the aberrations in declination and right ascension, they must depend on the position of the planet in the zodiac. The aberration in longitude, being determined by the geocentric motion, will be nothing at all when the planet is stationary; and greatest, in the *superiour* planets when they are in *opposition* to the sun, but in the *inferiour* planets when they are in their *superiour conjunction*. These *maxima* of aberration for the several planets, when their distance from the sun is least, are as follow: *georgium sidus*, 25″; *saturn*, 27″; *jupiter*, 29.″8; *mars*, 37.″8; *venus*, 43.″2; *mercury*, 59″: the *moon*, $\frac{2}{3}''$. Between these quantities and nothing the aberrations in longitude, of the respective planets, vary according to their situations. And as to the aberration of the sun, in longitude, although it varies not (as before observed), yet it causes a variation in the aberration in declination, which is greatest (about 8″) at the equinoxes, where the sun's motion is most inclined to the equator; and is least (or absolutely nothing) in the solstices, where the sun's motion in the ecliptic is for a short time parallel to the equator. From what is here shewn respecting the aberrations of the planets, it follows, that where great accuracy is required, they must be attended to in calculating the times of transits and other celestial appearances*.

* Dr. *Bradley*, by his continued observations on the stars, perceived each year the period of the aberrations confirmed, according to the rules he had lately discovered: but besides this, he found from year to year other differences, the consideration of which led him to another brilliant discovery, that of the *nutation of the earth's*

646. Scholium. The aberration of light has been considered by some philosophers, in a manner rather different from that of Dr. *Bradley*. About the year 1767, Mr. *Patrick Wilson*, assistant professor of astronomy at Glasgow, entertained an idea that the aberration of the fixed stars indicated the proportion between the orbital velocity of the earth, and the velocity of light *in the vitreous humour of the eye*. This opinion soon led him into various discussions, and in particular, made him suppose, that the aberration of the fixed stars, when determined by observations made with a telescope *filled with water*, would be different from the aberration ascertained by observations with a common telescope. About the year 1775, Mr. *Wilson* felt inclined to embrace an opinion (which has been started by various ingenious men at different times), that the centre of the solar system was in motion, and in 1777, communicated to Mr. *Professor*

axis. This nutation is a kind of libratory motion of the earth's axis, by which its inclination to the plane of the ecliptic is continually varying backwards and forwards, by a small number of seconds. The whole extent of this change in the inclination of the axis, or, which is a consequence of it, in the apparent declination of the stars, is about 19″, and the period of the change is little more than nine years; or, the space of time from its setting out from any point and returning to the same again, about eighteen years and seven months, being the same as the period of the moon's motions; on which, indeed, it chiefly depends: being the effect of the inequalities of the joint action of the sun and moon upon the spheroidal figure of the earth, by which its axis is made to revolve with a conical motion, so that the extremity of it describes a small ellipsis, having its diameters 19″·1 and 14″·2, each revolution being performed in the time above mentioned. This is a natural consequence of the Newtonian system of universal attraction, and had been hinted at by some, ever since the publication of the *Principia*; but for Dr. *Bradley* was reserved the honour of establishing the fact, and thus confirming (in a manner beyond controversy) the truth of the principles which *Newton* had laid down.

For more on the subject of nutation, see *Philos. Transactions*, 1748; Dr. *Maskelyne's Astronomical Observations*, 1776; *D'Alembert Recherches sur la Précession des Equinoxes*; and *La Lande's Astronomy*, vol. III. p. 210—226.

Robison, and others, a paper on this subject. The water-telescope had, by this time, become familiar to his thoughts; and it occurred to him, that it might be employed for deciding this important question, and even for determining the direction and velocity of this motion, by means of the difference between the observed aberration of the fixed stars, and the aberration which should result from the earth's orbital motion alone. But various objections and difficulties occurred in the prosecution of this attempt; and Mr. *Wilson* soon after thought of a different method, which as it may, perhaps, at some future period, lead to the discovery of some terrestrial or celestial motions with which we are at present unacquainted, is here described.

If the earth be carried towards a fixed star, with a great velocity, and the rays from the star be made to deviate a little by an achromatic prism, it will follow, that a constant angle of incidence will give different angles of total deviation, according to the velocity of the motion, and this difference will be both real and apparent. Therefore, 1. Let the telescope of a meridional quadrant be furnished with a prism, refracting a few degrees in altitude; then search, by meridional observations, for such stars as exhibit altitudes inconsistent with Dr. *Bradley*'s observations; the differences will indicate an aberration caused by a motion of the earth, different from its orbital motion round the sun: if, after the variations occasioned by nutation (Art. 645. *note*) are allowed for, there still remain some number of seconds, or parts, they will indicate some motion of the earth of which we have, as yet, no knowledge.

2. Furnish a telescope with a plain mirrour, inclined to its axis in an angle of 45°, and a series of achromatic prisms refracting 90°. Suppose the telescope to be directed to a point of the heavens 90° distant from a star which is viewed through it: sup-

pose also, the earth to be at rest, and the images of this star, formed by the refracted and by the reflected light, to coincide: then suppose the earth to be in motion towards this star; the images will separate, both on account of a change in the total deviation of the refracted light, and likewise on account of a transverse aberration, to which the refracted image is liable, by the motion of the telescope.

3. If a long achromatic telescope be directed to a fixed star, towards which the earth is moving, the focal distance will be lengthened; and the contrary. The augmentation may indeed be very small, but an observer, well acquainted with optics, will, on a little consideration, be able to apply such means of increasing it, as to render it sensible.

CHAPTER XXIII.

On the Latitude and Longitude.

ART. 647. BECAUSE the latitudes and longitudes of most of the principal places in this kingdom are determined with tolerable accuracy, they have been assumed as known in many of the discussions in the preceding part of this treatise: but as there are many places upon the earth, and indeed in England, respecting which these particulars are not ascertained, the determination of them is of considerable importance to geography and navigation as well as astronomy; various methods of finding each are, therefore, here explained.

648. Since the latitude of a place on the earth, is its distance from the equator, measured on an arc of the meridian passing through it (Art. 19.), the most simple method of determining the latitude is, to take the meridian altitude of the sun's centre, when it is in the equator (i. e. in the first point of Aries or Libra), or the meridian altitude of a star which has no declination; then the complement of the meridian altitude is equal to the latitude of the place. For, in fig. 7, Pl. I. if H R represent the horizon, E Q the equator, Z the zenith of the place, Z H A R the meridian; the latitude of the place will be E Z, and this is manifestly equal to the complement of H E, the altitude of the equator. In this method, and likewise in the following ones, the proper allowances

must be made for refraction, dip of the horizon (Art. 680.), and the sun's semidiameter, when the altitude of either its upper or lower limb is taken.

649. But, since the sun is in the equator only on two days in the year (and then, perhaps, not exactly at noon), and there are very few stars in the equator upon which observations can be made; we may more generally find the latitude by means of the correct zenith distance of the sun's centre, when on the meridian, its declination being known; or the correct zenith distance when on the meridian of any star whose declination is known: for, if the zenith distance and declination are *both north*, or *both south*, *add* them together; but if *one* be *north*, and the other *south*, *subtract* the less from the greater, and the sum or difference will be the latitude, of the same name with the greater. The truth of this rule may be shewn by referring to fig. 7, Pl. I. where H R, E Q, and Z, represent as in the last article: then, 1st, when the zenith distance and declination are both north, or both south, let a be the place of the sun, or star, on the meridian, and E a its declination; a Z + a E = E Z the latitude: 2dly, when the zenith distance and declination is one north, the other south; if the zenith distance be greater, let c be the place of the sun, or star, on the meridian, then c Z (its zenith distance) — c E (its declination) = E Z the latitude; but if the declination be greater than the zenith distance, as when the star is at y, then E y (the declination) — Z y (the zenith distance) = E Z the latitude.

The latitude may be found in a similar manner, from the observed meridional altitude of the moon's upper or lower limb; but farther corrections will be necessary for the moon's parallax, and her altitude, and great care must be used to obtain the declination accurately.

650. The latitude may be found very readily by

observations upon a circumpolar star, when on the meridian both above and below the pole; for half the sum of the two meridian altitudes will be equal to the latitude. Thus, let $y z$ (fig 7, Pl. I.) be the parallel of declination in which the star appears to move, y the star's place when on the meridian above the pole N, z its place below the pole; then $Ry + Rz = RN + Ny + RN - Nz \; (= Ny) = 2\,RN$, and consequently $\frac{Ry + Rz}{2} = RN = EZ$ the latitude.

651. It may sometimes happen that the latitude of a place may be required, when some particular circumstances prevent the adoption of either of the preceding methods: in such cases, other methods may be had recourse to, a few of which we shall explain by examples, as below:

Suppose that in a certain northern latitude the sun's azimuth, at six o'clock in the morning, was observed to be 13° 20′ *from the east northward, and that the sun was due east,* at 7^h 6^m *the same morning) what was the real latitude of the place of observation?*

In fig. 4, Pl. II. where $a\,b$ represents the parallel in which the sun appears to move, e will be his place at six o'clock, w his place when due east, and the angle $u\,O\,w$ the latitude which we wish to determine: for this purpose we may use the spherical triangle $u\,O\,w$, $e\,O\,n$, the first right-angled at u, the other at n. Put $s = \cdot2840153$ the sine of $u\,O = 16°\ 30'$, the distance from 6, measured on the equator at 15° to an hour; $t = \cdot2370044$ the tangent of $O\,n = 13°\ 20'$, the sun's azimuth at six o'clock; $x =$ tangent of $u\,w$ the sun's declination; then $\frac{1}{x}$ is the co-tangent of $w\,u$ or of $O\,e$: let $y =$ sine of $u\,O\,w$, then $\sqrt{1 - y^2}$, and $\frac{\sqrt{1-y^2}}{y}$ will be the co-sine and co-tangent of $u\,O\,w$. In the triangle $u\,O\,w$, it will be

as tang. uw : radius : : sine uO : co-tangent uOw, that is, $x : 1 :: s : \frac{\sqrt{1-y^2}}{y}$; hence $sy = x\sqrt{1-y^2}$. And in the triangle eOn, co-tang. eO : radius : : co-sine eOn : tang. On, that is $\frac{1}{x} : 1 :: \sqrt{1-y^2} : t$; whence $t = x\sqrt{1-y_2}$: therefore, sy and t, being equal to the same expression, are equal to each other, i. e. $sy = t$, or $y = \frac{t}{s} = \cdot 8344775 =$ sine of 56° 33′ 41″ the latitude required. And from this we may, if required, find the sun's declination, by saying, co-tang. latitude : radius : : sine 16° 30′ : tang. 23° 16′ the declination.

652. *One morning in spring, the sun's azimuth at six o'clock was observed to be* 9° 55′, *from the east northward, and his altitude when due east* 19° 56′. *Required the latitude of the place, and the sun's declination?*

Here we refer to the same figure as in the last article, and put $s = \cdot 3409265$ the sine of Ow, the sun's altitude when east; $t = \cdot 1748277$ the tangent of On, the sun's azimuth at six o'clock; and $x =$ the sine of the latitude. Then, in the triangle uOw, we have $1 : s :: x : sx =$ sine of uw the sun's declination; therefore $\frac{\sqrt{1-s^2x^2}}{sx} =$ co-tangent of uw, or of Oe. Also, in the triangle eOn, we have, tangent On : co-sine eOn : : radius : co-tang. Oe, that is, $t : \sqrt{1-x^2} :: 1 : \frac{\sqrt{1-s^2x^2}}{sx}$; therefore $\sqrt{1-x^2} = \frac{t\sqrt{1-s^2x^2}}{sx}$, which reduces to $s^2x^2 - s^2x^4 = t^2 - s^2t^2x^2$, or $x^4 - \overline{t^2+1} \times x^2 = -\frac{t^2}{s^2}$. Substitute $2m$ for t^2+1, then $x^4 - 2mx^2 = -\frac{t^2}{s^2}$; whence, by completing the square, &c. $x =$

$\sqrt{m+\sqrt{m^2-\frac{t^2}{s^2}}}=\cdot 7521853$, the sine of 48° 56′ 48″ the latitude required: and $s\,x$, or sine $u\,w$ = $s\sqrt{m+\sqrt{m^2-\frac{t^2}{s^2}}}=\cdot 2564399$, = sine of 14° 51′ 32″ the sun's declination north.

653. *At a certain place in north latitude, the sun was observed to rise exactly at four o'clock, and at six o'clock the same morning his altitude was found to be 17° 30′: what are the latitude of the place, and the sun's declination?*

Referring still to the same figure, we make use of the spherical triangles O $e\,n$, O $i\,z$, right angled at n and z respectively. Put $a = \cdot 3007058$ the sine of $n\,e$, the sun's altitude at six o'clock; $d = \cdot 5$ = sine of O z, his ascensional difference; and x = sine of e O n, the latitude; then $\frac{x}{\sqrt{1-x^2}}$ = tang. of the latitude. But, sine e O n : sine $n\,e$: : radius : sine O e, or sine $z\,i$, the sun's declination, that is, $x : a :: 1 : \frac{a}{x}$; therefore the tang. $z\,i = \frac{a}{x} \div \sqrt{1-\frac{a^2}{x^2}} = \frac{a}{\sqrt{x^2-a^2}}$. And in the triangle $z\,i$ O, it will be co-tangent i O z : sine O z : : radius : tang. $z\,i$, that is $\frac{x}{\sqrt{1-x^2}} : d :: 1 : \frac{a}{\sqrt{x^2-a^2}}$; therefore $\frac{a\,x}{\sqrt{1-x^2}} = d\sqrt{x^2-a^2}$, and, by reduction, $x^4 + \overline{\frac{a^2}{d^2} - a^2 - 1} \times x^2 = -a^2$. By substituting $-2\,m$ for $\frac{a^2}{d^2} - a^2 - 1$, we get $x^4 - 2\,m\,x^2 = -a^2$, which equation solved, gives $x = \sqrt{m \pm \sqrt{m^2-a^2}} = \cdot 75506556$, or $\cdot 3982152$; the former of these is the sine of 49° 1′ 51″, and the latter to 23° 28′, either of which would be the latitude according to the equation; but since the sun never

rises so early as four o'clock (or indeed so early as five) at either of the tropics, the first of these numbers, viz. 49° 1′ 51″, is the latitude; and $\frac{a}{x} = \cdot 3982152$ = sine 23° 28′ the declination.

654. *On a certain day in the forenoon, the sun's altitude was observed to be* 33° 41′ 40″, *and his azimuth from the north* 102° 40′ 52″; *some time after, on the same morning, his altitude was found* 48° 46′ 53″, *and azimuth* 134° 39′ 56″. *From hence the latitude is required, being north.*

From these *data*, an expression for the latitude (or for the declination, and time of the day, if required) may be best deduced by the following CONSTRUCTION. With radius, = sine of 90°, describe the primitive circle on the plane of the meridian H Z H N; (fig. 8, Pl. IX.); draw H H through the centre for the horizon, and Z N at right angles thereto for the east azimuth: with the versed sines of 102° 40′ 52″, and 134° 39′ 56″ set off H *t* and H T; and with the sines of 33° 41′ 40″, and 48° 46′ 53″, lay down *C k* and C K; then draw *k q* and K Q parallel to H H: make N *u* = *k q*, and N *e* = K Q; join N T and N *t*; and draw *w u* and *h e* parallel to H H; then make *k s* = *w u*, and K S = *h e*: through S *s* draw G S *s m n g*, and draw the equator E C E parallel thereto; then S C *p*, being drawn perpendicularly to E C E, will represent the axis of the earth: whence C *n* will be the sine of the sun's declination, P *b* that of the latitude.

Now to investigate a general theorem for the *latitude*, put C *t* = *z*, C T = *m*, co-sine of 33° 41′ 40″ = *k q* = N *u* = *q*, co-sine of 48° 46′ 53″ = K Q = N *e* = *c*, sine of 33° 41′ 40″ = C *k* = *p*, sine of 48° 46′ 53″ = C K = *r*. Then, by similar triangles, we have N C (=radius=1) : C *t* (*z*) : : N *u* (*q*) : *u w* = *k s* = *q z*; and N C (1) : C T (*m*) : : N *e* (*c*) : *e h* = K S = *m c*; therefore S *a* = *c m* − *q z*. Also, as *s a*

$(r-p):\mathrm{S}a\,(cm-qz)::cz\,(1):\mathrm{AZ}=\frac{cm-qz}{r-p}$ $=$ tang. $54^\circ\ 51'$, the required latitude.

For the sun's declination. Represent the sine of the latitude by x, its co-sine by y, and the sine of the declination by d; then, $\mathrm{P}b\,(x):\mathrm{C}b\,(y)::sk\,(qz):km=\frac{qzy}{x}$; and, as $\mathrm{C}m\,(p-\frac{qzy}{x}):\mathrm{C}n\,(d)::\mathrm{CP}\,(1):\mathrm{P}b\,(x)$. Consequently $px-qyz=d=$ sine of $20^\circ\ 24'$, the declination.

For the hour of the day. Put $v=$ co-sine of sun's declination, u and w for the sine and co-sine of the hour from noon, $n=$ sine of the sun's azimuth, the other letters remaining as before. Then $px-qyz=d$, and $dx+vyw=p$, and by substitution $x^2p-xqzy+vyw=p$; but $1-y^2=x^2$, therefore $vw=py+xqz$; and, $u:q::n:v$, hence $\frac{qn}{u}=v$; which value of v, substituted for it in the equation just before the analogy, gives $\frac{qnw}{u}=py+xqz$; consequently $\frac{w}{u}=\frac{py+xqz}{qn}=$ co-tangent of the hour from noon at the first observation, which gives 4 hours, or 8 A. M.

N. B. When the sun's azimuth is less than 90° from the north, tC must be taken on the contrary side of C, and will therefore be negative with respect to what it is in the present instance: hence $\frac{qz-cm}{r-p}=\mathrm{AZ}$, $px+qyz=d$, and $\frac{w}{u}=\frac{py-xqz}{qn}$, become general expressions by which the latitude, declination, and hour, may be determined, when the azimuth from the north is less than 90°.

655. Sometimes the latitude is found by three observations upon the sun on the same morning, or afternoon; and as this method is supposed to be of great use at sea, it has exercised the talents of many learned men, both in England and upon the continent:

the general problem may be stated thus: *Given three descending, or ascending, altitudes of the sun, taken on the same day at unequal known intervals of time; thence to find those times, the latitude of the place, and the sun's declination.*

Mr. *Robertson*, in his *Elements of Navigation*, gives a solution which he supposes to be both concise and intelligible to learners; as such it is here inserted: Let *m a n* (fig. 12, Pl. IX.) be the parallel of declination described on *m n*; and *a*, *b*, *c*, the places of the sun when observed; *a* A, *b* B, *c* C, the sines of the times from noon, to the radius E *m*; *m*, *n*, the places at noon and midnight: and let *f* F, *e* F, *d* F, *m* F, FK, represent the sines of the distances of those places from the horizon H R. Draw *c g* parallel to A C; then if the angle *g c* E, which is equal to the angle *m* E *c*, the time from noon when the greatest altitude was taken, could be found, the times from noon when the other two were taken would be known also. Now *c b* being one interval, and *b a* the other, *b c* and *b a*, the chords of these arcs, will be known, as also *c a*, which is the chord of their sum. Draw *b o* perpendicular to *a c*; then in the right-angled triangle *c b o*, the angle $bco = \frac{1}{2}$ the arc *b a*, and the side *c b* = twice the sine of $\frac{1}{2}$ the arc *b c*, consequently, radius : sine *b c o* (= sine of $\frac{1}{2}$ arc *b a*) : : *b c* (=twice sine of $\frac{1}{2}$ arc *b c*) : *b o*; and, radius: sine *c b o* (= cosine $\frac{1}{2}$ arc *b a*) : : *b c* (2 sine $\frac{1}{2}$ arc *b c*) : *c o*. Now, *d f*, *d e*, the difference of the sines of the altitudes, are known; and the lines *f d*, A C, *a c*, are cut proportionally in *e*, B, and *r*; consequently, *d f* : *d e* : : (C A : C B : :) *c a* : *c r*, and $co - cr = ro$: in the triangle *b o r*, *r o* : *b o* : : radius : co-tang. *rbo* = co-tang. *r c g*. But *a c* E (= comp. of $\frac{1}{2}$ whole interval *a c*) − *r c g* = *g c* E = *c* E *m* = hour angle from noon when the greatest altitude was observed; therefore the time is known, as well as C *m* the versed sine of that time from noon. . . . . Again, ra-

dius : co-sine $acg :: ac : cg =$ AC; and, AC : $mc :: fd : dm$; then $Fd + dm = Fm$ the sine of Hm. Also, AC : $mn :: fd : m$K, and Km — Fm = FK, the sine of Rn; wherefore, knowing Hm, and Rn, the sun's meridional altitude and midnight depression, we have $\frac{Hm \backsim Rn}{2} = Qm$ the sun's declination: or $\frac{Hm + Rn}{2} = HQ$, the altitude of the equator, or complement (Art. 26.) of the latitude.

When the altitudes are taken at equal intervals of time, the two first proportions become useless; for ab, bc, being equal, the point o falls in the middle of the chord ca, and bo, the versed sine, is known: then $df : de ::$ (CA : CB ::) $ca : cr$, and $co - cr = ro$; also $bo : ro ::$ radius : tang. $rbo =$ tang. acg; and acE $- acg = gc$E $= c$Em. Hence the times from noon, the latitude, &c. are found as before.

656. But we may obtain a theorem for the nearest time from noon, which will probably be more ready in practice than Mr. *Robertson*'s method, by the following algebraic process: Let $c =$ co-sine of ZC, $t =$ co-sine of ZB, $d =$ co-sine of ZA, $h =$ sine of CPB, $p =$ its co-sine, $m =$ sine of CPA, $n =$ its co-sine, $x =$ sine of mPC, $y =$ its co-sine, r and s = sine and co-sine of the latitude, a and $e =$ the sine and co-sine of the sun's distance from the pole. Then in the three triangles CPZ, BPZ, APZ, we have, by a well-known spheric theorem, (I.) $er + asy = c$, (II.) $er + aspy - ashx = t$, (III.) $er + asny - asmx = d$. From the first of these equations we have $er = c - asy$, which substituted in the other two, make (IV.) $c - asy + aspy - asxh = t$, (V.) $c - asy + asny - asmx = d$. By the fourth, $-asy + aspy - ashx = t - c$, which put $= g$; then $as = \frac{g}{-y + py - hx} = \frac{g}{-y \times \overline{1 - p} - hx}$: call

$1 - p$, v, then will $as = \frac{g}{-yv - hx}$: but by the fifth, $-asy + asny - asmx = d - c$, which call k; then $as = \frac{k}{-y + ny - mx} = \frac{k}{-y \times \overline{1 - n} - mx}$; if w be put for $1 - n$, then $as = \frac{k}{-yw - mx}$: hence we obtain $\frac{g}{-yv - hx} = \frac{k}{-yw - mx}$, or $\frac{g}{yv + hx} = \frac{k}{yw + mx}$, which, by reducing the fractions, gives $gyw + gmx = kyv + khx$; whence, by farther reduction, $\frac{x}{y} = \frac{kv - gw}{gm - kh} =$ tangent of the angle mP C. From which the times of the observations, and the other requisites, may be found by the method explained in Art. 655.

657. The following practical rule for *the time of the day when the greatest altitude was taken*, is deduced from the final equation in the foregoing investigation:

I. Take the difference between the sines of the *first* and *third* altitudes (reckoning from noon), and multiply it into the versed sine of the arch of time between the *first* and *second* observations; let this product be made less by the difference between the sines of the *first* and *second* altitudes multiplied into the versed sine of the arch of time, between the *first* and *third* observations, and call the remainder A.

II. Multiply the difference between the sines of the *first* and *second* altitudes into the sine of the arch of time between the *first* and *third* observations; let this product be lessened by the difference between the sines of the *first* and *third* altitudes, multiplied into the sine of the arch of time between the *first* and *second* observations: call this remainder B.

III. Divide A by B, the quotient will be the tangent of an arch, which, converted into time, will be

the interval between the first observation (or that where the sun's altitude is greatest) and noon *.

658. There are also various methods of finding the latitude by *known fixed stars*, besides those before given (Art. 648. 649. 650.), where it was necessary that the star observed should be on the meridian: but of these we shall only explain two, leaving others to be invented, according to circumstances, by the ingenious practical astronomer.

Given the altitude of one of two known fixed stars,

* In the *Nautical Almanac* for 1778 are given some astronomical problems by the late Mr. *Lyons*, among which are two for finding the latitude by observations on the sun, that may often be useful as approximations; and, as the almanac for that year is scarce, they are inserted below:—

I. "*Having the time the sun's diameter takes up to pass a horizontal line, to find the latitude of the place of observation.*

"From the proportional logarithm (Art. 673) of the observed time, the index being increased by 10, subtract the proportional log. of the time the sun's diameter takes up to pass the meridian, found by doubling the time of the semidiameter set down in the Nautical Almanac, the remainder is the log. sine of the angle of position.

"To the log. co-sine of this angle add the log. co-sine of the sun's declination, the sum, rejecting ten from the index, is the log. sine of the latitude; exactly, if the observation was made of the sun's rising through the horizon, otherwise, only nearly, to be corrected as follows:

"Add together the log. sine of the sun's altitude, the log. sine of the declination, and the log. secant of the latitude, the sum, rejecting twenty from the index, is the log. sine of the first correction; to be added, if the latitude and declination are both of the same name, otherwise subtracted.

"Add together the log. tangent of the latitude, and the log. versed sine of the altitude, the sum, rejecting ten from the index, is the log. sine of the second correction, to be always subtracted.

II. "*Having the time the sun's diameter takes up to pass a vertical line* (as the vertical wire of a telescope), *to find the latitude.*

"From the proportional log. of the observed time, the index being increased by ten, subtract the proportional log. of the time the sun's diameter takes up to pass the meridian, the remainder is the log. co-sine of the angle of position; whence the latitude may be found as before."

when they have the same azimuth, to find the latitude of the place.

Suppose, for instance, the stars to be *schedar* in Cassiopeia, and *almaach* in Andromeda, the altitude of the former being taken; then, in fig. 9, Pl. IX. where the primitive circle represents the equator, the pole of which is at P, A the point from whence the right ascension is reckoned, A *b*, A *a*, the respective right ascensions of almaach and schedar, the former star being at S, the latter at B; we know, in the triangle S P B, P S almaach's co-declination, P B schedar's co-declination, S P B their difference of right ascension; whence we find S B P, or P B Z its supplement, Z being the zenith, and Z B S the azimuth circle on which both the stars are found. Again, in the triangle P B Z, we have P B schedar's co-declination, B Z schedar's co-altitude, and P B Z the angle of position; whence we obtain P Z the co-latitude.

659. *Given the altitudes of two known stars, to determine the latitude of the place of observation.*

Let us suppose the stars to be *cor hydra* and *cor leonis*, the former having south declination: if this be constructed on the plane of the equator, of which P is the pole (fig. 10, Pl. IX.), A will represent cor hydra, B cor leonis, and Z the zenith, then we shall have three spherical triangles, A P B, Z A P, Z P B; in the first we know A P cor hydra's co-declination, B P cor leonis' co-declination, A P B their difference of right ascension; from which we obtain B A P and B A. In the triangle B A Z we have A Z *cor hydra's* co-altitude, B Z co-altitude of cor leonis, and B A just found; whence we find B A Z, then B A Z — B A P = P A Z. Lastly, in the triangle A P Z, the sides A P, A Z, and the included angle P A Z, are known; from which P Z, the co-latitude, is readily determined.

In nearly a similar manner may other solutions be traced out, the circumstances varying according to the

peculiar situations of the stars, with respect to the chief circles of the sphere, and to each other.

*** Some of the preceding methods of finding the latitude (Art. 651.—656.) will be subject to small degrees of inaccuracy, arising from the sun's change of declination in the interval between the observations; but if the observations are carefully made, and not too far asunder, it is very seldom that the errours occasioned by the change of declination will be of material consequence *.

* The methods above given determine the latitude, on the supposition that the earth is a sphere; or they ascertain the *apparent latitude*, that is, the angle formed between a plumb line at the given place, and a diameter of the equator: the latitude, or angle at the *centre* of the *spheroid*, subtended between the given place and the equator, may be correctly found by this rule: *divide the square of the polar axis of the spheroid by the square of the equatoreal axis, and multiply the tangent of the apparent latitude by the quotient, the product is the tangent of the latitude at the centre*; or say, *as the square of the equatoreal axis : the square of the polar axis : : the tangent of the apparent altitude : the tangent of that at the centre.* For, let P G E *p* (fig. 10, Pl. VI.) be a section of half the earth on the meridian of the given place G, whose apparent latitude is G B E. Put C A $= x$, A G (perpendicular thereto) $= y$, B G $= n$, C P (half the polar axis) $= b$, C E (half the equatoreal) $= a$, sine of apparent latitude $= s$; then, by the nature of the ellipsis $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$, and $n = \frac{b}{a}\sqrt{b^2 + \left(\frac{a^2 - b^2}{b^2}\right)y^2}$, or $n^2 = \frac{b^4}{a^2} - \frac{b^2}{a^2}y^2 + y^2$; but $n \times s = y$, therefore $n^2 = \frac{b^4}{a^2 + b^2 s^2 - a^2 s^2}$; whence n becomes known, and we readily find $x = \sqrt{\frac{a^4 - a^4 s^2}{a^2 + b^2 s^2 - a^2 s^2}}$, and $y = \sqrt{\frac{b^4 s^2}{a^2 + b^2 s^2 - a^2 s^2}}$. Now, if A C be supposed radius, A G will be the tangent of the angle A C G $= \frac{y}{x} = \sqrt{\frac{b^4 s^2}{a^4 - a^4 s^2}} = \frac{b^2 \times \text{sin. app. lat.}}{a^2 \times \text{co-s. app. lat.}} = \frac{b^2}{a^2} \times$ tang. app. lat. $=$ tang. latitude at the earth's centre: whence the above rule is manifest.

The authour was favoured with the investigation from which this rule is deduced by Mr. *Thomas Simpson Evans*, second mathe-

660. *To determine the longitude of places on the earth*, is a problem of such great difficulty in practice, that, although the efforts of some of the greatest mathematicians in Europe have been directed towards the invention of easy, practicable methods, yet none has been hitherto discovered that is not liable to errours; these errours though, it must be remarked, are not to be ascribed entirely to the theory (which is, at least, with respect to several of the methods, accurate), but to the practice, particularly when the observations for determining the longitude are taken at sea. Still, however, when a comparison is formed between the modern methods of solving this problem, and those which were practised two centuries ago, it will appear that very considerable advances have been made towards a perfect solution; and these are probably, in great measure, to be attributed to the very handsome rewards which have been offered by commercial nations to those who should propose the most accurate and practicable way of attaining so desirable an end.

661. Before we enter upon an account of different methods of finding the longitude, a few observations may be premised, which will serve to elucidate the nature of the problem. The earth revolving on its axis from west to east once in the course of a natural day, or twenty-four hours, it is manifest that the meridian of every place on the earth will have its plane directed to the sun once in that time; and because,

matical master at *Christ's Hospital.* If this investigation be compared with that given by *Bezout* at the end of his *Navigation*, it will be found much more simple and concise.

N. B. Since b is to a as 214 to 215 (Art. 10.), we have $\frac{b^2}{a^2} =$ ·9907193 for a common multiplier, by which, if the nat. tang. of the app. latitude be multiplied, the product will be the nat. tang. of the latitude at the centre; or ·9907, the first four figures of this multiplier, will generally give a result sufficiently accurate.

whenever the meridian of any place has its plane directed to the sun, or (as it is commonly expressed) comes opposite to the sun, it is noon at that place, it follows that when the first meridian, namely (Art. 20.), that of Greenwich, comes opposite to the sun, there will be noon at Greenwich, and a new day will commence, according to the astronomical mode of reckoning. Then, as the earth's motion on its axis is uniform, equal parts of the equator will pass by the sun in equal spaces of time; and, consequently, one-twenty-fourth part of it, or 15°, will pass by the sun in one hour. Hence, at the end of one hour after the time when it was noon at Greenwich, that meridian which is 15° *west* of the meridian of Greenwich will come opposite the sun, and cause it to be noon to all the places which are on that meridian, and a new day will commence at those places exactly one hour *after* it commenced at Greenwich. In like manner, *two* hours *after* the time when it was noon at Greenwich, the meridian which is 30° west of the meridian of Greenwich will come opposite to the sun, and make noon to all the places which are upon it; and a new day will commence at those places when it is two o'clock in the afternoon at Greenwich. The same reasoning will be equally applicable for places which are still farther west; it is therefore evident, that the difference of longitude between any two places is in the same proportion to the difference between the times at those two places, as 15° to an hour; consequently, if the difference between the times at two places be converted into degrees, minutes, &c. at the rate of 15° to an hour, it will be the difference of longitude between those places. It is also manifest, that if the time at the meridian of Greenwich be *greater than the time at any other place*, that place lies to the *west* of Greenwich; but if the time at any place *be greater than the time at Greenwich*, the meridian of the place is to the *east* of

that of Greenwich; because the time of noon at that place must have preceded the time of noon at Greenwich. Hence it appears, that to find the longitude of any place from another given one, as Greenwich, is the same thing in fact as to find the time of day at each place, and convert the difference of those times into degrees and minutes, in the proportion above stated.

662. Now, the *time* at the place for which the longitude is required may be found by means of observations upon either the sun or stars, according to some of the methods given in Chap. VI., or some which have been pointed out in the preceding part of this chapter: and the time at the first meridian may be ascertained either by *mechanical contrivances*, or by *astronomical observations*. With respect to the first of these, it is obvious, that if a clock or watch could be so constructed, as to go uniformly in all seasons, and in all places, such a machine being once set to the time at Greenwich, would always shew the real time at Greenwich, in whatever part of the earth it might be; and therefore, when the time under any other meridian was found, its longitude from Greenwich would be readily obtained. Various attempts have been made to construct watches or timekeepers, which, by the regularity and accuracy of their motions, might be rendered subservient to the determination of the longitude: the most successful of these attempts have been made by Mr. *John Harrison* and Mr *Arnold*, both of whom have produced time-keepers, the motions of which have erred but little from the truth when tried in long voyages at sea; and very ingenious methods have been contrived for finding the daily deviation of the time shewn by these instruments from the mean time at the first meridian, and thus rendering them of great utility, even when the rate of their going needs correction. But, although these mechanical contrivances

could be brought to the highest degree of perfection, yet, as they are liable to be injured by various accidents, it would frequently be necessary to have recourse to celestial observations for the determination of the longitude; several astronomical methods, therefore, still continue to be made use of; the descriptions of some of which are here given *.

663. *First method: by the sun's declination.* By some of the methods already explained, either in this chapter or the sixth, compute the declination of the sun at noon, from observations made upon him either at the meridian, or three or four hours from it: take the difference between this computed declination, and that for the noon of the same day at Greenwich, as shewn by the Ephemeris; from which take likewise the daily difference of declination at that time, and useth is analogy, *as the daily difference of declination, is to the difference above found; so are* 360°, *to the difference of longitude.* But here a small errour in the computed declination, will cause a very considerable one in the difference of longitude; and

* These methods, it will be seen, chiefly depend upon the use of an ephemeris or almanac, adapted to the first meridian; which ephemeris shall contain, the sun's longitude, right ascension, declination; the planets' longitudes, latitudes, times of passing the meridian; the times of solar and lunar eclipses, together with those of jupiter's satellites; the distances of the moon from the sun, and certain fixed stars; and, in general, the times when any remarkable celestial appearances may be seen at the place for which the ephemeris is calculated. Such ephemeris was proposed by Dr. *Maskelyne* to be calculated for the meridian of Greenwich, and, the scheme being adopted by the commissioners of longitude, the first *Nautical Almanac* was published in 1767, and they have been regularly published ever since, and are continued as far as the year 1804. Dr. *Maskelyne* has also published *Requisite Tables to be used with the Nautical Ephemeris*; which, together with the Ephemeris, are indispensably necessary in most of the methods of finding the longitude now used; and will, even if the Doctor had done nothing else for the promotion of Astronomy and Navigation, ever reflect upon him the highest honour, and most lasting celebrity.

as such an errour must inevitably arise from considering the change of declination, during a day, as regular, this method is not to be recommended in practice.

664. *Second method: by the moon's culminating.* Seek in the Ephemeris for the time of the moon's coming to the meridian on the given day, and on the day following, and take the difference; take also the difference between their times of culminating on the same day, as found in the Ephemeris, and as observed; then say, *as the daily difference in the Ephemeris, is to the difference between the times of southing found by the Ephemeris and by observation; so are* 360°, *to the difference of longitude.* Here the moon's motion is supposed uniform, which is not the case; therefore, since a small errour in this respect, or in the time of the moon's culminating, may occasion a great errour in the longitude, this method is no more to be recommended in practice than the former.

665. *Third method: by the distance between the moon and a known fixed star, when both are on the meridian.* Let one observer take the altitude of the moon's centre when on the meridian (which may be done by taking the altitude of the upper or lower limb, and allowing for the semi-diameter), and another the altitude of any such star in the zodiac as then happens to be on or very near the meridian. Now the right ascension of the moon will be equal to the right ascension of the star, if they were on the meridian exactly together; but if they are not on the meridian together, if their difference of culminating do not exceed 8 or 10 minutes, the right ascension of the moon may be found by adding or subtracting this difference to or from the right ascension of the star, according as the moon culminates after or before the star. Then, knowing the day of the month, with the moon's right ascension, examine the *Nautical Al-*

manac, and find, by proportion, at what time at Greenwich the moon had that right ascension. Take also the difference between the meridional altitudes of the moon and the star which is then on the meridian; this difference (when properly corrected for dip, refraction, and parallax) will be the difference of the declinations of the moon and star, whence, the declination of the star being known, that of the moon becomes known also. Then find, by means of the *Nautical Almanac*, at what time on the given day, the moon had this declination: if the time thus found agree either exactly or very nearly with the time deduced from the right ascension, take the difference between half the sum of them, and the time of the observation (determined by some of the methods before referred to), for the difference of longitude in time, whence the longitude in degrees, &c. may be found: but if the times thus deduced do not correspond tolerably accurately, recourse must be had to some of the following methods.

666. *Fourth method: by eclipses of the moon.* These eclipses are seen at the same instant of absolute time in all parts of the earth (Art. 544.): therefore, if in two or more distant places where an eclipse of the moon is visible, the times of the beginning or ending are carefully observed; as also the times when any number of digits were eclipsed; or, which is better, the times when the earth's shadow began to touch, or to leave any remarkable spot on the moon's face; then will the difference of the times when the observations of like kind were made, give the difference of longitude between the places of observation. Or, instead of comparing the observations at two different meridians, the times of the beginning and end of the eclipse, as observed at the place the longitude of which is required, may be compared with the times of beginning and end at Greenwich, as given in the *Nautical Almanac*, and the longitude

may be deduced from the difference of times as before: but the longitude thus found will not be so accurate as that determined by two observations, because of the inaccuracy of the lunar tables, and of the great difficulty of telling exactly the time of the first and last contact of the earth's shadow with the moon's limb.

667. *Fifth method: by the eclipses of jupiter's satellites.* The eclipses of jupiter's satellites afford one of the readiest, and for general practice one of the best, methods of determining the longitudes of places at land: and whenever jupiter is to be seen *,

* An eclipse will be visible in any place, if jupiter be 8° or more above the horizon, and the sun as much below it: whether this will, or will not, be the case at any place where the observation is intended to be made, may be readily found with sufficient accuracy by a celestial globe. Or if both a celestial and terrestrial globe be at hand, the space upon the earth where an eclipse of one of the satellites will be visible, may be found thus: First, knowing the place of the sun, with the longitude and latitude of jupiter, as given in the Ephemeris; find thence their right ascensions and declinations by the celestial globe: then convert the time of the eclipse from noon, into degrees and minutes, they shew the longitude of that meridian on the earth where it is noon when the satellite is eclipsed; which we therefore call the *meridional longitude* of the eclipse, and is either *west* or *east* according as the time of the eclipse is *after* or *before* noon at Greenwich. Bring this *meridional longitude* under the meridian of the terrestrial globe, and make the elevation of that pole which is nearest the sun equal to his declination: keep the globe in this position, and if jupiter be *eastward* of the sun draw a line along that part of the globe which coincides with the *eastern* horizon, it passes over all those places where the sun is *setting* at that time; but if jupiter be *westward* of the sun, draw the said line on the globe by the *western* edge of the horizon, it passes over all those places where the sun is then *rising*. Jupiter being *eastward* of the sun, *add* the difference of his and the sun's right ascensions to the *meridional longitude*, bring the degree of the equator answering to their *sum* under the meridian, raise the pole nearest jupiter equal to his declination, and detaining the globe in this position, draw a line again coinciding with the *eastern* horizon; the space intercepted betwixt this and the line of the *sun's setting* before described on the globe, comprehends all those places on the earth where this eclipse is seen in the interval between the sun's and

they might be applied at sea, oftener than they would be wanted, if they could be observed with sufficient accuracy in a ship under sail. Mr. *Irwin* invented a marine chair for the purpose of adding to the accuracy of telescopic observations at sea; and Dr. *Maskelyne*, in a voyage to Barbadoes in 1763, made a full trial of this chair; but he informs us he "could not derive any advantage from the use of it; and, considering the great power requisite in a telescope for making these observations well, and the violence, as well as irregularity, of the motions of a ship, he is afraid the complete management of a telescope on shipboard will always remain among the *desiderata*." In order to find the longitude of a place at land by an eclipse of one of Jupiter's satellites, the following directions must be observed:—On the day preceding the evening on which it is proposed to observe the eclipse, note the time it will happen at Greenwich, as given in the *Nautical Almanac*. Let the estimated longitude of the place of observation be converted into time, and, if *eastward* of Greenwich, *added* to the time of the beginning of the eclipse, as given in the *Nautical Almanac;* but if the place be *westward, subtracted;* and it will give the time nearly when the eclipse is to be expected at that place. Begin to observe 20 or 30 minutes sooner than the time thus estimated, and let the in-

jupiter's setting. But if jupiter were *westward* of the sun, *subtract* the difference of his and the sun's right ascensions from the *meridional longitude*, set the degree of the equator corresponding to the *remainder* under the meridian, and elevate the pole nearest jupiter till it is equal to his declination: keeping the globe in this position, draw a line upon it by the *western* edge of the horizon; the space included betwixt this and the line of the sun's rising, contains all those places on the earth where this eclipse is visible between jupiter's and the sun's rising. It is manifest that the eclipse will be seen the best (other circumstances being supposed the same) at those parts of these spaces which are farthest from the bounding lines.

stant when the eclipse begins or ends be noted, as shewn by a watch previously regulated to the time at the place of observation: then the difference between this time and the Greenwich time, converted into degrees, will shew the longitude from Greenwich. If the time at Greenwich be *less* than the time observed, the longitude is *east*; otherwise, it is *west*. . . . It is to be observed, that a correspondent observation of an eclipse of one of jupiter's satellites, made under a well-known meridian, is preferable to the calculations of the Ephemeris for comparing with an observation made in a meridian whose longitude is required: but if no corresponding observation can be obtained, as is frequently the case, it will be best to find what correction the calculations of the Ephemeris require by the nearest observations to the given time that can be obtained; which correction applied to the calculation of the given eclipse in the Ephemeris, renders it nearly equivalent to an actual observation. . . The best eclipses for finding the longitude are those of the first satellite, because its theory is most accurately settled; but it should be observed, that its *emersions* are not visible from the time of jupiter's conjunction with the sun to the time of his opposition; and the *immersions* are not visible from his opposition to his conjunction. The satellites may be distinguished one from another, by means of the configurations given in the 12th page of each month of the *Nautical Almanac*, which exhibit the apparent positions of the satellites with respect to each other and to the primary, at such an hour as they are most likely to be observed.

668. *Sixth method: by the differences between the observed transits of the moon, and a fixed star over the meridian at each place.* Let P (fig. 11, Pl. IX.) be the pole of the earth RQ, P G the meridian of Greenwich passing through the moon at M, P D the meridian of any other place; which, when it is brought

by the earth's rotation into the situation P *d*, passes through the moon at *m*. Determine by the times of transitting the meridian, the differences M P S, *m* P S, between the right ascensions of the moon and a fixed star S, the difference of these is the angle M P *m*, which is the increase of the moon's right ascension in the interval of the transits. Find, from the *Nautical Almanac*, the increase (A) of the moon's right ascension in twelve hours apparent time, thus: Let *a* represent the variation of the equation of time in twelve hours on the day of observation, then twelve hours apparent time is $12^h \pm a$ of mean time, where the sign + is used if the equation of time be *increasing* and *additive*, or *decreasing* and *subtractive*; and the sign — when the equation is *increasing* and *subtractive*, or *decreasing* and *additive*. Now A : M *p m* : : $12^h \pm a : x$ the angle (expressed in mean time) described by a meridian of the earth in the time the moon describes M *p m*. Then, since the earth revolves through very nearly $180° \times \frac{366}{365}$ of longitude, in twelve hours of mean time, we shall have $x \times \frac{366}{365}$ = the angle D P *m* of longitude described by a meridian in the time the moon describes M *p m*: consequently D P G, the difference of meridians, $= \frac{366}{365} x -$ M *p m*. If the places do not differ much in longitude, we may for $\frac{366}{365} x$ substitute $x + \frac{x}{6 \times 60}$, and the result will still be nearly accurate: in this case also, the apparent may be used for the mean time. . . . . This method of finding the longitude, was proposed by Dr. *Maskelyne*, in the *Nautical Almanac* for 1769: the above investigation of it is given in Mr. *Vince's System of Astronomy*. It is certainly very easy in practice, and is

found to be more accurate than the method by the eclipses of jupiter's satellites.

Ex. On August 4, 1800, after midnight, at a certain place, having west longitude, the centre of the moon (which was then nearly in the full) was observed to be on the meridian $6^m\ 34^s{\cdot}6$ before the star β *aquarii*: from hence it is required to determine the longitude of the place of observation.

On the given day the moon come to the meridian at Greenwich at $12^h\ 9^m$; and the right ascension of β *aquarii* in August, 1800, was $21^h\ 21^m\ 2^s{\cdot}9$, from which take $8^h\ 56^m\ 57^s{\cdot}8$, the sun's right ascension at noon, the remainder $12^h\ 24^m\ 5^s{\cdot}1$ is the estimate time of the star's culminating, which made less by $1^m\ 55^s{\cdot}5$, half the daily variation in the sun's right ascension (Art. 171.), gives $12^h\ 22^m\ 9^s{\cdot}6$, correct time of the star's culminating at Greenwich. Therefore $12^h\ 22^m\ 9^s{\cdot}6 - 12^h\ 9^m = 13^m\ 9^s{\cdot}6$, difference between the times of the moon and the star's culminating at Greenwich. Then $13^m\ 9^s{\cdot}6 - 6^m\ 34^s{\cdot}6 = 6^m\ 35^s$ in time, or $1^\circ\ 34'\ 45''$ in space $=$ M· *p m*, the increase of the moon's right ascension in the time of its passage from the first meridian, to that whose longitude is sought. The variation of the equation of time for twenty-four hours on Aug. 4, was $5^m\ 43^s{\cdot}3$, *decreasing* and *additive*; therefore $12^h \pm a = 12^h - 2^m\ 51^s{\cdot}6 = 11^h\ 57^m\ 8^s{\cdot}4$. To find the increase of the moon's right ascension, take from the *Nautical Almanac* the right ascensions both at noon and midnight, for two or three days, and after arranging them, take their differences, as below:

				1st. dif.	2d. dif.
Aug.	3, noon . .	293°	53′		
	3, midnight .	301	49	7° 56′	
	4, noon . .	309	30	7 41	15′
	4, midnight .	316	54	7 24	17
	5, noon . .	324		7 6	18
	5, midnight .	330	48	6 48	18

Here, as the second differences are evidently nearly uniform, we may take $7° 15' = \frac{7° 24' + 7° 6'}{2}$, for the rate of increase for the twelve hours, the middle of which correspond with midnight on the fourth day. Therefore, $7° 15' : 1° 34' 45'' :: 11^h 57^m 8^s \cdot 4 : x = 156^m \cdot 20463$: consequently $\frac{366}{365} x = 156 \cdot 63261 =$ D P *m*; and D *p m* — M *p m* = D P G $= 156^m \cdot 63261 - 6^m \cdot 58333 = 150^m \cdot 04928$ in time, or $37° 30' 44'' \cdot 35$ in space, the longitude west from Greenwich.

It is a peculiar advantage attending this method, that it requires the taking of no altitudes or distances; neither is their needed the *true* times at which the transits are observed, but merely the true interval of time between the observations at the place where the longitude is required. Now the difference of times between the transits of the moon and star over the meridian, will seldom exceed a quarter of an hour, and such an interval may certainly be measured with sufficient accuracy by either a watch or pendulum, although its motions may not be sufficiently regular to measure much longer intervals of time. If the observations are made when the moon is not near the full, it will be proper to observe when the enlightened limb is on the meridian, and allow for the passage of the semidiameter by means of the *Nautical Almanac*: when the required longitude is great, and is to be determined with all possible accuracy, an allowance must be made for the change of the moon's semidiameter (in the interval of the passages), arising from its change of distance, also for the change of semidiameter in right ascension caused by the change of declination.

669. *Seventh method: by the distance between the moon and the sun, or a fixed star.* This method has, since the invention of Mr. *Hadley's octant*,

by which altitudes and distances may be taken on ship-board, been found more practicable and accurate for determining the longitude at sea, than any other method which has yet been tried. On this account it has very much engaged the attention of the best Astronomers, ever since the time of Dr. *Halley*, who strongly recommended it. Dr. *Maskelyne* in particular, in the explanation and use of the *Requisite Tables*, has laid down rules of his own invention, and Mr. *Witchell*'s, which very much facilitate the computations; and has thus removed, what was formerly looked upon as a material objection to this method, namely, the tediousness and difficulty of calculating the moon's place, and thence its distance from the sun or a star.

The nature of the process in this method of finding the longitude may be thus explained: 1. The altitudes of the moon and sun, or fixed star, are carefully taken with an Hadley's octant, and likewise the distance between them: then, from the observed altitudes and distance, the true distance is computed, making proper allowances for parallax and refraction. 2. The time is found at Greenwich, when the moon's distance from the sun or star agrees with the distance above computed. 3. If the latitude be known, the time at the place of observation is determined by some of the problems in Chap. VI.: or if the latitude is not known, both it and the time may be found by some of the methods in the former part of this chapter. 4. The difference of the times at Greenwich, and the place of observation, gives the longitude from Greenwich.

670. Having the apparent distance of the moon from the sun or a star, and their respective zenith distances, the true distance may be readily found: thus, in fig. 9, Pl. VI. let M be the apparent place of the moon, and S that of the sun, or star; *m* and *s* their true places: M *m* being the moon's parallax

lessened by refraction, and S *s* the star's refraction, or the sun's refraction lessened by parallax. Then, Z being the zenith, in the spherical triangle M Z S, there are given the apparent distance S M, with the apparent zenith distances Z S, Z M, to find the angle Z: and in the triangle *m* Z *s*, we then know, Z *m*, Z *s*, the correct zenith distances, and *m* Z *s* the included angle, from which the true distance *m s* may be determined.

671. But the calculations of these triangles by the common methods being tedious, different approximations and rules have been invented for the value of *m s*; one or two of which are here given. Thus, first: find the segments of the base M I, SI, and take $\text{M}\,o = \frac{\text{tang. MI}}{\text{tang. MZ}} \times \text{M}\,m$, and $\text{S}\,p = \frac{\text{tang. SI}}{\text{tang. SZ}} \times \text{S}\,s$; then M S — M *o* + S *p* will be very nearly equal to *m s* the true distance: if the angle M or S be obtuse, take + M *o*, or — S *p*. For, in the right-angled spherical triangle Z I M, we have tang. MZ : radius : : tang. MI : co-sine $\text{IMZ} = \frac{\text{tang. MI}}{\text{tang. MZ}} \times \text{radius}$; and, in the right-angled triangle M *m o*, which, on account of the minuteness of its sides, may be reckoned a plane triangle, we have radius : M *m* : : co-sine M $\left(= \frac{\text{tang. MI}}{\text{tang. MZ}} \times \text{radius}\right)$: $\text{M}\,o = \frac{\text{tang. MI}}{\text{tang. MZ}} \times \text{M}\,m$. By a similar kind of reasoning it will appear that $\text{S}\,p = \frac{\text{tang. SI}}{\text{tang. SZ}} \times \text{S}\,s$. Now, since the arches M *m*, S *s*, are very small, D *m* will be very nearly equal to D *o*, or D M — M *o*; and D *s* nearly equal to D *p*, or D S + S *p*. Therefore D *m* + D *s* = D M — M *o* + DS + S *p*, or *m s* = M S — M *o* + S *p*.

672. Or the true distance may be found very compendiously by the following precepts:

1. To the arith. comp. of the log. sine of half the apparent distance, add the log. sine of half the dif-

ference of the apparent altitudes; the sum is the log. sine of a first arc.

2. To the log. sine of half the apparent distance, add the log. co-sine of the first arc, reject radius, and the sum is the log. sine of a second arc.

3. Add together twice the log. sine of the second arc, and the arith. complements of the log. co-sines of the two apparent altitudes, their sum is the log. of a third arc.

4. To the log. of the third arc, add the log. co-sines of the correct altitudes, the sum is the log. of a fourth arc.

5. From half the log. of the fourth arc, take the log. sine of half the difference of the corrected altitudes, there remains the log. tangent of a fifth arc.

6. From half the log. of the fourth arc, take the log. sine of the fifth arc, the remainder is the log. sine of half the true distance *.

673. The true distance from the moon's enlightened limb to the sun or star, being determined by some of these rules, or those given with the *Requisite Tables*, the next step is to find the time at Greenwich, when the given objects had this distance. Now, if it were necessary to go through the whole of the calculations by which the time when the moon was at a given dis-

* While this work has been in the press, "*An Introduction to the Theory and Practice of Plane and Spherical Trigonometry*" has been published, by Mr. *Thos. Keith*, at page 295 of which he has investigated a general rule for determining *the true distance of the moon from the sun or a star*, having given the apparent altitudes and distance: as this rule is perfectly correct, and requires but few numbers to be taken out of the tables, and those at no great distance from each other, it is here inserted. RULE. "To the natural co-sine of the difference between the apparent altitudes, *add* the natural co-sine of the apparent distance, if *more* than 90°, or *subtract* it, if *less* than 90°, and find the common logarithm of the remainder; to which add the logarithm secants of the apparent altitudes, the logarithm co-sines of the true altitudes, and reject the tens from the index. The difference between the natural number answering to this sum in the table of logarithms, and the natural co-sine of the difference between the true altitudes, will give the natural co-sine of the true distance."

tance from any other celestial object was to be ascertained, this part of the business would be extremely complex and embarrassing; but, in the *Nautical Almanac*, the distances of the moon from the sun, and certain fixed stars which lie nearest her path, are set down to every three hours of apparent time at Greenwich; so that, assuming the moon's motion in its orbit, or its motion with respect to the object to or from which it is approaching or receding, to be uniform for a small portion of time (Art. 677.), the time corresponding to the given distance may be nearly found by simple proportion. Thus, seek in the Ephemeris amongst the computed distances of the proposed objects, on the day given for the given distance D; which, if found there, shews the time by inspection: but if the given distance fall between two computed ones, A, B, say, *as the difference of the distances* A, B, *to the difference between* A *and* D; *so are three hours of time, to the proportional time to be added to that of the distance* A. In the *Requisite Tables* there is a table of *proportional logarithms*, which are calculated from the common logarithms, by subtracting the log. of the given number, from the log. of 3; in which, therefore, the log. of 3 is = 0: of course, by the help of this table, the fourth term in the above proportion is found by subtraction only*.

* The advantage, as well as the accuracy of applying proportional logarithms, will obviously appear, after performing an operation by common logarithms, and observing in what manner the log. of 3 affects the others with which it is connected. Thus,

As $1^\circ\ 30'\ 50'' = 1{\cdot}51388$.	Log.	$\cdot 1800914 = a$	
To $32'\ 24'' = \cdot 54$ . . .	Log.	$1{\cdot}7323938 = b$	
So are 3 hours	Log.	$\cdot 4771213 = p$	
		$\cdot 2095151$	
		$\cdot 1800914$	
To $1^h\ 4^m\ 12^s = 1{\cdot}070098$.	Log.	$\cdot 0294237 = c = p + b - a$	

Now

674. Having, agreeably to these directions, found the time at Greenwich corresponding to the true distance calculated; and the time at the place of observation, by some of the methods so often alluded to; the difference of longitude, in time, and thence in degrees of the equator, may be readily ascertained. But to render the whole process more obvious, we shall illustrate the preceding rules by an example, as below.

Ex. Suppose that on the night of the 24th of April, 1801, when the star *pollux* was to the west of the moon; the apparent distance of the moon and star was found to be 56° 40′, the apparent altitude of the moon 33° 16′, of the star 20° 12′, and the apparent time of observation $12^h\ 26^m\frac{1}{2}$: from hence it is required to determine the longitude of the place where the observations were made.

On the 24th of April, at midnight, the moon's horizontal parallax was 56′ 45″; therefore (Art. 83.), as radius : co-sine 33° 16′ : : sine 56′ 45″ : sine 47′ 27″ parallax in altitude, and the respective refractions at the altitudes 33° 16′, 20° 12′, are (Table VIII.) 1′ 27″ and 2′ 34″. Hence, by the rules in Art. 672., proceed thus:

Apparent distance	56°	40′	its $\frac{1}{2}$ = 28° 20′
☽ apparent altitude	33	16	
∗ app. alt. . . .	20	12	
Difference .	13	4	
Half diff. app. alt.	6	32	

Now, by the construction of these proportional logs. it is manifest that $p - a$ = proportional log. of a
$p - b$ = proportional log. of b
$p - c$ = proportional log. of c
And, by the application of these logarithms, $\overline{p - b} - \overline{p - a} = \overline{p - c}$: but by actually subtracting $p - a$ from $p - b$, we have $a - b = p - c$, and by transposition $c = p + b - a$, which exactly corresponds with the result above given by common logarithms.

☽ app. alt.	33°	16′	″	* app. alt.	20°	12′	″
— refrac.		1	27	— refrac.		2	34
	33	14	33	* correct alt.	20	9	26
+ paral.		47	27				

From	34	2		☽ correct alt.
Take	20	9	26	* correct alt.
Diff.	13	52	34	
	6	56	17	. . . ½ diff. true alt.

To log. sine 28° 20′ arith. comp.	0·3236719
Add log. sine 6 32	9·0560706
Sum log. sine 1st arc	9·3797425
To log. sine 28° 20′	9·6763281
Add log. co-sine 1st arc	9·9871465
Sum, log. sine 2d arc	9·6634746
log. sine 2d arc	9.6634746
log. co-sine 33° 16′ arith. comp.	0·0777279
log. co-sine 20 12 arith. comp.	0·0275690
Sum, log. 3d arc	19·4322461
To log. 3d arc	19·4322461
Add log. co-sine 34° 2′ . . .	9·9184037
And log. co-sine 20° 9′ 26″ . .	9·9725501
Sum, log. 4th arc	39·3231999
From half log. 4th arc	19·6615999
Take log. sine 6° 56′ 17″ . .	9·0619229
Rem. log. tang. 5th arc	10·5996770

From half log. 4th arc	. . .	19·6615999
Take log. sine 5th arc	. . .	9·9866943
Rem. log. ½ dist. 28° 13′ 56″	.	9·6749056

Therefore . . 56 27 52 is the true distance.

Now, by the *Nautical Almanac*, 1801, it appears that, on April 24th, the distances of the moon's centre from pollux are as follows:

At 12 h.	57° 10′ 43″	. .	56° 27′ 52″ = D
At 9 h.	55 33 48	. .	55 33 48 = A
A ∽ B	1 36 55	A ∽ D	54 4

Hence, by Art. 673., as 1° 36′ 55″ : 54′ 4″ : : 3^h : 1^h 40^m 25^s. Or, if we use the proportional logarithms given in the 15th of the *Requisite Tables*, then,

From Prop. log. 0° 54′ 4″	. .	5223
Take Prop. log. 1 36 55	. .	2688
Rem. Prop. log. 1^h 40^m 25^s	.	2535

Consequently, $9^h + 1^h\ 40^m\ 25^s = 10^h\ 40^m\ 25^s$ apparent time at Greenwich, when the true distance of the moon from pollux is 56° 27′ 52″; but the apparent time at the place of observation is 12° 26^m 30^s; therefore, $12^h\ 26^m\ 30^s - 10^h\ 40^m\ 25^s = 1^h\ 46^m\ 5^s$ in time, or 26° 31′ 15″ in space, longitude from Greenwich; and, as the time at Greenwich is *less* than that at the place of observation, the longitude is *east* (Art. 661.)*.

* The most correct and practicable rules now extant, for finding the longitude by the lunar distances, are those invented by Dr. *Maskelyne*, Mr. *Lyons*, Mr. *Dunthorne*, and Mr. *Witchell*, and printed with the *Requisite Tables*. None of those admirable rules are inserted in this treatise, because they cannot be put in practice without the tables which they accompany; these tables and rules must, however, be strongly recommended, as what no practical astronomer or navigator ought to be without.

675. In order to arrive at the greatest possible precision in determining the longitude from the distance between the moon and the sun, or a star, it is absolutely necessary to make the observations with considerable care, for an errour of only one minute in the apparent distance will, at a mean, occasion an errour of twenty-seven miles in the longitude deduced. This will readily appear, from considering that the errour in longitude, resulting from an errour in apparent distance, is as 360° to the diurnal motion of the moon; thus, taking the moon's mean diurnal motion at 13° $10\frac{1}{2}'$, an errour in distance of 4′ will produce an errour in longitude of 1° 49′; and the corresponding errour in longitude to 1′ in distance is about $27\frac{1}{4}'$ or geographical miles on the equator. On account of the errours in longitude bearing so great a proportion to those in distance, a few remarks, tending to render the observations more accurate, are here added.

676. It has been shewn in Art. 132., that when the time is determined from observations on the sun, the errour in time will be least when the sun is on the prime vertical, provided a proper allowance be made for refraction: but this allowance must not always be the same, for the sun is often not far from the horizon when on the prime vertical, and near the horizon the refraction is very variable. From the notation made use of in that article, it readily follows, that the *perpendicular ascent of a body is greatest* when on the prime vertical; for, when $p\,q$ and the latitude are given, $r\,s$ (fig. 4, Pl. II.) varies as the sine of the azimuth, which is a *maximum* when the body is on the prime vertical. Or, if the object be situated within the circle of perpetual apparition, it changes its altitude the most speedily (Art. 180.) when it comes in contact with an azimuth circle. It may also be remarked, that the sun, or a star, changes its altitude most slowly when it is near the meridian; therefore,

the best times of the day for taking the distance between the moon and sun is from about $\frac{1}{2}$ or $\frac{3}{4}$ of an hour after sun-rise to about ten o'clock in the morning, and in the afternoon, from nearly two o'clock to nearly $\frac{1}{2}$ an hour before sun-set. If a watch be used, the altitudes for regulating it may be taken during any part of the morning or afternoon, when the sun is not higher than 18° or 20°, or lower than 5°. These observations may be taken during the four days preceding the moon's first quarter, and the four days following the last quarter, which make eight days in each lunation, provided a *quadrant* be used; but if a *sextant* be made use of, observations may be made on the four days preceding, and the four days following the quadratures also; that is, fifteen or sixteen days in each lunation.

677. At sea, the best time for making observations between the moon and a star is during the morning or evening twilight, or when the sea horizon can be seen sufficiently distinct to take an accurate altitude: here also the altitude of the moon should be between 5° and 18°, or at most 20°; but, with a distinct horizon at land, any altitude may be used, which is not too near the meridian. The star, which is chosen, should be at least two hours from the meridian, particularly if the apparent time is to be deduced from it. As to its situation, with respect to the moon, it should be such that the moon should approach to it, or recede from it, with a velocity as nearly uniform as possible. In order to this, it must be situated either in or near the moon's path, that is, very nearly in the direction of a line conceived to be drawn so as to pass through the moon's centre perpendicular to the right line joining her cusps (Art. 470.); or, between the full moon and her quadratures, in the direction of a line passing through her centre perpendicular to her longest diameter; and, if the distance from the moon to the star chosen be more than 25° or 30°,

although the star should deviate in a small measure from the moon's path, still the moon will approach to it, or recede from it, for an hour or two, with nearly uniform velocity. The stars whose distances are set down in the *Nautical Almanac*, on the respective days for which the distances are computed, lie very near the moon's path, so that the chief irregularity in the motion with which the moon approaches to them, or recedes from them, arises from her absolute motion in her orbit; therefore, in cases where extreme accuracy is required, the velocity should be taken into the account, as well as the space, in determining the time taken by the moon to move any given distance, and the proper measure of the velocity is such a quantity as has the same ratio to the space described, as three hours have to the time that has been actually taken to move the given distance: to find this quantity correctly would require interpolation, but it will be sufficient in practice to find the time first by the common method (Art. 673.), and then to correct the interval for three hours to that time, by taking a proportional part of the *second difference* of the moon's distance at the beginning of each three hours, supposing the *first differences* to answer to the middle of each interval. This correction will often bring the result nearer to the truth by four or five, and sometimes even by six miles, which, in geographical determinations, is of consequence.

678. If two, three, or more sets of observations be made as near to each other as possible, and the means of the times by the watch, of the distances, and of the respective altitudes, be taken, these means may be esteemed a set of observations more to be depended on than any single set; or, the longitudes may be computed to each set, and the mean of the results taken for the true longitude.

679. In taking the distances between the moon and the sun, or a star, with a Hadley's sextant, it is

usual to move about the index till the moon's *enlightened limb* is made to touch the limb of the sun, or the middle of the star. Now, by observations which have been made in a place whose longitude was previously determined with accuracy by the eclipses of jupiter's satellites, it has been found that bringing the object to the moon's enlightened limb is liable to errour. A little before and after the conjunction, when the whole hemisphere of the moon is visible (Art. 467.), several sets of distances of a star, both from the bright and dusky parts of the circumference, were taken; and of the results it appeared, that those deduced from the dusky part were considerably nearer the truth than the others: the nature of the errour plainly shewed that the star was really some distance (generally less than half a degree) from the enlightened limb, when it seemed to be in contact with it. Indeed it must be well known, that near the conjunction the outward arc of the moon's enlightened crescent seems to be a portion of a larger circle than the limb of the dusky part: this is the source of the errour, and it is the more necessary to attend to it, because it is of such a kind as is not to be remedied by increasing the number of observations. To avoid the errours which arise from this deception, the distances may be taken from the dusky part of the limb when the moon is near the conjunction; or, at other times, instead of making the star appear to touch the moon *outwardly*, or *at the centre*, to cause it to touch the moon *inwardly*, so that the whole star shall just be hid.

680. When the altitudes of the moon and the sun, or a star, are taken at sea, they require to be corrected for what is called *the dip of the horizon*; for the observer, standing upon deck, is raised above the level of the sea, and observes an horizon below the level of the true one. Thus (fig. 13, Pl. IX.), let *a* S *f* be the surface of the sea, E S the height of the

eye above that surface, and A F the observer's horizon; then the observer, deeming E *a* the horizon, which, in fact, dips below the true one by the arc A *a*, the apparent altitude *a* B of an object B must manifestly be lessened by the arc *a* A. Or, if the observer's back be towards the object B, the line *f* E *c* becomes his apparent horizon, which dips below the true one by the arc F *f* = A *c*, it is obvious the apparent altitude *c* B is too small, and must be augmented by the dip E *f* or A *c*; consequently, in the *fore* observations with the sextant, the altitude must be *lessened* by the horizontal dip; and in the *back* observations, the altitude must be *increased* by the dip: but in *zenith distances add* the dips *forward*, and *subtract* them *backward*. The dips may be easily computed; for, let C be the earth's centre, E the eye, and the tangent E *f* meeting the surface in *f*, where the sky and water seem to meet; then C *f* E is a triangle right-angled at *f*, in which are known C *f* = C S = the earth's semidiameter, and C E = C S + S E = the sum of the earth's radius and the height of the eye; then, as C E : C *f* : : radius : sine C E *f*, and because C E F is a right angle, the complement of C E *f* is *f* E F measured by E *f* the dip sought. . . . . Among the *requisite tables* there are two which relate to the *dip of the horizon*; the one (Tab. II.) expresses the dip to different heights of the eye, from 1 foot to 100, when the horizon is entirely open and free from intervening objects, as supposed above; the other (Tab. V.) when the shore is nearer the ship than the visible horizon would be, if it were unconfined, in which the dip is given as it corresponds to altitudes between 5 and 40 feet, and distances between 1 of a mile and 6 miles.

681. Whenever recourse is had to the *Nautical Almanac*, it will be necessary to attend to the proper distinction between the astronomical, civil, and nautical mode of reckoning time; the nautical or sea

reckoning is twenty-four hours, or one whole day, and the civil twelve hours, in advance of the astronomical, at the commencement of the day. Thus, April 27th, $8^h\ 40^m$ in the morning, civil time, is April 26th, $20^h\ 40^m$, astronomical time (Art. 109.), and April 27th, $8^h\ 40^m$, A.M. nautical time; also, April 27th, $8^h\ 40^m$ in the afternoon, civil time, is April 27th, $8^h\ 40^m$ astronomical, and April 28th, $8^h\ 40^m$, P.M. nautical: the beginning of the astronomical day, the noon of the civil, and the ending of the nautical, being at one and the same instant, under the same meridian. . . . . Other directions and remarks might be added; but the above will, we hope, suffice to give a clear view of the subject; further information may be met with in works written expressly on the longitude *.

* On this subject the performances of Dr. *Mackay* and Mr. *Wales* merit particular commendation; Mr. *Robertson's Navigation*, and Mr. *H. Clarke's Seaman's Desiderata*, may be advantageously consulted; and several papers on the longitude in the *Philosoph. Transactions* and *Asiatic Researches* contain very useful directions. An extensive set of *Tables for facilitating the Calculations of Nautical Astronomy*, has been just published by *Joseph de Mendoza Rios*, esq. The methods of finding the longitude by a *solar eclipse*, and by an *occultation of a fixed star by the moon*, are not given in this treatise, for the necessary computations in each are much more tedious and irksome than in any of the methods explained in this chapter; and, as they are not of universal application, and even when they can be applied are not more accurate than either the 5th, 6th, or 7th method, there is nothing gained to compensate for the additional labour and fatigue.

CHAPTER XXIV.

On the Distances, Magnitudes, and Number of the fixed Stars, with Reflections on their Nature, and on the Immensity of the Universe.

ART. 682. WHEN treating upon the relative situations and appearances of the *fixed stars*, in the fourth chapter of this work, we did not attempt to assign their *real distances* or *magnitudes*, because it could not possibly have been attempted in that place, without assuming, as known, many principles, with which a person on the entrance of his astronomical studies must be in great measure unacquainted; we therefore deferred this part of the enquiry until other truths were established, which will enable us to pursue it with greater probability of success.

683. That the fixed stars are placed in situations extremely remote from us, will be manifest from the consideration that even their *annual parallax* is quite insensible, or at least, it requires the most ingenious contrivances to bring it in any way under our notice; that is (Art. 630.), the diameter of the earth's orbit, which has been shewn to be 190 millions of miles, bears no sensible proportion to the immense distance of the fixed stars. Various methods have been invented for investigating the annual parallax, but none of them have as yet afforded us more than a distant approximation. The method, though, which was practised by *Hook*, *Flamsteed*, *Molyneux*, and *Brad-*

ley, has given us a much more accurate idea of the distance of the stars from us than was ever derived from any previous method. Dr. *Bradley* says, that if this parallax had amounted to a single second, or two at most, he should have discovered it in the great number of observations which he made upon stars near the zenith, particularly upon γ *Draconis;* and that it seemed to him very probable that the annual parallax of this star does not amount to a single second; and, consequently, that it is about four hundred thousand times farther from us than the sun.

684. Mr. *Michell** instituted an enquiry into the probable parallax and magnitude of the fixed stars, from the quantity of light which they afford us, and the peculiar circumstances of their situation. With this view, he supposed that they are, on a medium, equal in magnitude and natural brightness to the sun; and then he proceeded to enquire, what would be the parallax of the sun if he were to be removed so far from us, as to make the quantity of the light which we should then receive from him, no more than equal to that of the fixed stars. Adopting the generally received principle that *the intensity of light is reciprocally as the square of the distance from the luminous point*, he found that if the sun were removed to 220000 times his present distance, he would still appear as bright as saturn, and his whole parallax, upon the diameter of the earth's orbit, would be less than 2″; and this he assumed for the parallax of the brightest of the fixed stars, upon the supposition that their light does not exceed that of saturn. This ingenious gentleman imagines, that the quantity of light we receive from *sirius*, does not exceed the light we receive from the least fixed star of the sixth magnitude, in a greater ratio than that of 1000 to one, nor less than that of 400 to one; and the smaller stars of the second mag-

* *Philosophical Transactions*, vol. 57.

nitude seem to be about a mean proportional between the other two. Hence the whole parallax of the least fixed stars of the sixth magnitude, supposing them to be of the same size and native brightness with the sun, should be from about 2‴ to 3‴, and their distance from about eight or twelve million times that of the sun; upon the same supposition, the parallax of the smaller stars of the second magnitude should be about 12‴, and their distance about two million times that of the sun.

685. Dr. *Herschel*, to whose great sagacity and industry astronomy is, on many accounts, so highly indebted, has proposed a method by means of double stars, which is free from most of the errours peculiar to other methods, and is of such a nature, that the annual parallax, even if it should not exceed the tenth part of a second, may still become visible, and be ascertained with much more precision than heretofore *. This method was first suggested in an imperfect manner by *Galileo*, and has been since mentioned by Mr. *Emerson* in his *Astronomy*, and by some other authours; it is capable of every improvement which the telescope and mechanism of micrometers can furnish. Dr. *Herschel* premises as postulata two principles; first, *that each of the stars is about the size of the sun;* and secondly, *that the difference in the apparent magnitudes of the stars is owing to their different distances*, so that *a star of the second, third, or fourth magnitude, is about two, three, or four times farther from us than one of the first.* These postulata will not, we suppose, be readily granted by every one; but, if the mode of reasoning be adopted which is pursued in the *Doctrine of Chances*, it will appear that they have considerable probability in their favour, and may therefore be admitted in an enquiry, respecting which, if the result should not be perfectly accurate, it will in no shape

* See *Philosophical Transactions*, vol. 72. Also Dr. *Hutton's Math. and Phil. Dictionary*, Art. STAR.

affect the truth of any of our previous discussions, all of which are totally independent of the present speculation. But to proceed to an elucidation of this method: Let E O (fig. 16, Pl. IX.) be the whole diameter of the earth's annual orbit, and let I and B be two stars of unequal apparent magnitudes, in the same plane with E O; then the difference between the angles I O B, I E B, subtended by these stars at two opposite points of the earth's orbit, or between the two angles, measured at the distance of half a year from each other, will lead to the determination of the parallax: and in order to this, two stars, or more, should be chosen which appear as near as possible to each other, and at the same time have the greatest difference in apparent magnitude. Let A, B, C, be three stars situated in the ecliptic, in such a manner that they may appear all in one line O A B C, when the earth is at O. Now, if O A, A B, B C, be equal to each other, A will (by the second postulatum) be a star of the first magnitude, B of the second, and C of the third. Let us next suppose the angle O A E, or annual parallax, to be $1''$; then, because very small angles having the same subtense E O may be estimated in the inverse ratio of the lines O A, O B, O C, &c. we shall have E B O $= \frac{1}{2}''$, E C O $= \frac{1}{3}''$, &c.: also, because E A $=$ A B nearly, the angle A E B $=$ A B E $= \frac{1}{2}''$; and because B C $= \frac{1}{2}$ B O $= \frac{1}{2}$ B E nearly, the angle B E C $= \frac{1}{2}$ BCE $= \frac{1}{6}''$, and hence A E C $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}''$: from which it follows that, when the earth is at E, the stars A and B appear at $\frac{1}{2}''$ distance from each, the stars A and C at $\frac{2}{3}''$, and the stars B and C only $\frac{1}{6}''$ distant. In a similar manner may be deduced a *general* expression for the parallax that will become visible in the change of distance between the two stars, by the removal of the earth from one extreme of her orbit to the other. Let P denote the total parallax of a fixed star of the magnitude of the M order, and *m* the number of the order of a smaller star, *p* denoting the partial paral-

lax, to be observed by the change in the distance of a double star; then drawing a A perpendicular to E C, we have $p : P :: a A : E O :: C A : C O ::$ $m - M : m$; hence $p = \frac{m - M}{m} P$, or $P = \frac{m p}{m - M}$, which gives P when p is found by observation.

Ex. Suppose a star of the first magnitude should have a small star of the twelfth magnitude near it; then will the partial parallax we are to expect to see, be $\frac{12 - 1}{12} P = \frac{11}{12} P$, or $\frac{11}{12}$ of the total parallax of the larger star: and if we should, by observation, find the partial parallax between two such stars to amount to 1″, then will the total parallax $P = \frac{12}{11} p = 1''\frac{1}{11}$. Again, if the stars be of the third and twenty-fourth magnitude, the total parallax will be $P = \frac{24}{24 - 3} p = \frac{24}{21} p = \frac{8}{7} p$; so that, if, by observation, p be found to be $\frac{1}{10}$ of a second, the whole parallax P will be $\frac{8''}{70} = 0{\cdot}11428''$.

686. Farther, the stars being still in the ecliptic, suppose they should appear in one line, when the earth is in some other part of her orbit between E and O; then will the parallax be still expressed by the same algebraic formula, and one of the maxima will still lie at E, the other at O; but the whole effect will be divided into two parts, which will be in proportion to each other, as radius — sine, to radius + sine, of the star's distance from the nearest conjunction or opposition.

687. When the stars are any-where out of the ecliptic, situated so as to appear in one line O A B C perpendicular to E O, the maximum of parallax will still be expressed by $\frac{m - M}{m} P$; but there will arise another additional parallax in the conjunction and opposition, which will be to that which is found 90°

before or after the sun, as the sine (s) of the latitude of the stars seen at O, is to radius (1); and the effect of this parallax will be divided into two parts; half of it lying on one side of the large star, the other half on the other side of it. This latter parallax will also be compounded with the former, so that the distance of the stars in the conjunction and opposition, will then be represented by the diagonal of a parallelogram, whose sides are the two semiparallaxes; a general expression for which will be $\frac{m - M}{2m} P \sqrt{1 + s^2}$, or $\frac{1}{2} p \sqrt{1 + s^2}$. When the stars are in the pole of the ecliptic, s will be $= 1$, and the last formula will be $\frac{1}{2} p \sqrt{2} = \cdot 7071\, p$.

688. Again, let the stars be at some distance, as 5″, from each other, and let them be both in the ecliptic. This case is resolvable into the first: for imagine the star A to stand at I; then the angle A E I may be accounted equal to A O I; and, as the first formula $p = \frac{m - M}{m} P$, gives us the angles A E B, A E C, we are to add A E I, or 5″, to A E B, which will give I E B. In general, let the angular distance of the stars be d, and let the observed distance at E be D, then will $D = d + p$, and therefore the whole parallax of the annual orbit will be expressed by $\frac{D - d}{m - M} D d = P$.

689. Suppose now, the stars to differ only in latitude, one being in the ecliptic, the other at some distance, as 5″ north, when seen at O. This case, also, may be resolved by referring to the former; for imagine the stars B and C to be elevated at right angles above the plane of the figure, so that A O B or A O C may make an angle of 5″ at O; then, instead of the lines O A B C, E A, E B, E C, imagine them all to be planes at right angles to the figure; and it will appear that the parallax of the stars in lon-

gitude, must be the same as if the small star had been without latitude. And, since the stars B C, by the motion of the earth from O to E, will not change their latitude, we shall have the following construction for finding the distance of the stars A B, and A C at E, and thence the parallax P.——Let the the triangle $a\,b\,\beta$ (fig. 14, Pl. IX.) represent the situation of the stars; $a\,b$ is the subtense of 5″, the angle under which they are supposed to be seen at O. The quantity $b\,\beta$ is found by a former theorem $= \frac{m - M}{m} P$, which is the partial parallax that would have been seen by the earth's moving from O to E, if both stars had been in the ecliptic; but, on account of the difference in latitude, it will now be represented by $a\,\beta$, the hypothenuse of the triangle $a\,b\,\beta$: therefore in general, putting $a\,b = d$, $a\,\beta = D$, we have $\frac{m}{m - M} \sqrt{D^2 - d^2} = P$. Hence, D being found by observation, and the three d, m, M, being given, the total parallax is obtained.

690. When the stars differ in longitude as well as latitude, a formula for the parallax may be obtained thus: Let the triangle $a\,b\,\beta$ (fig. 15, Pl. IX.) represent the situation of the stars, $a\,b = d$ being their distance seen at O, $a\,\beta = D$ their distance seen at E. That the change $b\,\beta$, which is produced by the earth's motion, will be truly expressed by $\frac{m - M}{m} P$, may be proved as before, by supposing the star a to have been placed at α. Now let the angle of position $b\,a\,\alpha$ be taken by a micrometer, or by any other method sufficiently exact; then, by resolving the triangle $a\,b\,\alpha$, we obtain the longitudinal and latitudinal differences $a\,\alpha$ and $b\,\alpha$ of the two stars. Put $a\,\alpha = x$, $b\,\alpha = y$, and it will be $x + b\,\beta = a\,q$, whence $\sqrt{(x + \frac{m - M}{m} P)^2 + y^2} = D$; and $P = \frac{\sqrt{D^2 - y^2} - x}{m - M} m$.

691. If neither of the stars should be in the ecliptic, nor have the same longitude or latitude, the last theorem will still serve to calculate the total parallax, whose maximum will lie in E. There will also arise another parallax, whose maximum will be in the conjunction and opposition, which will be divided, and lie on different sides of the large star; but as the whole parallax is extremely small, it is not necessary to investigate every particular case of this kind; for by reason of the division of the parallax, which renders observations taken at any other time, except where it is greatest, very unfavourable, the formulæ would be of little use.

692. Dr. *Herschel* closes his account of this theory, with a general observation on the time and place where the maxima of parallax will happen. Thus, when two unequal stars are both in the ecliptic, or, not being in the ecliptic, have equal altitudes, north or south, and the *largest* star has the greatest longitude, the *maximum* of the apparent distance will be when the sun's longitude is 90° *greater* than the star's, or when observed in the morning: and the *minimum*, when the longitude of the sun is 90° *less* than that of the star, or when observed in the evening. But when the *small* star has greatest longitude, the maximum and minimum, as well as the time of observation, will be the reverse of the former. And when the stars differ in latitude, this makes no alteration in the place of the maximum or minimum, nor in the time of observation; that is, it is immaterial which of the two stars has the greater latitude.

693. Upon the whole, there is reason to conclude that the decision of Dr. *Bradley* (Art. 683.) is within the limits of truth: we may, therefore, set down the distance of γ *draconis* at 400000 times that of the sun, and the distance of the nearest fixed star, 80000 times that of the sun, or 40000 times the diameter of the earth's orbit. These distances are immensely great, as will more manifestly appear, by

finding the time which would be occupied by some moving body of great velocity in passing through them. Light passes from the sun to the earth in about 8 minutes; yet this would take up more than six years in travelling from γ *draconis* to the earth; and nearly a year and a quarter in passing from the nearest fixed star to the earth. A cannon ball moving with the velocity of 1760 feet per second, or 20 miles in a minute, would not traverse the distance from the nearest fixed star to us in a shorter period of time than *one million one hundred and twenty-eight thousand years!!*

694. As to the *real magnitudes* of the fixed stars, there is but little prospect of our arriving at any thing like certainty on that head. All that we have any reason to expect is a mere approximation, and that not a very accurate one: for if we could determine the annual parallax of any of them with precision, which has not been done hitherto, still the best and most complete instruments will not enable us to determine their diameter accurately. Conjectures have sometimes been made, from a comparison of the light they afford us with that of the sun, and it has been concluded that the magnitude of the stars does not differ considerably from that of the sun: this is probably not far from the truth; but it would not become those who study a science reared on demonstration, to lay much stress on the result of mere conjecture.

695. With respect to the *number* of the fixed stars, we could, almost without hesitation, pronounce it infinite. The number that may be seen by the naked eye does not exceed 3000, if it extend to so many: but, since the invention of telescopes, it has been found, that to the greater perfection these instruments are brought, the greater, in a very high proportion, is the number of the stars which may be observed. A good telescope, directed almost indiffer-

ently to any point of the heavens, discovers multitudes that are lost to the naked eye. Dr. *Hook*, with a telescope twelve feet long, reckoned seventy-eight stars in the single constellation of the *pleiades*: and *F. de Rheita* with a better telescope discovered 188; whereas we cannot reckon above seven or eight seen by the naked eye. *Galileo* found eighty stars in the space of the belt of orion's sword; and *F. de Rheita* observed more than 2000 in the whole constellation of *orion*, of which not more than seventy or eighty (Art. 70.) can ever be seen without glasses. That bright irregular zone the *milky way* (Art. 70.) has been very carefully examined by Dr. *Herschel*; who has, in the space of a quarter of an hour, seen the astonishing number of 116000 stars pass through the field of view of a telescope of only 15′ aperture. Every improvement of his telescopes has led to the discovery of innumerable stars not seen before; so that it seems improper for us to set bounds to their number.

696. The fixed stars do not appear to be all regularly disseminated through the heavens, but the greater part of them are collected into clusters; and it requires a large magnifying power, with a great quantity of light, to distinguish separately the stars which compose these clusters. With a small magnifying power, and small quantity of light, they only appear as minute whitish spots, much like small light clouds, and thence they are called *nebulæ*. The numbers of nebulæ was formerly imagined to be about 103; but Dr. *Herschel*, early in the year 1784, had discovered 469 more, and since then has given a catalogue of 2000 *nebulæ* which he has discovered. The most careful and accurate observations give great reason to conclude, that they all consist of large masses or clusters of stars at prodigious distances from our system. Dr. *Herschel* is of opinion that the starry heaven is replete with these nebulæ, and that each of

them is a distinct and separate system independent of the rest. The milky way he supposes to be that particular nebula in which our sun is placed; and, in order to account for the appearance it exhibits, he supposes its figure to be much more extended towards the apparent zone of illumination, than in any other direction; which is a supposition that he thinks allowable, from the observations he has made on the figures of other nebulæ.

697. This idea of the stars being formed into systems *, had indeed been previously started by the ingenious Mr. *Michell*, in a paper we before referred to (Art. 684.): he observed that there were many large spaces in the heavens where no stars were to be seen; and others in which a number of very considerable ones appear near together, in the midst of several smaller ones. Now he apprehends that those stars which are found in clusters, and surrounded by many others at a small distance from them, belong probably to other systems, and not to ours. Those stars which are surrounded with nebulæ, are probably only very large stars, which, on account of their superiour magnitude, are singly visible, while the others, which compose the remaining parts of the same system, are so small as to escape our observation. And those nebulæ in which we can discover either none, or only a few stars, even with the assistance of the best telescopes, are probably systems that are still more distant than the rest. This gen-

* M. *Lambert*, too, in the second part of his *Cosmogony*, has gone far in the illustration and defence of a similar opinion. His mode of discussion is too diffuse to admit of extracts in a ſmall compass; the work is therefore recommended to the attention of the reader. But the recommendation must be qualified: for, at the same time that the ſublimity and extent of this author's astronomical knowledge enlighten the understanding, and the brilliancy of his genius calls forth our admiration, his proneness to indulge the most romantic conceptions, compels us to read with caution, and sometimes with regret.

tleman conducted part of his reasoning nearly as follows: The pleiades are composed of six remarkable stars, which are placed in the midst of a number of others, that are all between the third and sixth magnitudes; and comparing this number *six* with the whole number visible in the heavens to the naked eye, he calculated, by the doctrine of chances, that, among all this number, if they had been dispersed arbitrarily through the celestial vault, it was about five hundred millions to one, that six of them should be placed together in so small a space. It is, therefore, so many chances to one that this distribution was the result of design; or that there is a reason or cause for such an assemblage: and in a universe where every thing appears governed by immutable laws, this degree of probability is exceedingly strong. The stars, therefore, which are thus grouped are most probably systems analogous to the solar one; and our sun, who "appears to be the lord of the universe, is most likely only a star which belongs to some of those systems that are interspersed through the regions of the infinite expanse. This is conformable to the designs of Nature in all her operations. Our planetary system demonstrates, that she unites and connects several bodies together in order to compose a whole; and it is highly probable that all her works are conducted upon the same plan."

698. Now, a necessary consequence of this hypothesis is, that these different systems should be mutually balanced among themselves, by some general principle which, with its influence, pervades the whole: such principle we have in the law of universal gravitation, which may connect together these innumerable bodies, and reduce their motions to harmony and order. The learned Dr. *Halley*, conformably to this idea, conceived the whole solar system, together with all the systems of the stars, to be in motion

round some point which is the common centre of gravity of the whole. In pursuing this opinion he makes the following reflections: "If the number of stars be finite and occupy only a part of space, it will follow that they must be surrounded by a void. But as this void can have no action upon the bodies which it environs, those bodies must exert all their force upon each other, without equilibrium, and without compensation. Those which are at the extremities, or near the borders of the void, will be strongly and continually attracted by those near the centre; and these efforts, continued and multiplied through a number of ages, must at length draw all the suns and planets into that point, and form one immense mass, which must for ever remain there, without action and without motion. But if, on the contrary, the number of stars be infinite, and the system without bounds, all the forces will be balanced among themselves; the suns and planets will preserve the paths prescribed them, and the order of the universe will be perpetually the same *." Opinions in great measure similar to these of Dr. *Halley*, appear to have been entertained by philosophers nearly 2000 years ago, though their ideas were probably very confused and indefinite; but that they had some notion of the celestial bodies being influenced considerably by gravity, is evident from the observation of *Lucretius*, who thinks the world to be infinitely extended, "For otherwise," says he, "the bodies that are on the outside, would, by their gravity, be drawn towards those on the inside, and would long since have all fallen together in a choatic mass in the middle."

699. Whether all the bodies in the universe partake of some general motion round a common centre, as well as continue to move, by the established laws,

* *Philosophical Transactions*, No. 364.

about the particular centre of each system? or whether our system changes its situation with respect to absolute space, while the stars continue fixed, or the stars have a proper motion of their own, while the centre of the solar system remains at rest? are questions, the decision of which probably falls not within the scope of human intelligence. That the fixed stars have a small apparent motion, besides those which are occasioned by the earth's diurnal and annual revolutions by the precession of the equinoxes, the aberration of light, and the nutation of the earth's axis, is now completely ascertained by the observations of *Roemer*, *Mayer*, *Muskelyne*, and *Herschel*. Some of the motions observed are reconcileable with the supposed motion of the solar system, but others are of a nature quite contrary to what ought to arise from this supposition; nor do they entirely correspond with any other hypothesis. On these complicated and mysterious points we shall probably always float in uncertainty; our scanty views and feeble conceptions fall far below the range of such enquiries; that which incontrovertibly approaches so near to infinity, is not, perhaps, to be comprehended by any thing inferiour to *infinite wisdom*.

700. In determining the *nature of the fixed stars*, we are able to make somewhat nearer approaches; but even here most of our conclusions will be deduced from analogical reasoning. That the fixed stars are at immense distances from us, is manifest, from the fruitless attempts which have been made to determine their annual parallax: we may safely state their distance from us to be several thousand times greater than the distance of georgium sidus, yet they shine with much more brightness and splendour than that planet, whence we have great reason to conclude that they derive not their light from the same source, viz. from the sun: and, if they derived their light from any luminous body which was near them, that body

would certainly be visible to us, as well as those which shone with the borrowed light: it is natural, then, to suppose, that each fixed star shines, not with a reflected light, but with its own native lustre. To strengthen this opinion, let it be considered that the stars appear less in a telescope which magnifies two or three hundred times, than they do to the unassisted eye; for they appear, when viewed through the instrument, as mere points: now, if they shone by borrowed light, they would be as invisible without telescopes, as the satellites of jupiter or of saturn; for these satellites, when observed with a good telescope, appear larger than fixed stars of the first magnitude, when viewed in like manner. Hence we infer, that the fixed stars shine with their own light: and, when their immense distances are taken into account, we cannot hesitate to admit that their real magnitude considerably exceeds that of any of the planets. The fixed stars, then, seem to be of a nature very similar to the sun; nay, that they are suns in reality will scarcely admit of a doubt, for the analogy between them may be traced in numerous particulars: the sun turns on his axis, so do many of the stars, as is known from observation; the sun has spots on his surface, so have many of the stars; on the sun these spots are changeable, so they are on several of the stars, as has been frequently observed by Dr. *Herschel* and other astronomers. If, then, the stars are of a nature similar to the sun in these particulars, as well as others which are not now mentioned, is it not highly probable that each of them is designed to answer purposes like those for which the sun is so admirably fitted? Is it not reasonable to conceive that each star is *the centre of a system*, and has planets, which are illuminated, warmed, and cherished by its light and heat? We surely cannot imagine that the stars were formed for no other purpose than to cast a faint light around the earth, especially when we

consider that we have incomparably more light from the moon than from all the stars together, and that innumerable stars, so far from affording us light, are not even discernible, without the aid of the best telescopes. Let it also be considered, that if our sun, with the whole system of planets, moons, and comets, were to be removed to the distance of the nearest fixed star, nothing would be seen but the sun, which would then appear, not as the lord of the system, the source of universal light and heat, but as *sirius* or as *procyon*, brilliant though minute, luminous though diffusing no warmth. Instead, therefore, of looking upon the fixed stars as a curious species of twinkling flames, hung up in the spacious canopy of heaven merely to ornament it, or to add to the delight of man; let us suffer our conceptions to expand till they embrace a wider field of contemplation; let us not be accused of soaring into the regions of fancy, when we speak of the fixed stars as bodies of magnitude and brilliancy equal to, or greater than, the sun, placed not only at vast distances from us, but from each other, occupying each its proper situation in the unbounded immensity of space, and each dispensing light and heat, and controlling the motions of a system of planets and satellites which traverse their respective orbits under the influence of laws similar to those which regulate the motions of the bodies in our system.

701. But if we allow the existence of these innumerable planetary and stellar worlds, shall we be denied the pleasure of conceiving that they have inhabitants? When every part of matter, upon which we have opportunities of making observations, is found fitted for the production, nourishment, and growth of living creatures peculiar to itself; when we cannot meet with even a minute portion of nature which lies waste and useless; but when not only seas, lakes, and rivers, mountains and valleys, trees and herbs, grasses,

and the animals which feed upon them, teem with life, and even the blood and humours of animals, nay putrefactions, all produce or support their respective inhabitants; shall we, as it were, in direct opposition to the universal voice of nature, indulge the unreasonable notion, that the most numerous and most stupendous bodies in the universe are barren and void? No: let us rather, since it is so consonant to all that is grand and noble and reasonable, look upon the greater and less luminaries, sun and stars, planets and satellites, as an innumerable multitude of habitable worlds; let us consider them as furnished with plants, herbs, animals, and every thing necessary for the support or delight of rational beings; let us not be unwilling to admit that these rational beings have faculties, perceptions, senses; that they are capable of enjoying pleasure, and suffering pain; that they are not devoid of a knowledge of useful inventions, or sublime sciences; but that they are formed with every capacity necessary for social intercourse, for the employments of active life, and for the exalted exercises of devotion. But we forbear to proceed farther in the path of speculation; though on a subject which places before us such unbounded prospects, it is next to impossible to avoid it: even the known and acknowledged truths of astronomy almost stagger our belief; but when we reflect upon millions of millions of *worlds*, placed at the distance of millions of miles from each other, and the whole inhabited by millions of millions of millions of *rational creatures*, formed for never-ending felicity, the idea, though delightful, must necessarily be vague and indefinite; really, when indulging such contemplations, we can scarcely help acknowledging, that all our science is as nothing, and that we have yet to be initiated in the principles of a new notation, which may enable us to comprehend these astonishing numbers *.

* The inquisitive reader, who wishes to see such speculations farther pursued, will be charmed with the perusal of *Huygens's Cosmotheoris*, or *Conjectures concerning the Planetary Worlds.*

CONCLUSION.

ART. 702. WE have now gone through our enquiry into the nature, magnitudes, and motions of the heavenly bodies: it remains for us to make a few reflections. And here we cannot help remarking that, besides the obvious tendency which the study of astronomy has to improve the powers of reason, to lead to a nearer acquaintance with the most stupendous and amazing scenes in the universe, and thus to enlarge our conceptions, and cherish an ardent desire for more copious draughts of real and useful knowledge; it also, when properly pursued, directly tends to meliorate and improve the heart. We do not say that these beneficial consequences will result from a mere acquaintance with the properties of lines, and angles, and triangles, and circles, or with a facility in applying the principles of geometry and analysis to the determination of celestial distances and magnitudes; though this will be far from a useless exercise for the faculties of the mind: but what we mean to affirm is, that the truths which astronomy places before us, when rightly attended to, lead to the advancement of wisdom and good morals; indeed we could almost hazard the opinion, that they inculcate a morality inferiour only to the excellent morality of the gospel. By means of astronomy we may, as the ingenious *Huygens* remarks, " mount from this dull earth, and viewing it from on high, consider whether Nature has laid out all her cost and finery

upon this small speck of dirt. So, like travellers into other distant countries, we shall be better able to judge of what is done at home, know how to make a true estimate of, and set its own value upon, every thing: we shall be less apt to admire what this world calls great, and shall nobly despise those trifles the generality of men set their affections upon." Surely, if any thing can eradicate the sentiments of self-conceit, and instil a principle of humility, we may hope that this desirable end will be effected, at least in a considerable degree, by recollecting that an individual is only one among the millions of intelligent agents who inhabit one among the many millions of habitable worlds; one who, to an eye that can take in the whole of creation, is but as an insect existing upon an atom of earth. Can the baneful passions of envy and avarice be fostered in the breast of him who is convinced that even the wealth of the Indies is as nothing, when set in competition with the riches of the universe? Can he who has frequently contemplated the splendour of the sun, the silent majesty of the moon, and the combined lustre and glory of the starry worlds, give way to ambition, or evince an eagerness to attain the shadowy distinctions of human grandeur? or can he who has adopted the persuasion that worlds are innumerable and their inhabitants infinite, when he reflects upon the blessings and benefits which fall to the share of one minute object in the immense catalogue, do otherwise than repress all repining, to make room for the swelling sensations of gratitude? But gratitude, it must be acknowledged, implies an object to whom it is due; we therefore proceed to enquire, whether the study of astronomy does not furnish us with the most satisfactory proofs of the existence of a Being to whom gratitude is justly due for all we have enjoyed, or ever can expect to enjoy.

703. If an ancient heathen poet, who dealt in fic-

tion, was so convinced of the dependency of the works of Nature and of all second causes, upon some First Cause, as to make the inference that "*The highest link of Nature's chain must be attached to the foot of Jupiter's throne*";* it is not unreasonable to expect that a modern enquirer after truth, with present advantages, will find ample grounds for believing that there exists, distinct from the universe, a supreme intelligent First Cause. The arguments generally adduced to prove that there is a God, are distributed into two kinds: arguments *à priori*, or those taken from the necessity of the divine existence; and arguments *à posteriori*, or those taken from the works of Nature. The first kind fall not peculiarly within the compass of this undertaking; nor, if they did, would they be exclusively employed, as the proof resulting from them is not always of the most satisfactory nature: the second kind, on the contrary, afford such evidence of the being of a God, as cannot well be doubted by any who are at all swayed by the force of truth. And when the existence of this great Being is shewn, his attributes may be inferred from a union of both these kinds of arguments, which forms a *mixed* proof. It is manifest that the universe must either be eternal, or brought into existence by chance, or formed by the power and wisdom of a supreme intelligent Being. Now, if the universe be *eternal*, since nothing could exist prior to it, it must be *self-existent*; and if self-existent, it must be *immutable*: for since a being is the same with all its properties taken together, if any property were taken away from it, a part of the being would perish, which is inconsistent with its being necessary; or, if any properties or parts were added, the being itself would not be eternal, and therefore not necessarily existent. But the universe has, we

* *Homer*. See *Iliad*, Book IX.

are certain, undergone considerable changes: many stars of great magnitude which were formerly visible, are now no-where to be found; and many stars are now visible which have every appearance of new ones: besides this, the earth we inhabit, as appears from the most authentic histories, has undergone material changes; and from our own observations we may assert, that it is incessantly changing: hence, then, we infer that the universe is not *self-existent*, and, consequently, not *eternal*. This being admitted, it follows that the universe is either the offspring of chance, or the result of design. A slight survey of a few parts of the universe will compel us to acknowledge that there exists in nature a certain constant order, which is totally irreconcileable with all our ideas of chance: for instance, all the planets and satellites in our system have, ever since accurate observations have been made upon them, been found to move in conformity to a regular established proportion between their distances from the sun or their respective primaries, and the times of their revolutions: they are always found to revolve about their respective centres of force, with such a variable velocity, as to describe equal trilineal areas in equal times, whatever their distances from their central bodies may be: all those celestial bodies upon which sufficient observations have been made (and probably all the rest), are of a spheroidical form: the constant return of the day and night on the earth, and each of the planets, is occasioned, not by the sun being carried round the planet, but by the rotation of the planet upon its axis: the cause of the variety of seasons is very simple, for it is occasioned by the axis of the planet being inclined to the plane of its orbit; and the regular return of the seasons is maintained by the continuance of the planet's axis in a situation very nearly parallel to itself: the innumer-

able multitude of the fixed stars seem, as far as we can determine, to be of the same nature, and each calculated to answer similar purposes. Is it possible to conceive that such marks of order, such invariable laws, such general relations, as are here mentioned, and many others which must suggest themselves to every intelligent mind, should spring out of confusion, or that such beauty and harmony should be produced by the fortuitous concourse of atoms? "As well, nay better, and more easily, might we suppose, that an earthquake might happen to build towns and cities; or the materials carried down by a flood, fit themselves up without hands to make a regular fleet. For what are towns, cities, or fleets, in comparison of the vast and amazing fabric of the universe!" The contemplation of these various relations, of these unequivocal marks of power and of design, must lead to the persuasion of an intelligent First Cause: for the greater number and variety of parts there are in a whole, all concurring to a common end, the greater is the probability that the whole is not the result of a blind chance: since we must be satisfied that motion is not essential to matter, we can place neither in matter nor in motion the efficient reason of what exists; because assigning the efficient reason of a thing is not simply giving a cause to that thing, it is assigning a principle by which we may clearly conceive why a thing is, and for what reason it is *as* it is, and not otherwise. Now it is only in an intelligent self-existent cause, that we find sufficient reason for the mode of being of the universe; and it is only in the power of the First Cause, that we find the efficient reason of the existence or actuality of the universe: we therefore conclude that there is a separate invisible Being who formed all worlds, and this great Being we distinguish by the appellation—God.

704. The preceding reasoning is so strongly confirmed by Dr. *Young* in his *Night Thoughts*, that we cannot forbear extracting a passage:

" Whence earth and these bright orbs?—Eternal?
" Grant matter was eternal; still these orbs
" Would want some other father;—much design
" Is seen in all their motions, all their makes;
" Design implies intelligence and art:
" That can't be from themselves—or man: That art
" Man scarce can comprehend, could man bestow?
" And nothing greater yet allow'd than man.—
" Who, motion, foreign to the smallest grain,
" Shot through vast masses of enormous weight?
" Who bid brute matter's restive lump assume
" Such various forms, and gave it wings to fly?
" Has matter innate motion? then each atom
" Asserting its indisputable right
" To dance, would form an universe of dust:
" Has matter none? then whence these glorious forms,
" And boundless flights, from shapeless and repos'd?
" Has matter more than motion? has it thought,
" Judgment, and genius? is it deeply learn'd
" In mathematics? has it fram'd such laws,
" Which, but to guess, a NEWTON made immortal?
" If so, how each sage atom laughs at me,
" Who think a clod inferiour to a man!
" If art to form; and counsel, to conduct;
" And that with greater far than human skill,
" Resides not in each block; a GODHEAD reigns."

705. If, from the arguments which have been adduced, we are constrained to admit the being of a God, we must also allow his chief attributes; for they are manifestly deducible from his existence, or his workmanship. Thus, as to his *eternity*: if God made all things, he could not receive being from another; and, as a being, cannot spring up from absolute nothing into existence; and besides, as something must of necessity be eternal, otherwise nothing could have been at all; therefore the Maker of the World must be eternal. His *omnipotence* appears from creation itself; for no effect can be assigned so great, but he is able to produce it: it is impossible to

conceive of any power greater than that which created the universe; and, besides this, there is no external power to limit the operations of the Divine Being; he must, therefore, be omnipotent. That he is *immortal* and *immutable* obviously follows: for, since neither his existence, nor any thing pertaining to it, depends upon another being, we cannot imagine that he depends on any other being for his continuance or conservation: and, as he has power superiour to all things, having conferred upon them whatever power they possess, nothing can oppose him, or make any prevalent impression upon him, so as to destroy or change any thing in his nature. The *perfect knowledge* of God appears from considering, that, since he made all creatures, he must know all that relates to them; and he must, by consciousness, know himself; he therefore knows all things. He is also possessed of the *highest wisdom*: for we cannot possibly conceive of any more exquisite degrees of wisdom than are displayed in every part of the universe; we see a most astonishing subordination of means to ends; and the more man becomes enlightened, the more traces he discovers of wisdom and contrivance; which are exhibited, either in the not multiplying means without occasion, or in the choice of the best possible means to arrive at the best possible end. The *unity* of God will be manifest from the consideration, that if there were any other self-existent being besides him, he must, in all respects, be equal to him; and this would, in effect, limit the omnipotence of God; for if the volitions of the two should in any respect contradict each other, which is, at least, *possible*, the one would be a restraint upon the other, and so neither would be omnipotent. Or, indeed, there is no reason, from the light of nature, to conclude, or even to *imagine*, that there are more Deities than one; for such an hypothesis would not solve any appearances, or remove any difficulties, but what

may be solved or removed upon the belief of one God:—one *almighty* and *alwise* Being, is able to perform as stupendous and as perfect things, as could be performed by a thousand such beings, if we could permit ourselves to conceive of so many. The unbounded *glory* and *majesty* of God, must also be acknowledged; for what can be so glorious, what so majestic, as he who is the authour of all that is glorious, as the great Being who created every thing that is majestic? And as to the *goodness* of God, it is loudly proclaimed in the admirable laws established—to prevent the celestial bodies from falling upon each other, and thus to avert the destruction of their inhabitants;—to maintain the regular returns of the seasons, and thus contribute so effectually to the comfort, the nourishment, and the delight of human creatures;—and, in short, to prevent all that would be ultimately prejudicial, and promote every thing which in reality tends to man's welfare and final good. If we reflect upon the excellency of his nature, the unlimited extent of his wisdom and power, the abundance of his goodness; and consider that these, and all his attributes, are continually * exerted

* The authour is well aware that some gentlemen of very considerable talents and genius, are unwilling to admit the necessity of the continual superintendence of the Deity: he, therefore, thinks it incumbent upon him to assign one of the most forcible of those arguments which have induced him to embrace an opinion that differs from the opinion of gentlemen to whose judgment he is always inclined to pay the utmost deference. It is demonstrable, from the principles of mechanics, that *from all action of body upon body, motion is still impaired, and the quantity of it constantly decaying in the universe.* (See *Newton's Optics*, p. 373. 375.) Now, since matter cannot re-excite the motion in itself; it follows that, as an immaterial power first impressed motion on matter, so it still re-produces the motion lost, and makes up the decays sustained. This argument is urged with great force and ability in *Andrew Baxter's Enquiry into the Nature of the Human Soul*, and concluded by the following observation: "I wish men would examine this argument on all sides; and if they cannot

in preserving every part of that immense universe, to which he gave existence; we cannot deny his rightful title to *supreme dominion:* but must immediately acknowledge, that all adoration, obedience, and veneration, are justly due to so great and glorious a Being.

706. To pursue the subject in the language of *Sir Isaac Newton:* "This Being governs all things, not as the Soul of the World, but as *Lord over all;* and, on account of his dominion, he is wont to be called Lord God, or Universal Ruler. For God is a relative word, and has respect to servants; and Deity is the dominion of God, not over his own body, as those imagine who fancy him to be the Soul of the World, but over servants. The supreme God is a Being eternal, infinite, and absolutely perfect; but a being, *however perfect, without dominion, cannot be said to be Lord God:* for we may say my God, your God, the God of Israel, the God of Gods; my Eternal, your Eternal, the Eternal of Israel, the Eternal of Gods: but we do not say my Infinite, or

find a fault in it, that they would embrace the consequences of it with a philosophical ingenuity. I am of opinion, and think it would be easy to shew it, if one had leisure to run through the several particulars, that unless an *immaterial power* continually re-excited motion in the material universe, all motion would stop in it *in a very short time,* perhaps in less than half an hour, except that the planets would run out in streight-lined directions. And then what a face of things should we have! Darkness, silence, and unsuccessive rest. For certainly matter of itself *can pursue no end, obey no law, nor change the direction* of its motion, nor any way *alter its present state:* and since in all motions in the universe *an end is pursued, a law obeyed, the direction changed;* and since in all the remarkable phenomena of nature there is a circulation of motion; the truth of what I here suggest, must be pretty obvious to any thinking person." See vol. I. p. 88.

Having thus referred to Mr. *Baxter*'s book, the authour cannot help remarking, that the first section of it contains the most ingenious and satisfactory demonstration of the *vis inertiæ* of matter he recollects having ever met with.

my Perfect; these are titles which have no respect to servants. The word God usually signifies Lord; but every Lord is not a God. *It is the dominion of a spiritual Being which constitutes a God*; a true, supreme, or imaginary dominion, makes a true, supreme, or imaginary God. And from his *true dominion* it follows that *the true God is a living, intelligent, and powerful Being*; and from his other perfections, that he is *supreme, or most perfect*. He is eternal and infinite, omnipotent and omniscient; that is, *his duration reaches from eternity to eternity, his presence from infinity to infinity; he governs all things, and knows all things that are or can be done.* He is not eternity or infinity, but eternal and infinite; he is not duration or space, but he endures and is present. He endures for ever, and is every-where present; and by existing always and every-where, constitutes duration and space. Since every particle of space is always, and every indivisible moment of duration is every-where, certainly the Maker and Lord of all things cannot be never and no-where. Every soul that has perception is, though in different times, and in different organs of sense and motion, still the same indivisible person. There are given, successive, parts in duration, and co-existent parts in space; but neither the one nor the other in the person of a man, or his thinking principle; and much less can they be found in the thinking substance of God. Every man so far as he is a thing that has perception, is one and the same man during his whole life, in all and each of his organs of sense. *God is one and the same God, always and every-where.* He is omnipresent, not virtually only, but also substantially; for virtue cannot subsist without substance. In him are all things contained and moved; yet neither affects the other: God suffers nothing from the motion of bodies; bodies find no resistance from the omnipresence of God. It is allowed by all, that the supreme

God exists necessarily, and by the same necessity he exists always and every-where. Hence also he is all similar, all eye, all ear, all brain, all arm, all power to perceive, to understand, and to act; but in a manner not at all human, in a manner not at all corporeal, in a manner utterly unknown to us. As a blind man has no idea of colours, so have we no idea of the manner by which the allwise God perceives and understands all things. He is utterly void of all body and bodily figure, and can therefore neither be seen, nor heard, nor touched; nor ought he to be worshipped under the representation of any corporeal thing. We have ideas of his attributes, but what the real substance of any thing is, we know not. In bodies we see only their figures and colours, we hear only the sounds, we touch only their outward surfaces, we smell only the odours, and taste the savours, but their inward substances are not to be known, either by our senses, or by any reflex act of our minds; *much less, then, have we any idea of the substance of God.* We know him only by his properties and attributes, by his most wise and excellent contrivances of things, and by final causes; we *admire* him for his *perfections*, but we *reverence* and *adore* him on account of *his dominion.* For we adore him as his servants; and *a God without dominion, providence, and final causes, is nothing else than fate and nature.*"

707. Before we conclude, we cannot help taking notice of one circumstance which appears to have been designed by the Authour of Nature: he has so constituted things, that it is impossible for the inhabitants of the earth to have any communication with any other bodies in the solar system; nor, on the supposition that those other bodies are inhabited with creatures whose faculties and perceptions are not much superiour to ours, can they have any communication with this or any other planet: in like manner, all

communication between system and system, appears to be totally precluded. Even in respect of the body with which the earth is most nearly connected, namely, the moon, almost all that we know is little more than conjecture, which telescopic observations have induced us to indulge: we conclude without hesitation, that the moon has mountains and volcanoes, valleys and seas, and there is much reason to suppose it is surrounded with an atmosphere; nay, farther than this the analogy between this body and the earth might be drawn: but whether the analogy is rendered complete, by the moon being the habitation of rational creatures, is an enquiry, respecting which we cannot attempt to advance beyond the bounds of probable conjecture, at least in the present state of our existence. So also, with regard to jupiter and saturn, and in some measure to all the planets, we observe much that raises an ardent curiosity, which cannot here be gratified: What, for instance, are the *belts* upon the body of jupiter? What is the *ring* of saturn? What beneficial consequences do they produce? If there are any inhabitants upon these planets, how are they affected by these *belts*, or this *ring*? To such questions we can receive no satisfactory solution. So that, after all the inventions of men of genius, the investigations of men of science, and the reiterated experiments and enquiries of men of industry and skill, our knowledge is very limited and imperfect. Yet there exists in the human mind an earnest desire to make farther advances in knowledge, to enlarge the sphere of understanding: either, then, this desire is planted in the mind in order to be frustrated, and the most exalted part of the creation is always to be imperfect and defective, or we may look forward to a future state, as a sequel to the present, where our desires will be completely gratified, and the beautiful scheme of nature fully unfolded and explained in every part. The latter is most consistent with all our

ideas of the wisdom and goodness of the supreme Being: we must therefore consider our present state as only the dawn of our existence, a state of preparation for something far more exalted and grand than any thing the eye hath beheld, or the most lofty imagination hath conceived. We may, however, even now, form extensive and noble conceptions of the immensity of the universe, and of the unbounded power and wisdom of the Almighty Creator; and we may look forward to the happy period when we shall drink full draughts from the fountain-head of knowledge; rejoicing, above all things, that our hope rests not upon ingenious speculations, but that "*God hath brought life and immortality to light* "*through the Gospel.*"

TABLE I.

Sun's Longitude for the Noon of every Day in the Year 1802.

	JAN.			FEB.			MARCH			APRIL		
	♑			♒			♓			♈		
	°	′	″	°	′	″	°	′	″	°	′	″
1	10	26	59	12	0	15	10	14	48	11	3	8
2	11	28	12	13	1	7	11	14	56	12	2	14
3	12	29	25	14	1	59	12	15	3	13	1	19
4	13	30	38	15	2	49	13	15	8	14	0	21
5	14	31	50	16	3	37	14	15	11	14	59	22
6	15	33	2	17	4	24	15	15	11	15	58	20
7	16	34	14	18	5	10	16	15	10	16	57	15
8	17	35	25	19	5	54	17	15	7	17	56	9
9	18	36	36	20	6	36	18	15	1	18	55	0
10	19	37	46	21	7	16	19	14	53	19	53	49
11	20	38	55	22	7	55	20	14	43	20	52	35
12	21	40	3	23	8	32	21	14	31	21	51	19
13	22	41	11	24	9	7	22	14	16	22	50	1
14	23	42	18	25	9	40	23	13	59	23	48	40
15	24	43	23	26	10	11	24	13	40	24	47	18
16	25	44	28	27	10	41	25	13	18	25	45	53
17	26	45	33	28	11	9	26	12	54	26	44	26
18	27	46	36	29	11	35	27	12	28	27	42	58
19	28	47	39	♓	12	0	28	11	59	28	41	27
20	29	48	41	1	12	23	29	11	29	29	39	55
21	♒	49	42	2	12	45	♈	10	57	♉	38	21
22	1	50	43	3	13	5	1	10	23	1	36	45
23	2	51	43	4	13	24	2	9	48	2	35	8
24	3	52	43	5	13	42	3	9	10	3	33	30
25	4	53	42	6	13	58	4	8	31	4	31	50
26	5	54	40	7	14	12	5	7	50	5	30	9
27	6	55	37	8	14	26	6	7	7	6	28	26
28	7	56	35	9	14	38	7	6	23	7	26	41
29	8	57	31				8	5	37	8	24	56
30	9	58	26				9	4	49	9	23	8
31	10	59	21				10	3	59			

TABLE I. *(Continued.)*

	MAY			JUNE			JULY			AUG.		
	♉			♊			♋			♌		
	°	′	″	°	′	″	°	′	″	°	′	″
1	10	21	19	10	12	2	8	50	40	8	26	19
2	11	19	29	11	9	31	9	47	54	9	23	47
3	12	17	37	12	6	58	10	45	7	10	21	16
4	13	15	43	13	4	24	11	42	21	11	18	45
5	14	13	47	14	1	49	12	39	34	12	16	16
6	15	11	49	14	59	12	13	36	47	13	13	46
7	16	9	50	15	56	35	14	33	59	14	11	18
8	17	7	48	16	53	56	15	31	12	15	8	50
9	18	5	45	17	51	17	16	28	24	16	6	23
10	19	3	40	18	48	36	17	25	36	17	3	57
11	20	1	33	19	45	55	18	22	48	18	1	32
12	20	59	24	20	43	12	19	20	1	18	59	8
13	21	57	13	21	40	29	20	17	13	19	56	45
14	22	55	1	22	37	44	21	14	25	20	54	23
15	23	52	47	23	34	59	22	11	38	21	52	3
16	24	50	32	24	32	14	23	8	52	22	49	44
17	25	48	15	25	29	28	24	6	6	23	47	26
18	26	45	56	26	26	42	25	3	20	24	45	11
19	27	43	37	27	23	56	26	0	35	25	42	57
20	28	41	17	28	21	10	26	57	51	26	40	44
21	29	38	55	29	18	23	27	55	8	27	38	34
22	♊	36	33	♋	15	36	28	52	26	28	36	25
23	1	34	9	1	12	50	29	49	45	29	34	18
24	2	31	45	2	10	4	♌	47	5	♍	32	13
25	3	29	20	3	7	17	1	44	26	1	30	10
26	4	26	54	4	4	31	2	41	48	2	28	9
27	5	24	28	5	1	45	3	39	11	3	26	9
28	6	22	1	5	58	59	4	36	35	4	24	11
29	7	19	32	6	56	13	5	34	0	5	22	14
30	8	17	3	7	53	27	6	31	25	6	20	19
31	9	14	33				7	28	52	7	18	26

TABLE I. *(Continued.)*

	SEPT.			OCT.			NOV.			DEC.		
	♍			♎			♏			♐		
	°	′	″	°	′	″	°	′	″	°	′	″
1	8	16	34	7	34	17	8	22	17	8	38	32
2	9	14	43	8	33	24	9	22	26	9	39	27
3	10	12	54	9	32	33	10	22	36	10	40	22
4	11	11	6	10	31	43	11	22	48	11	41	18
5	12	9	20	11	30	55	12	23	1	12	42	14
6	13	7	35	12	30	9	13	23	16	13	43	12
7	14	5	51	13	29	24	14	23	32	14	44	10
8	15	4	9	14	28	41	15	23	50	15	45	8
9	16	2	29	15	28	1	16	24	9	16	46	8
10	17	0	50	16	27	22	17	24	30	17	47	9
11	17	59	13	17	26	45	18	24	53	18	48	10
12	18	57	38	18	26	10	19	25	17	19	49	12
13	19	56	4	19	25	37	20	25	44	20	50	15
14	20	54	33	20	25	6	21	26	12	21	51	19
15	21	53	3	21	24	38	22	26	42	22	52	25
16	22	51	36	22	24	12	23	27	15	23	53	30
17	23	50	11	23	23	48	24	27	49	24	54	37
18	24	48	48	24	23	27	25	28	24	25	55	45
19	25	47	28	25	23	8	26	29	2	26	56	54
20	26	46	10	26	22	51	27	29	42	27	58	3
21	27	44	54	27	22	37	28	30	23	28	59	13
22	28	43	41	28	22	25	29	31	6	♑	0	24
23	29	42	30	29	22	15	♐	31	51	1	1	36
24	♎	41	21	♏	22	8	1	32	37	2	2	47
25	1	40	14	1	22	2	2	33	24	3	3	59
26	2	39	10	2	21	59	3	34	13	4	5	12
27	3	38	7	3	21	58	4	35	3	5	6	24
28	4	37	7	4	21	58	5	35	54	6	7	36
29	5	36	9	5	22	1	6	36	46	7	8	48
30	6	35	12	6	22	5	7	37	38	8	10	0
31				7	22	10				9	11	12

TABLE II.

The Sun's Declination to every Degree of his Longitude.

Deg.	0 ♈ North 6 ♎ South ⊙ Declin.	1 ♉ North 7 ♏ South ⊙ Declin.	2 ♊ North 8 ♐ South ⊙ Declin.	Deg.
	° ′ ″	° ′ ″	° ′ ″	
0	0 0 0	11 29 5	20 10 25	30
1	0 23 53	11 50 6	20 22 57	29
2	0 47 47	12 10 56	20 35 7	28
3	1 11 39	12 31 34	20 46 55	27
4	1 35 30	12 51 59	20 58 20	26
5	1 59 20	13 12 12	21 9 21	25
6	2 23 8	13 32 12	21 19 59	24
7	2 46 54	13 51 58	21 30 13	23
8	3 10 37	14 11 30	21 40 3	22
9	3 34 17	14 30 48	21 49 29	21
10	3 57 54	14 49 52	21 58 30	20
11	4 21 17	15 8 40	22 7 6	19
12	4 44 57	15 27 13	22 15 17	18
13	5 8 22	15 45 30	22 23 3	17
14	5 31 42	16 3 31	22 30 24	16
15	5 54 57	16 21 16	22 37 18	15
16	6 18 6	16 38 44	22 43 47	14
17	6 41 9	16 55 55	22 49 50	13
18	7 4 6	17 12 48	22 55 27	12
19	7 26 57	17 29 23	23 0 38	11
20	7 49 41	17 45 40	23 5 22	10
21	8 12 17	18 1 38	23 9 39	9
22	8 34 45	18 17 18	23 13 29	8
23	8 57 5	18 32 38	23 16 53	7
24	9 19 17	18 47 38	23 19 50	6
25	9 41 19	19 2 18	23 22 20	5
26	10 3 12	19 16 37	23 24 22	4
27	10 24 56	19 30 35	23 25 57	3
28	10 46 30	19 44 13	23 27 5	2
29	11 7 53	19 57 30	23 27 46	1
30	11 29 5	20 10 25	23 28 0	0
Deg.	⊙ Declin. 11 ♓ South 5 ♍ North	⊙ Declin. 10 ♒ South 4 ♌ North	⊙ Declin. 9 ♑ South 3 ♋ North	Deg.

TABLE III.

Sun's Right Ascension in Time to every Degree of his Longitude.

[Greatest Obliq. Eclip. 23° 28′.]

	♈			♉			♊			♋		
	h	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*
0	0	0	0	1	51	37	3	51	15	6	0	0
1	0	3	40	1	55	27	3	55	25	6	4	22
2	0	7	20	1	59	17	3	59	36	6	8	43
3	0	11	0	2	3	8	4	3	48	6	13	5
4	0	14	41	2	6	59	4	8	0	6	17	26
5	0	18	21	2	10	51	4	12	13	6	21	48
6	0	22	2	2	14	44	4	16	26	6	26	9
7	0	25	42	2	18	37	4	20	40	6	30	30
8	0	29	23	2	22	31	4	24	55	6	34	51
9	0	33	4	2	26	25	4	29	10	6	39	11
10	0	36	45	2	30	20	4	33	26	6	43	31
11	0	40	26	2	34	16	4	37	42	6	47	51
12	0	44	8	2	38	13	4	41	59	6	52	11
13	0	47	50	2	42	10	4	46	16	6	56	31
14	0	51	32	2	46	8	4	50	34	7	0	50
15	0	55	14	2	50	7	4	54	52	7	5	8
16	0	58	5	2	54	7	4	59	10	7	9	26
17	1	2	40	2	58	7	5	3	29	7	13	44
18	1	6	23	3	2	8	5	7	49	7	18	1
19	1	10	7	3	6	9	5	12	9	7	22	18
20	1	13	51	3	10	12	5	16	29	7	26	34
21	1	17	35	3	14	15	5	20	49	7	30	50
22	1	21	20	3	18	19	5	25	9	7	35	5
23	1	25	6	3	22	23	5	29	30	7	39	20
24	1	28	52	3	26	29	5	33	51	7	43	34
25	1	32	38	3	30	35	5	38	12	7	47	47
26	1	36	25	3	34	41	5	42	34	7	52	0
27	1	40	12	3	38	49	5	46	55	7	56	12
28	1	44	0	3	42	57	5	51	17	8	0	24
29	1	47	48	3	47	6	5	55	38	8	4	35
30	1	51	37	3	51	15	6	0	0	8	8	45

TABLE III. *(Continued.)*

	♌	♍	♎	♏
	h m s	h m s	h m s	h m s
0	8 8 45	10 8 23	12 0 0	13 51 37
1	8 12 54	10 12 12	12 3 40	13 55 27
2	8 17 3	10 16 0	12 7 20	13 59 17
3	8 21 11	10 19 48	12 11 0	14 3 8
4	8 25 19	10 23 35	12 14 41	14 6 59
5	8 29 26	10 27 22	12 18 21	14 10 51
6	8 33 31	10 31 8	12 22 2	14 14 44
7	8 37 37	10 34 54	12 25 42	14 18 37
8	8 41 41	10 38 40	12 29 23	14 22 31
9	8 45 45	10 42 25	12 33 4	14 26 25
10	8 49 48	10 46 29	12 36 45	14 30 20
11	8 53 51	10 49 53	12 40 26	14 34 16
12	8 57 52	10 53 37	12 44 8	14 38 13
13	9 1 53	10 57 20	12 47 50	14 42 10
14	9 5 53	11 1 3	12 51 32	14 46 8
15	9 9 53	11 4 46	12 55 14	14 50 7
16	9 13 52	11 8 28	12 58 57	14 54 7
17	9 17 50	11 12 10	13 2 40	14 58 7
18	9 21 47	11 15 52	13 6 23	15 2 8
19	9 25 44	11 19 34	13 10 7	15 6 9
20	9 29 40	11 23 15	13 13 51	15 10 12
21	9 33 35	11 26 56	13 17 35	15 14 15
22	9 37 29	11 30 37	13 21 20	15 18 19
23	9 41 23	11 34 18	13 25 6	15 22 23
24	9 45 16	11 37 58	13 28 52	15 26 29
25	9 49 9	11 41 39	13 32 38	15 30 35
26	9 53 1	11 45 19	13 36 25	15 34 41
27	9 56 52	11 49 0	13 40 12	15 38 49
28	10 0 43	11 52 40	13 44 0	15 42 57
29	10 4 33	11 56 20	13 47 48	15 47 6
30	10 8 23	12 0 0	13 51 37	15 51 15

TABLE III. *(Continued.)*

	♐			♑			♒			♓		
	h	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*
0	15	51	15	18	0	0	20	8	45	22	8	23
1	15	55	25	18	4	22	20	12	54	22	12	12
2	15	59	36	18	8	43	20	17	3	22	16	0
3	16	3	48	18	13	5	20	21	11	22	19	48
4	16	7	0	18	17	26	20	25	19	22	23	35
5	16	12	13	18	21	48	20	29	26	22	27	22
6	16	16	26	18	26	9	20	33	31	22	31	8
7	16	20	40	18	30	30	20	37	37	22	34	54
8	16	24	55	18	34	51	20	41	41	22	38	40
9	16	29	10	18	39	11	20	45	45	22	42	25
10	16	33	26	18	43	31	20	49	48	22	46	9
11	16	37	42	18	47	51	20	53	51	22	49	53
12	16	41	59	18	52	11	20	57	52	22	53	37
13	16	46	16	18	56	31	21	1	53	22	57	20
14	16	50	34	19	0	50	21	5	53	23	1	3
15	16	54	52	19	5	8	21	9	53	23	4	46
16	16	59	10	19	9	26	21	13	52	23	8	28
17	17	3	39	19	13	44	21	17	50	23	12	10
18	17	7	49	19	18	1	21	21	47	23	15	52
19	17	12	9	19	22	18	21	25	44	23	19	34
20	17	16	29	19	26	34	21	29	40	23	23	15
21	17	20	49	19	30	50	21	33	35	23	26	56
22	17	25	9	19	35	5	21	37	29	23	30	37
23	17	29	30	19	39	20	21	41	23	23	34	18
24	17	33	51	19	43	34	21	45	16	23	37	58
25	17	38	12	19	47	47	21	49	9	23	41	39
26	17	42	34	19	52	0	21	53	1	23	45	19
27	17	46	55	19	56	12	21	56	52	23	49	0
28	17	51	17	20	0	24	22	0	43	23	52	40
29	17	55	38	20	4	35	22	4	33	23	56	20
30	18	0	0	20	8	45	22	8	23	24	0	0

TABLE IV.

Sun's apparent Semidiameter and hourly Motion to every ten Degrees of his mean Anomaly.

Signs	0		1		2		3		4		5		
	Sem. Diam.	Ho. Mot	Sem. Diam	Ho. Mot.	Sem. Diam.	Ho. Mot	Sem. Diam	Ho. Mot.	Sem. Diam.	Ho. Mot.	Sem. Diam.	Ho. Mot.	
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
0	15 47	2 23	15 49	2 24	15 55	2 25	16 3	2 28	16 11	2 30	16 17	2 32	30
10	15 47	2 23	15 51	2 24	15 57	2 26	16 5	2 29	16 13	2 31	16 18	2 33	20
20	15 49	2 23	15 52	2 25	16 0	2 27	16 8	2 30	16 15	2 32	16 19	2 33	10
30	15 49	2 24	15 55	2 25	16 3	2 28	16 11	2 30	16 17	2 32	16 19	2 33	0
	11		10		9		8		7		6		Signs

TABLE V.

Sun's mean Parallax for every five Degrees of his Altitude corrected for Refraction.

Altitude . .	° 0	° 5	° 10	° 15	° 20	° 25	° 30	° 35	° 40	° 45	° 50
Parallax . .	″ 8·75	″ 8·71	″ 8·62	″ 8·45	″ 8·22	″ 7·94	″ 7·58	″ 7·17	″ 6·70	″ 6·18	″ 5·62
Altitude . .	° 55	° 60	° 65	° 70	° 75	° 80	° 85	° 90			
Parallax . .	″ 5·02	″ 4·37	″ 3·70	″ 3·00	″ 2·26	″ 1·52	″ 0·76	″ 0·00			

TABLE VI.

Equation of Time for each Degree of the Sun's Longitude.

[Place of Apoge 9° of Cancer.]

	♈ 0	♉ 1	♊ 2	♋ 3	♌ 4	♍ 5
D.	+ —	—	— +	+	+	+ —
	m s	*m s*	*m s*	*m s*	*m s*	*m s*
0	7 36	1 9	3 51	1 13	5 57	2 20
1	7 17	1 23	3 47	1 26	5 59	2 4
2	6 58	1 36	3 41	1 40	6 0	1 48
3	6 39	1 48	3 37	1 55	6 1	1 31
4	6 20	2 0	3 32	2 7	6 1	1 14
5	6 1	2 11	3 26	2 20	6 0	0 56
6	5 42	2 22	3 19	2 33	5 59	0 38
7	5 24	2 32	3 12	2 45	5 57	0 20
8	5 5	2 42	3 4	2 58	5 54	+ 1
9	4 47	2 51	2 56	3 11	5 51	— 18
10	4 28	3 0	2 47	3 23	5 47	0 37
11	4 9	3 8	2 38	3 35	5 42	0 57
12	3 50	3 16	2 29	3 46	5 37	1 17
13	3 32	3 23	2 19	3 58	5 31	1 38
14	3 13	3 30	2 8	4 9	5 24	1 58
15	2 55	3 36	1 57	4 19	5 17	2 19
16	2 37	3 41	1 46	4 29	5 9	2 40
17	2 19	3 46	1 35	4 39	5 1	3 1
18	2 1	3 50	1 23	4 48	4 52	3 22
19	1 43	3 53	1 11	4 57	4 43	3 44
20	1 26	3 56	0 59	5 5	4 33	4 5
21	1 9	3 58	0 46	5 13	4 22	4 26
22	0 52	4 0	0 34	5 20	4 11	4 47
23	0 36	4 1	0 21	5 27	3 50	5 9
24	0 20	4 1	— 8	5 33	3 46	5 30
25	+ 4	4 1	+ 5	5 39	3 33	5 32
26	— 11	4 0	0 19	5 44	3 19	6 13
27	0 26	3 59	0 31	5 48	3 4	6 35
28	0 40	3 57	0 46	5 52	2 50	6 56
29	0 53	3 54	0 59	5 55	2 35	7 17
30	1 9	3 51	1 13	5 57	2 20	7 28

TABLE VI. *(Continued.)*

D.	♎ 6 —	♏ 7 —	♐ 8 —	♑ 9 — +	♒ 10 +	♓ 11 +
	m s	m s	m s	m s	m s	m s
0	7 38	15 31	13 33	1 11	11 28	14 19
1	7 58	15 39	13 17	0 42	11 45	14 13
2	8 19	15 46	13 0	— 12	12 1	14 6
3	8 40	15 52	12 42	+ 17	12 17	13 59
4	9 1	15 57	12 23	0 46	12 32	13 51
5	9 21	16 2	12 4	1 16	12 46	13 43
6	9 41	16 6	11 44	1 45	12 59	13 34
7	10 1	16 9	11 23	2 14	13 12	13 24
7	10 20	16 11	11 1	2 43	13 24	13 14
9	10 39	16 13	10 39	3 11	13 35	13 3
10	10 57	16 13	10 16	3 39	13 45	12 51
11	11 15	16 13	9 53	4 7	13 54	12 39
12	11 33	16 12	9 29	4 35	14 2	12 27
13	11 51	16 10	9 5	5 2	14 9	12 14
14	12 8	16 7	8 40	5 29	14 16	12 0
15	12 25	16 4	8 14	5 56	14 22	11 46
16	12 41	16 0	7 48	6 22	14 27	11 31
17	12 57	15 55	7 22	6 48	14 31	11 16
18	13 12	15 49	6 55	7 13	14 35	11 1
19	13 27	15 42	6 28	7 37	14 38	10 46
20	13 42	15 35	6 0	8 1	14 40	10 30
21	13 56	15 26	5 32	8 24	14 41	10 14
22	14 9	15 17	5 4	8 47	14 42	9 58
23	14 21	15 7	4 36	9 9	14 41	9 41
24	14 33	14 56	4 8	9 31	14 40	9 24
25	14 44	14 44	3 39	9 53	14 39	9 6
26	14 53	14 31	3 10	10 14	14 37	8 48
27	15 5	14 17	2 41	10 34	14 34	8 30
28	15 14	14 3	2 11	10 53	14 30	8 12
29	15 23	13 48	1 41	11 11	14 25	7 54
30	15 31	13 33	1 11	11 28	14 19	7 36

TABLE VII.

The Moon's apparent Diameter to every 10 *Seconds of the Equatoreal Parallax.*

Equat. Par.		☽'s Diam.		Equat. Par.		☽'s Diam.	
'	"	'	"	'	"	'	"
54	0	29	26	58	0	31	37
54	10	29	31	58	10	31	42
54	20	29	37	58	20	31	47
54	30	29	42	58	30	31	53
54	40	29	48	58	40	31	58
54	50	29	53	58	50	32	4
55	0	29	58	59	0	32	9
55	10	30	4	59	10	32	15
55	20	30	9	59	20	32	20
55	30	30	15	59	30	32	26
55	40	30	20	59	40	32	31
55	50	30	26	59	50	32	36
56	0	30	31	60	0	32	42
56	10	30	37	60	10	32	47
56	20	30	42	60	20	32	53
56	30	30	47	60	30	32	58
56	40	30	53	60	40	33	4
56	50	30	58	60	50	33	9
57	0	31	4	61	0	33	15
57	10	31	9	61	10	33	20
57	20	31	15	61	20	33	26
57	30	31	20	61	30	33	31
57	40	31	26	61	40	33	36
57	50	31	31	61	50	33	42
58	0	31	37	62	0	33	47

TABLE VIII.

Mean astronomical Refractions in Altitude.

Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt	Refr.
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	°	′ ″	°	″	°	″
0 0	33 0	1 55	19 0	4 10	11 29	8 0	6 29	13 0	4 3	25	2 2	48	51	71	19
0 5	32 10	2 0	18 35	4 20	11 8	8 10	6 22	13 20	3 57	26	1 56	49	49	72	18
0 10	31 22	2 5	18 11	4 30	10 48	8 20	6 15	13 40	3 51	27	1 51	50	48	73	17
0 15	30 35	2 10	17 48	4 40	10 29	8 30	6 8	14 0	3 45	28	1 47	51	46	74	16
0 20	29 50	2 15	17 26	4 50	10 11	8 40	6 1	14 20	3 40	29	1 42	52	44	75	15
0 25	29 6	2 20	17 4	5 0	9 54	8 50	5 55	14 40	3 35	30	1 38	53	43	76	14
0 30	28 22	2 25	16 44	5 10	9 38	9 0	5 48	15 0	3 30	31	1 35	54	41	77	13
0 35	27 41	2 30	16 24	5 20	9 23	9 10	5 42	15 30	3 24	32	1 31	55	40	78	12
0 40	27 0	2 35	16 4	5 30	9 8	9 20	5 36	16 0	3 17	33	1 28	56	38	79	11
0 45	26 20	2 40	15 45	5 40	8 54	9 30	5 31	16 30	3 10	34	1 24	57	37	80	10
0 50	25 42	2 45	15 27	5 50	8 41	9 40	5 25	17 0	3 4	35	1 21	58	35	81	9
0 55	25 5	2 50	15 9	6 0	8 28	9 50	5 20	17 30	2 59	36	1 18	59	34	82	8
1 0	24 29	2 55	14 52	6 10	8 15	10 0	5 15	18 0	2 54	37	1 16	60	33	83	7
1 5	23 54	3 0	14 36	6 20	8 3	10 15	5 7	18 30	2 49	38	1 13	61	31	84	6
1 10	23 20	3 5	14 20	6 30	7 51	10 30	5 0	19 0	2 45	39	1 10	62	30	85	5
1 15	22 47	3 10	14 4	6 40	7 40	10 45	4 53	19 30	2 39	40	1 8	63	29	86	4
1 20	22 15	3 15	13 49	6 50	7 30	11 0	4 47	20 0	2 35	41	1 5	64	28	87	3
1 25	21 44	3 20	13 34	7 0	7 20	11 15	4 40	20 30	2 31	42	1 3	65	26	88	2
1 30	21 15	3 25	13 20	7 10	7 11	11 30	4 34	21 0	2 27	43	1 1	[illegible]6	25	89	1
1 35	20 46	3 30	13 6	7 20	7 2	11 45	4 29	21 30	2 24	44	0 59	67	24	90	0
1 40	20 18	3 40	12 40	7 30	6 53	12 0	4 23	22 0	2 20	45	0 57	68	23		
1 45	19 51	3 50	12 15	7 40	6 45	12 20	4 16	23 0	2 14	46	0 55	69	22		
1 50	19 25	4 0	11 51	7 50	6 37	12 40	4 9	24 0	2 7	47	0 53	70	21		

TABLE IX.

The Magnitudes, Right Ascensions, and Declinations, of Sixty-four fixed Stars, for the Beginning of the Year 1800, with their annual Variations.

Names and Characters of the Stars.	Magnitude.	Right Ascens. in Time.	An. Var. in do.	Right Ascens. in Degrees.	An. Var. in do.	Declination North and South.	An. Var. in do.
		h m s	s +	° ′ ″	″ +	° ′ ″	″
γ Pegasi	2	0 2 56·79	3·063	0 44 11·85	45·95	14 4 10 N.	+ 20·1
β Ceti	2·3	0 33 31·83	3·001	8 22 57·40	45·01	19 4 50 S.	— 19·8
Stella Polaris	2	0 52 20	12·53	13 5 0· 0	187·9	88 14 25 N.	+ 19·6
Mirach	2	0 58 34·23	3·297	14 38 33·38	49·46	34 33 29 N.	+ 19·5
Almach	2	1 51 41·05	3·615	27 55 15·76	54·23	41 22 0 N.	+ 17·7
α Arietis	2	1 55 55·27	3·335	28 58 49·05	50·02	22 31 0 N.	— 17·6
ο Ceti (Variab.)	2	2 9 14·70	3·019	32 18 40·50	45·29	3 54 2 S.	— 17·1
Menkar	2	2 51 50·07	3·119	42 57 31·06	46·66	3 18 8 N.	+ 15·2
Algol	2·3	2 55 12·07	3·846	43 48 1·04	57·69	40 10 56 N.	+ 14·6
Algenib	2	3 10 6·85	4·203	47 31 42·77	63·05	49 8 15 N.	+ 13·6
η Lucida Plei	3	3 35 17·17	3·555	53 54 17·56	53·03	23 29 4 N.	+ 11·9
γ Eridani	2·3	3 48 42·25	2·786	57 10 33·77	41·79	14 5 0 S.	
Aldebaran	1	4 24 27·29	3·421	66 6 49·38	51·31	16 6 4 N.	+ 8·0
Capella	1	5 1 56·16	4·414	75 29 2·40	66·21	45 46 50 N.	+ 5·2
Rigel	1	5 4 55·54	2·867	76 13 53·10	43·01	8 26 54 S.	— 5·1
β Tauri	2	5 13 39·38	3·778	78 24 50·70	56·67	28 26 0 N.	+ 4·1
γ Orionis	2	5 14 24·54	3·209	78 36 8·16	48·13	6 9 21 N.	+ 4·0
δ Orionis	2	5 21 47·38	3·057	80 26 53·69	45·86	0 28 4 S.	— 3·5
ε Orionis	2	5 26 4·15	3·037	81 31 2·19	45·55	1 20 27 S.	— 3·0
ζ Orionis	2	5 30 40·57	3·020	82 40 8·57	45·30	2 4 0 S.	
α Columbæ	2	5 32 20·83	2·167	83 6 15·45	32·50	34 11 6 S.	— 2·5

TABLE IX. (*Continued.*)—*Most remarkable fixed Stars.*

Names and Characters of the Stars.	Magnitude.	Right Ascens. in Time.	An. Var. in do.	Right Ascens. in Degrees.	An. Var in do.	Declination North and South.	An. Var. in do.
		h m s	s +	° ′ ″	″ +	° ′ ″	″
Betelguese	1	5 44 20·57	3·239	86 5 8·55	48·59	7 21 30 N.	+ 1·7
β Aurigæ	2·3	5 44 51·68	4·398	86 12 55·22	65·97	44 55 0 N.	
β Canis Majoris	2·3	6 13 53·76	2·638	93 28 26·33	39·57	17 52 0 S.	
γ Geminorum	2·3	6 26 9·35	3·463	96 32 20·32	51·95	16 34 0 N.	
Sirius	1	6 36 19·91	2·647	99 4 58·65	39 71	16 26 20 S.	+ 4·4
ε Canis Majoris	2·3	6 50 46·21	2·354	102 41 33·20	35·31	28 43 0 S.	
Castor	1·2	7 21 48·81	3·855	110 27 12 15	57·83	32 19 0 N.	— 6·8
Procyon	1·2	7 28 49·10	3·137	112 12 16·50	47·06	5 44 4 N.	— 8·7
Pollux	2	7 33 3·18	3·687	113 15 47·70	55·31	28 30 0 N.	— 7·6
Alphard	2	9 17 44·97	2·935	139 26 14·55	44·03	7 47 48 S.	+ 14·8
Regulus	1	9 57 42·02	3·204	149 25 30·30	48·06	12 56 29 N.	— 16·9
γ² Leonis	2·3	10 8 55·22	3·306	152 13 48·23	49·60	20 51 0 N.	
β Ursæ Majoris	2	10 49 39·53	3·709	162 24 54·93	55·63	57 27 3 N.	— 19·1
α Ursæ Majoris	1 2	10 51 15·84	3·847	162 48 57·61	57·70	62 50 0 N.	— 19·1
δ Leonis	2·3	11 3 26·39	3·199	165 51 35·91	47·98	21 37 0 N.	
Denebola	1·2	11 38 50·49	3·062	174 42 37·35	45·93	15 41 36 N.	— 19·8
γ Ursa Majoris	2	11 43 14·22	3·212	175 48 33·33	48·18	54 48 0 N.	
Allioth	2·3	12 45 12·58	2·746	191 18 8·67	41·19	57 3 0 N.	— 19·7
Spica Virginis	1	13 14 40·11	3·137	198 40 1·66	47·06	10 6 52 S.	+ 18·8
η Ursæ Majoris	2·3	13 39 38·85	2·355	204 54 42·80	35·88	50 19 0 N.	— 18·2
α Draconis	2·3	13 58 58·88	1·628	209 44 43 24	24 42	65 20 8 N.	— 17·4

TABLE IX. *(Concluded.)—Most remarkable fixed Stars.*

Names and Characters of the Stars.	Magnitude.	Right Ascens. in Time.	An. Var. in do.	Right Ascens. in Degrees.	An. Var. in do.	Declination North and South.	An. Var. in do.
		h m s	s +	° ′ ″	′ +	° ′ ″	
Arcturus	1	14 6 32·21	2·722	211 38 3·16	40·83	20 15 0 N.	− 18·4
α^2 Libræ	2·3	14 39 49·97	3·289	219 57 29·55	49·34	15 12 0 S.	+ 15·3
β Libræ	2·3	15 6 15·61	3·215	226 33 54·21	48·22	8 38 3 S.	+ 13·8
γ^2 Ursæ minoris	2·3	15 21 11·76	− 0·209	230 17 56·39	− 3·14	72 33 0 N.	
Gemma	2	15 26 13·29	2·543	231 33 19·35	38·15	27 24 0 N.	− 12·3
α Serpentis	2	15 34 25·21	2·936	233 36 18·00	44·04	7 3 56 N.	− 12·0
β Scorpii	2	15 53 49·71	3·465	238 27 25·65	51·97	19 15 0 S.	
Antares	1	16 17 9·69	3·645	244 17 25·35	54·68	25 58 21 S.	+ 8·6
ζ Ophiuchi	2·3	16 26 9·55	3·287	246 32 23·24	49·30	10 9 0 S.	
η Ophiuchi	2·3	16 58 55·15	3·424	254 43 47·26	51·36	15 28 0 S.	
α Herculis	2·3	17 5 31·76	2·726	256 22 56·40	40·89	14 38 2 N.	
α Ophiuchi	2	17 25 38·97	2·768	261 24 44·55	41·52	12 43 9 N.	− 4·7
γ Draconis	2·3	17 51 57·79	1·389	267 59 26·85	20·83	51 31 5 N.	− 3·0
ε Sagittarii	2·3	18 10 53·67	3·984	272 43 25·11	59·76	34 28 0 S.	− 0·7
Wega	1	18 30 9·89	1·994	277 32 28·35	29·91	38 36 26 N.	+ 3·1
Altair	1·2	19 41 1·02	2·918	295 15 15·30	43·78	8 21 11 N.	+ 9·1
Deneb	1·2	20 34 36·68	2·034	308 39 10·20	30·51	44 34 0 N.	− 12·4
Fomalhaut	1	22 46 33·60	3·330	341 38 24·00	49·95	30 40 56 N.	− 19·1
β Pegasii	2	22 54 5·50	2·874	343 31 22·47	43·11	27 0 0 N.	
Markab	2	22 54 47·99	2·964	343 41 59·85	44·46	14 8 0 N.	+ 19·2
α Andromadæ	2	23 58 4·52	3·065	359 51 4·95	45·97	27 59 34 N.	+ 20·1
β Cassiopeiæ	2·3	23 58 34·32	3·051	359 38 34·75	45·76	58 3 0 N.	

TABLE X.

The Longitudes, Latitudes, and Magnitudes, of the most remarkable fixed Stars that the Moon can Eclipse, or make a near Appulse to; exactly rectified to the Beginning of the Year 1800.

Con.	Cha.	Long.	Lat.	Mag	Con.	Cha.	Long.	Lat.	Mag.
		° ′ ″	° ′ ″				° ′ ″	° ′ ″	
♓	δ	♈ 11 21 34	2 9 44 N.	4	♎	α	♏ 12 17 52	0 21 48 N.	2
	ε	14 44 33	1 5 37 N	4		12	18 13 0	1 49 14 S.	3
	ζ	17 4 48	0 13 11 S.	4		γ	22 20 32	4 24 41 N.	3
♈	δ	♉ 18 3 7	1 48 7 N.	4		η	24 34 56	4 2 52 N.	4
♉	η	27 12 7	4 1 56 N.	3		κ	24 58 10	0 1 1 N.	4
	γ	♊ 3 0 23	5 45 30 S.	3		θ	27 4 35	3 29 24 N.	4
	ε	5 40 0	2 35 37 S.	3		λ	27 41 9	0 6 53 N.	4
	α	6 59 43	5 29 2 S.	1	♏	δ	29 46 51	1 57 17 S.	3
	β	19 47 0	5 21 59 N.	2		π	♐ 0 8 58	5 26 15 S.	3
	ζ	21 59 38	2 13 29 S.	3		β	0 23 56	1 2 18 N.	2
♊	η	♋ 0 39 0	0 55 4 S.	4		ν	1 51 13	1 39 52 N.	4
	μ	2 30 25	0 50 34 S.	3		σ	5 0 36	4 0 23 S.	4
	γ	5 18 43	6 46 12 S.	2		α	6 58 21	4 32 17 S.	1
	ε	7 8 53	2 2 28 N.	3		τ	8 40 5	6 5 21 S.	4
	δ	15 43 52	0 12 19 S.	3	♐	γ	28 28 26	6 56 48 S.	3
	β	20 27 57	6 40 4 N.	1		μ	♑ 0 25 21	2 22 24 N.	4
♋	γ	♌ 4 15 14	3 10 22 N.	4		λ	3 31 54	2 5 31 S.	4
	δ	5 55 32	0 4 13 N	4		φ	7 33 11	3 55 22 S.	3
♌	ξ	18 51 46	3 1 57 S.	4		σ	9 35 40	3 24 55 S.	3
	ο	21 28 1	3 46 1 S.	4		τ	12 2 25	5 2 33 S.	3
	η	25 6 46	4 51 9 N.	4		ο	12 11 59	0 53 36 N.	3
	α	27 3 12	0 27 27 N	1		π	13 27 44	1 28 7 N.	4
	ρ	♍ 3 35 48	0 8 29 N.	4	♑	β	♒ 1 15 18	4 36 46 N.	3
	τ	18 43 10	0 31 21 S.	4		ε	17 24 23	4 57 31 S.	4
	υ	22 14 55	3 2 51 S.	4		γ	18 59 16	2 32 6 S.	4
♍	β	24 19 10	0 41 36 N.	3		δ	20 44 28	2 33 40 S.	3
	ο	♎ 0 34 33	5 4 42 N.	3	♒	ι	25 55 40	2 3 47 S.	4
	η	2 2 39	1 22 24 N	3		θ	♓ 0 28 5	2 43 22 N.	4
	γ	7 23 4	2 48 57 N.	3		λ	8 47 6	0 22 57 S.	4
	α	21 3 13	2 2 11 S.	1		φ	14 20 58	1 2 8 S.	4

EXPLANATION

OF THE

TABLES.

TABLE I. contains the *sun's longitude* for the noon of every day in the year 1802: this year was chosen, because it is the second from leap year, and will, therefore, correspond more nearly to the sun's real longitude for each day of a series of a few years, than a table would which contained the longitudes for every day on the year before or after leap year. If tables of the sun's longitude for four years had been given, they would not answer for any great length of time, because the sum of four years, one of which is a bissextile, does not exactly equal four sidereal years. This table, however, though not perfect, may be useful; for, from the longitudes here given, the sun's right ascension may be deduced sufficiently exact to find the culminating of any star, by subtracting its R. A. from that of the sun; and by these means the star may be known. The longitude for any intermediate time between the noons of two given days may be readily found by proportion; thus, to find the sun's longitude on May 3d, 1802, at 4 o'clock, P.M.:

Long. May 4, noon ♉	13° 53′ 43″	As 24h : 1° 46′ 6″
Long. May 3, noon ♉	12 17 37	: : 4h : 17′ 41″
Difference for 1 day	1 46 6	

Hence 12° 17′ 37″ + 17′ 41″ = 12° 35′ 18″ of ♉, sun's longitude at the given hour.

TABLE II., which contains the *sun's declination* to each degree of his longitude, will be very useful in conjunction with the former. Each of the 2d, 3d, and 4th columns contains the declinations corresponding to every degree of four signs of the sun's longitude; thus, the 2d column shews the declination answering to each degree of aries and libra, by reckoning from the top of the page to the bottom, and the declination to each degree of virgo and pisces, estimating from the bottom to the top: it will also be seen, that the declinations in this column, corresponding to the different points of aries and virgo, are *north*, and those answering to the several points of libra and pisces, *south*. The method of finding the declination for any given time may be shewn by an example; thus, to determine the declination on May 3d, 1802, at 4h P.M.: the longitude we have found to be 12° 35′ 18″ of ♉, and the declination agreeing to 13° of ♉ is 15° 45′ 30″ N. the dec. to 12° of ♉ 15° 27′ 13″, their difference 18′ 17″; then 60′ : 18′ 17″ : : 35′ 18″ : 10′ 13″; wherefore 15° 27′ 13″ + 10′ 13″ = 15° 37′ 26″ north, the declination sought. . . . When the place is not on the meridian of Greenwich, the declination may be found by adding to, or subtracting from, the given time, the time corresponding to the difference of longitude (according to the kind of longitude and declination), and then proceeding as above. Where the differences are not tolerably uniform, and when great accuracy is required, recourse must be had to the method of interpolation.

TABLE III., containing the *sun's right ascensions*, requires no explanation; the R. A.'s for any given time may be determined in the same way as the declinations: if they are wanted in degrees, &c. they may be found by multiplying the tabular R. A.'s by 15.

TABLE IV. contains the *sun's semidiameter* and *hourly motion* for every ten degrees of mean anomaly; they may be readily found for any intermediate degrees by proportion. From the table it appears that the semidiameter of the sun, when it is greatest in perigee, is 16′ 19″, and when least in apogee, 15′ 47″, their difference being 32″; the difference between the greatest hourly motion, when the sun is in perigee, and the least, when he is in apogee, is 10″. The use of this table is greatest in the calculation of eclipses; it may be exemplified, by finding the sun's apparent semidiameter and hourly motion, on Aug. 27, 1802, when there will be a solar eclipse, partly visible at Greenwich. The place of the sun's apogee is nearly 9° of cancer, which answers to the 2d of July (Tab. I); this day is 56 days before the day of the eclipse: the mean anomaly may be found sufficiently near, by saying, as $365^d : 360° :: 56^d : 55\frac{1}{2}°$ $= 1^s\ 25\frac{1}{2}°$: to this we, by proportion, find $15'\ 53\frac{1}{2}''$ the corresponding semidiameter, and $2'\ 25''$ the hourly motion.

TABLE V. exhibits the *sun's mean parallax* for each 5° of apparent altitude: in cases of great accuracy, the parallax for any intermediate altitude may be determined by proportional differences, as in the other tables. The parallax must be *added* to the apparent altitude, to give the true altitude.

TABLE VI. of the *equation of time*, shews that equation for every degree of the sun's longitude, and will answer tolerably nearly for the days when the sun has those longitudes. The equations with the sign + are to be *added* to the apparent time, to have

the mean time; those with the sign —, are to be *subtracted* from the apparent for the mean time.

TABLE VII. shews the *moon's diameter* to every 10″ of the *equatoreal parallax*; by it we may also find the equatoreal parallax to different diameters. Where great precision is wished for, the equatoreal parallax must be reduced to the horizontal parallax in any given latitude, or the contrary. This may be done with all desireable accuracy by the following theorem: put $t = \frac{1}{2}$ the semi-axis major, $c = \frac{1}{2}$ the semi-axis minor, s and r the sine and co-sine of the latitude of the place of observation, and p the sine of the horizontal polar parallax, the equatoreal parallax being always to the polar parallax, as 100438 to 100000; then $\frac{tcp}{\sqrt{t^2 s^2 + c^2 r^2}}$ = the sine of the horizontal parallax at the given latitude. The parallax obtained by this theorem will sometimes be nearly 16″ less than the equatoreal parallax; but this is in *high* latitudes: in the latitude of London, the reduction can never exceed 10″, on the supposition that t is to c, as 230 to 229. . . . See also Art. 553.

TABLE VIII. contains the *mean astronomical refractions*, as calculated by Dr. *Bradley*, according to the theorem given in Art. 105.: the use of this table is so manifest, that it needs no example, the refraction, it is well known, being always subtracted from the apparent, to find the true, altitude. This table shews the refractions in the medium state of the atmosphere, *i. e.* when the barometer stands at 29·6 inches, and Fahrenheit's thermometer at 50 degrees. The refractions agree very nearly with those in Dr. *Maskelyne*'s table, given with his *Observations*, 1796.

TABLE IX. contains the *right ascensions* (both in time and degrees) and the *declinations* of sixty-four of the principal *fixed stars* for the beginning of the year 1800, with their annual variations in right

ascension and declination, where such variations were well ascertained. The greater part of this table was taken from Dr. *Zach*'s catalogue of 381 stars, given in Dr. *Hutton's Dictionary*. Some errours are corrected, and most of the variations in declination supplied, from a comparison of the tables of Dr. *Maskelyne*, *La Caille*, *Mayer*, and others of acknowledged accuracy. When the places of these stars are wanted for any time after the beginning of the year 1800, multiply the annual variation, both in right ascension and declination, by the years and decimals which have elapsed since that time; the product arising from multiplying the R. A. must be added to the R. A. for 1800, for the R. A. sought: but the product arising from multiplying the declination must be added or subtracted, according as the sign + or — is found against the annual variation in the last column; the sum or difference gives the declination. But if the places of the stars be wanted for any time previous to the year 1800, the variation in R. A. must be subtracted from the R. A. found in the table (except in one instance where the variation is —), and the variation in declination must be applied with a contrary sign to that which stands against it. In this table the *mean* places of the stars are exhibited; and, to obtain their apparent places, the mean places should be corrected for aberration, and the equation of equinoctial points; this we merely mention, to shew the reason of the difference between the tabular and the observed places.

TABLE X. shews the *mean longitudes* and the *latitudes* of sixty *stars* for the beginning of the year 1800: their mean longitudes for any other time may be found, if $50\frac{1}{4}$ seconds (Art. 57.) be *added* for each *succeeding*, or *subtracted* for each *preceding*, year, and proportional parts of $50\frac{1}{4}''$ for parts of years. The latitudes vary by such insensible degrees, that the table will answer pretty well for a few years,

without regarding the variations. By means of this table, and a table of the moon's longitude for every twelve hours, we may ascertain how often a given fixed star may be eclipsed by the moon in a given year; for instance, *spica virginis*, or α ♍ in the year 1801: the longitude of the star for 1801 is nearly 21° 4′ of ♎, the latitude 2° 2′ 11″S. Now, on Mar. 30, at 12 P.M. the long. ☽ is 19° 37′ 44″ of ♎, and the lat. ☽ 58′ 14″S. and the long. and lat. are so increasing, that at about 2^h past midnight the moon and star are in conjunction, and the difference of their latitudes is less than 57′; whence (Art. 545.) it appears there will be an occultation about that time. In a similar manner it may be found that there will be occultations of the same star behind the moon on May 24, between eight and ten o'clock, P.M. and on July 18, about noon: either of these occultations may then be calculated, by following the precepts in Art. 546, *et seq.*

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27	*line* 3	*for* Art. 73	*read* Art. 37.
43	6, *from b.*	Zubenich	Zubenisch.
48	10 . . *do.*	his hole	this hole.
69	1	refraction of	refractions at.
102	*in the running title*	Latitude	Altitude.
105	11, *from b.*	Art. 140	Art. 160.
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201	6 . . *do.*		$r+\sqrt{r^2 c^2}+$
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270	3		$\sqrt{r^2+r+1}$
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298 The reasoning which is adopted in this page, to remove the apprehension that the moon, at her conjunction, ought to abandon the earth, in consequence of her greater gravitation towards the sun than towards the earth, may not, perhaps, carry conviction to the mind of every reader; it is, therefore, thought adviseable to insert likewise the very ingenious and satisfactory remarks of Mr. ROBINS on this subject, as given in the Second Volume of his TRACTS, p. 190. "There is," says he, "no proposition in the whole theory of motion more incontestible and obvious, than that the relative motions of any system of bodies will be no ways affected by equal accelerating forces applied in parallel directions to each of the bodies that compose it. Therefore, if to the earth and moon equal degrees of gravitation towards the sun were applied in parallel directions, no change in the relative motion of the moon about the earth would be thence produced, even though the gravitation to the sun were in each of them a thousand times greater than the moon's gravitation to the earth. . . . . Consequently, from the absolute quantity of the moon's gravitation to the sun, no inequalities will arise in her motion, unless that force be different, or differently directed, from the force of the sun on the earth. But the diameter of the moon's orbit is so small, compared with the distance of

the sun, that lines, drawn from any points of that orbit, to the sun, differ but little from being parallel to the line joining the earth and sun, and her gravitation to the sun in the several parts of her orbit is but little different from the gravitation of the earth to the sun. It is *by these differences only*, and not by the whole gravitation of the moon to the sun, that the relative motion of the moon about the earth can be disturbed; and from them arise the inequalities in that motion observed by astronomers."

309 . . *line* 8, *from b. for* $\frac{EC \times E\dot{r}}{Eo}$. *read* $\frac{EC \times Er}{Eo}$

317 Mr. ROBINS, at p. 245, vol. II. of his TRACTS, has shewn how to determine the apparent concavity of the sky in an accurate geometrical manner, by means of the hyperbola; he has also given a concise method of computation, and has recited various opinions on the subject of the horizontal luminaries, between p. 234 and 249 of the same volume, which may be consulted by such as wish to obtain farther information on this point.

352 *line* 11 . . *for* 2·180 . . . *read* 2180.
353 *in the note* . . penumbra pass . . penumbra to pass.
378 3 *b. in the note* to the north . . . to be the north.
389 *line* 30 . . . . *Hippocrates, Chios* . *Hippocrates of Chios.*
396 *bottom line* . . Art. 528. . . . . Art. 588.
417 *in the note* . . . vol. IV. . . . . . vol. VI.
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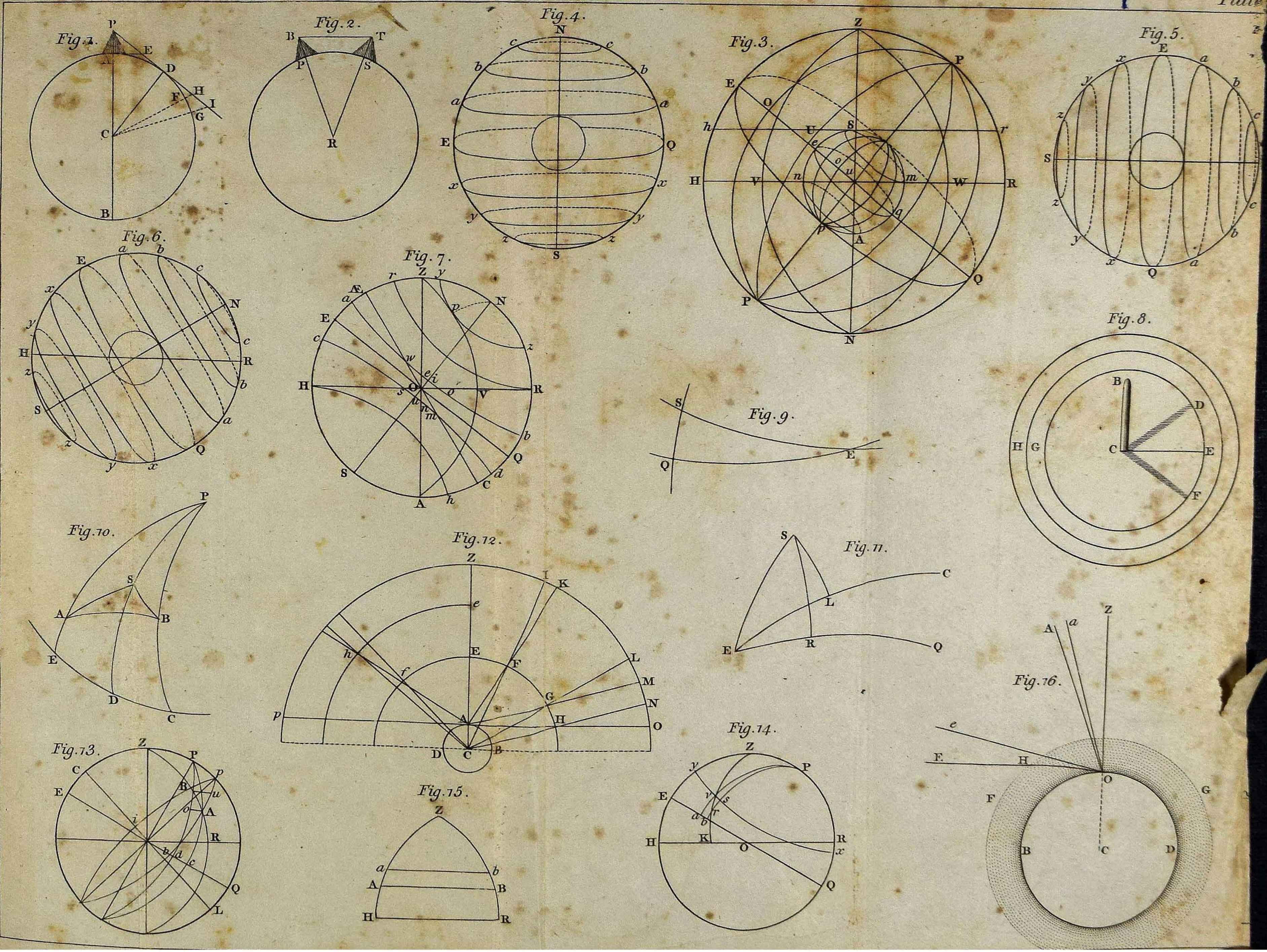
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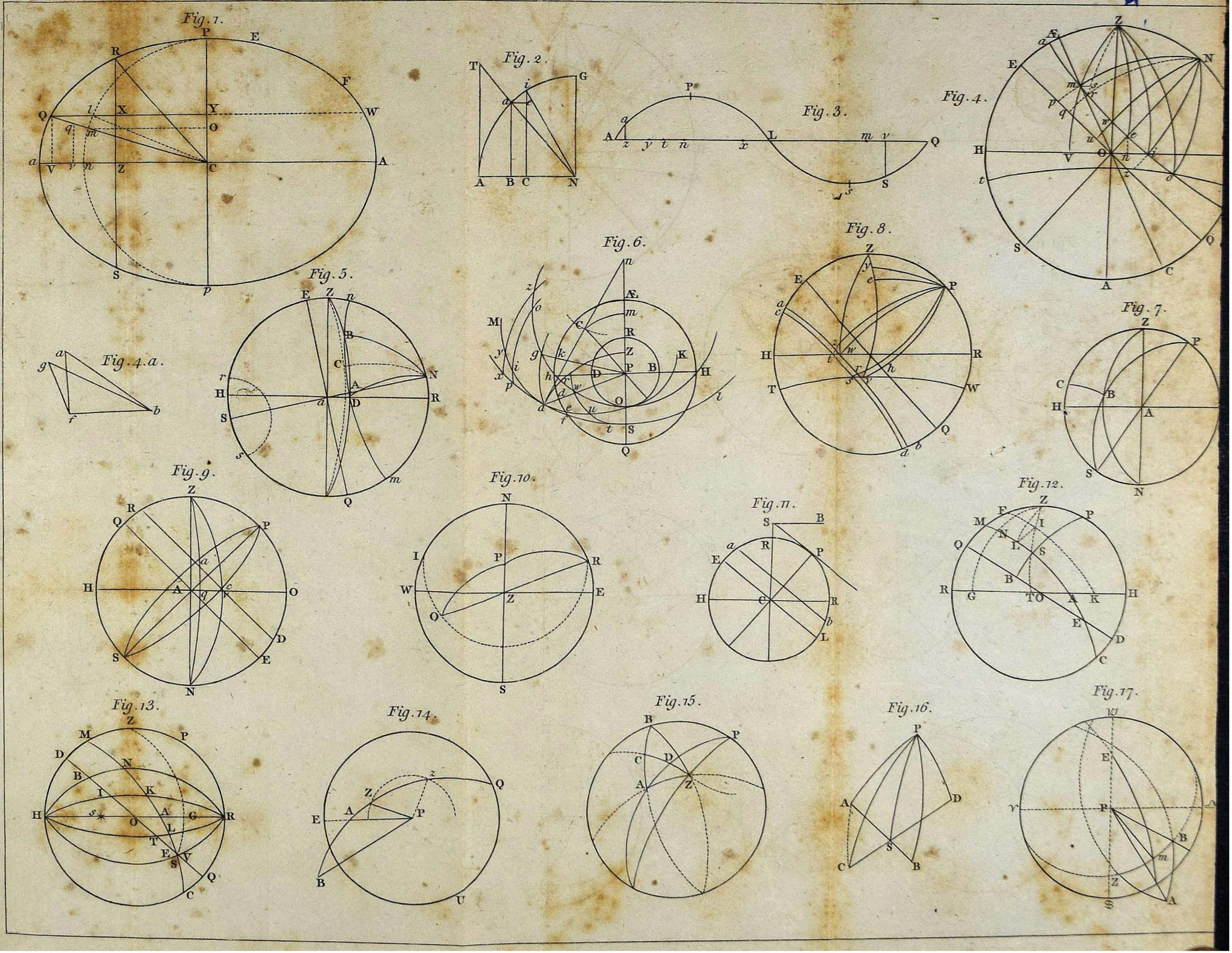
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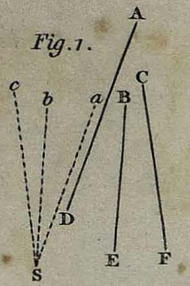

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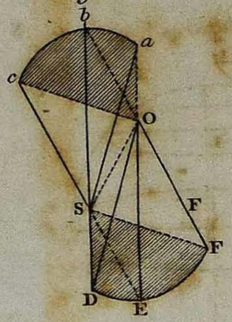

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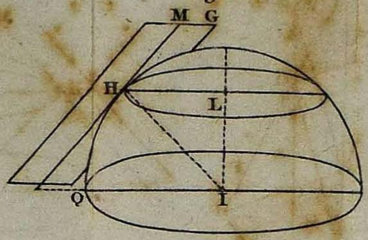

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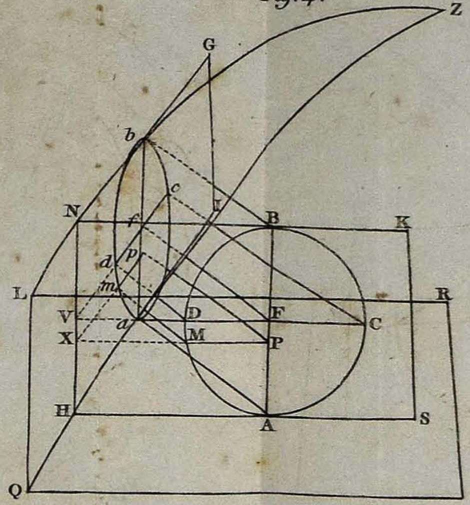

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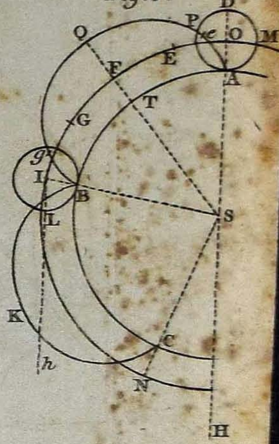

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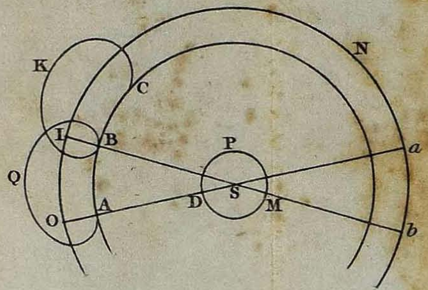

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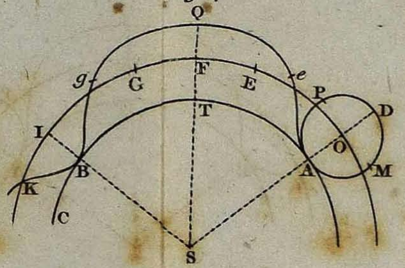

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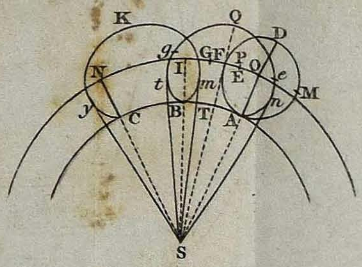

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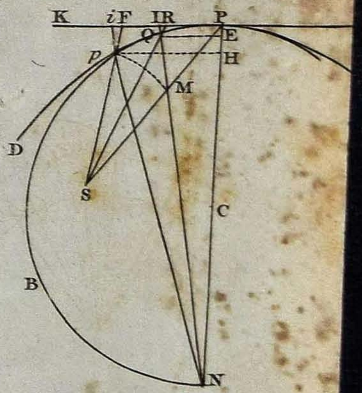

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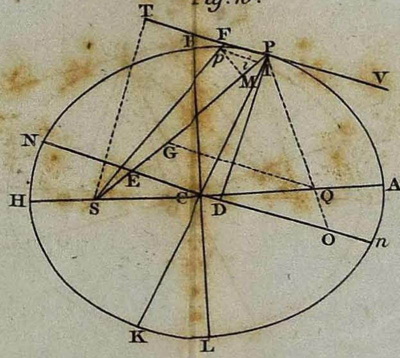

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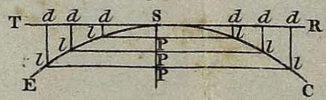

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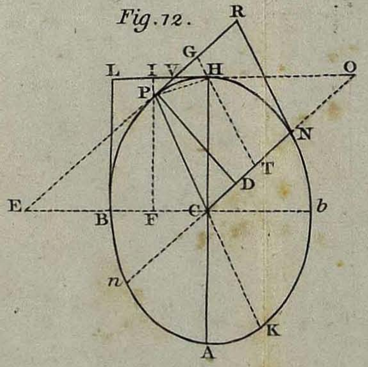

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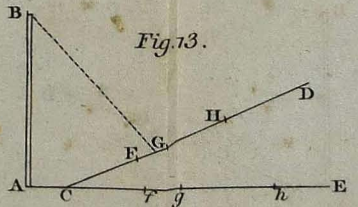

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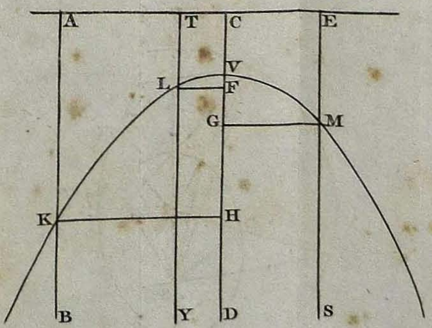

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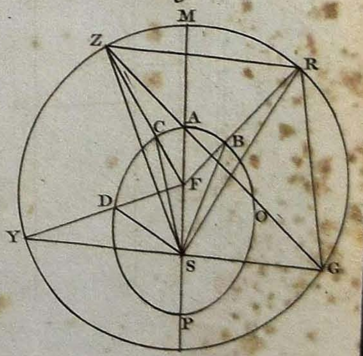

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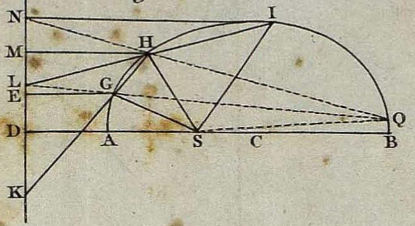

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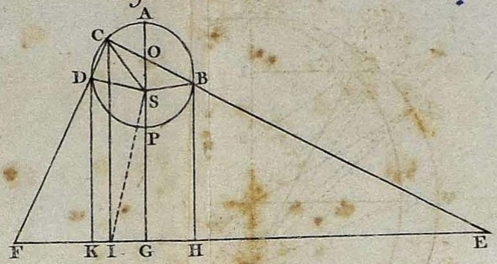

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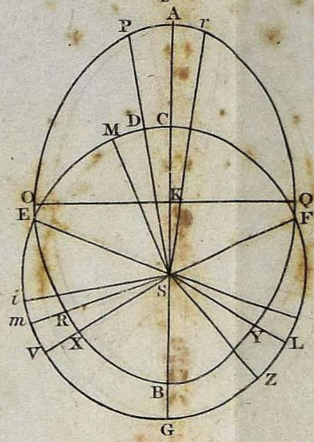

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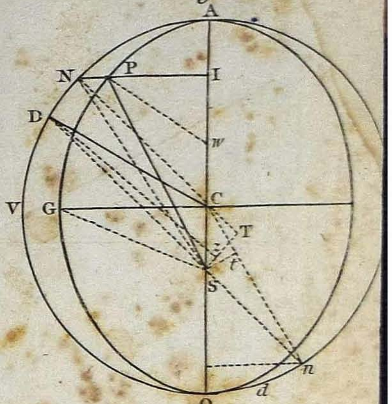

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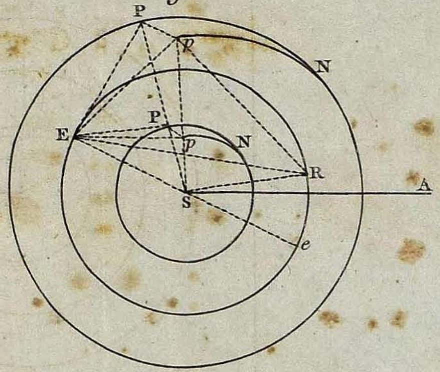

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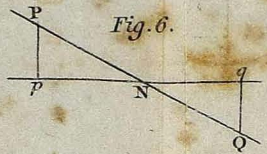

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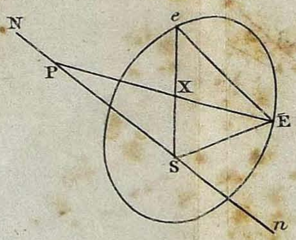

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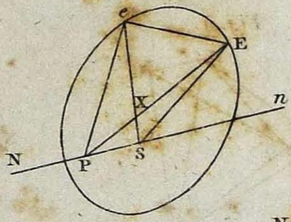

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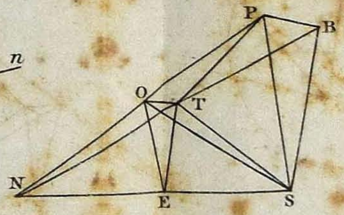

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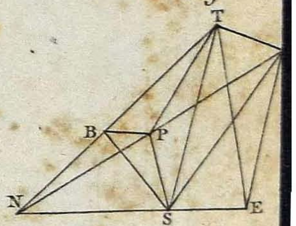

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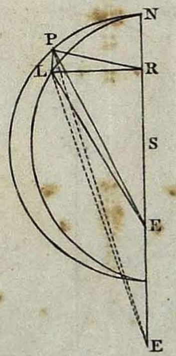

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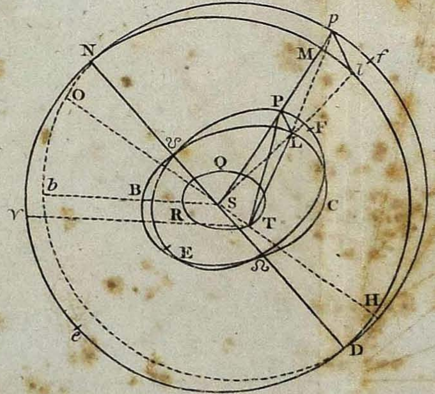

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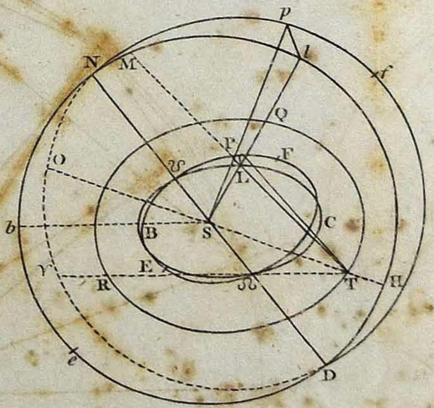

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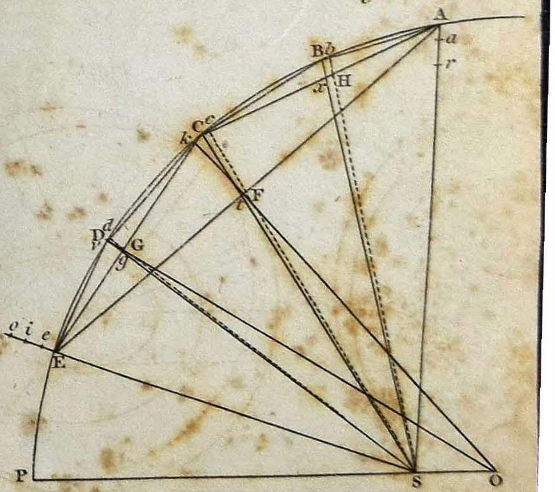

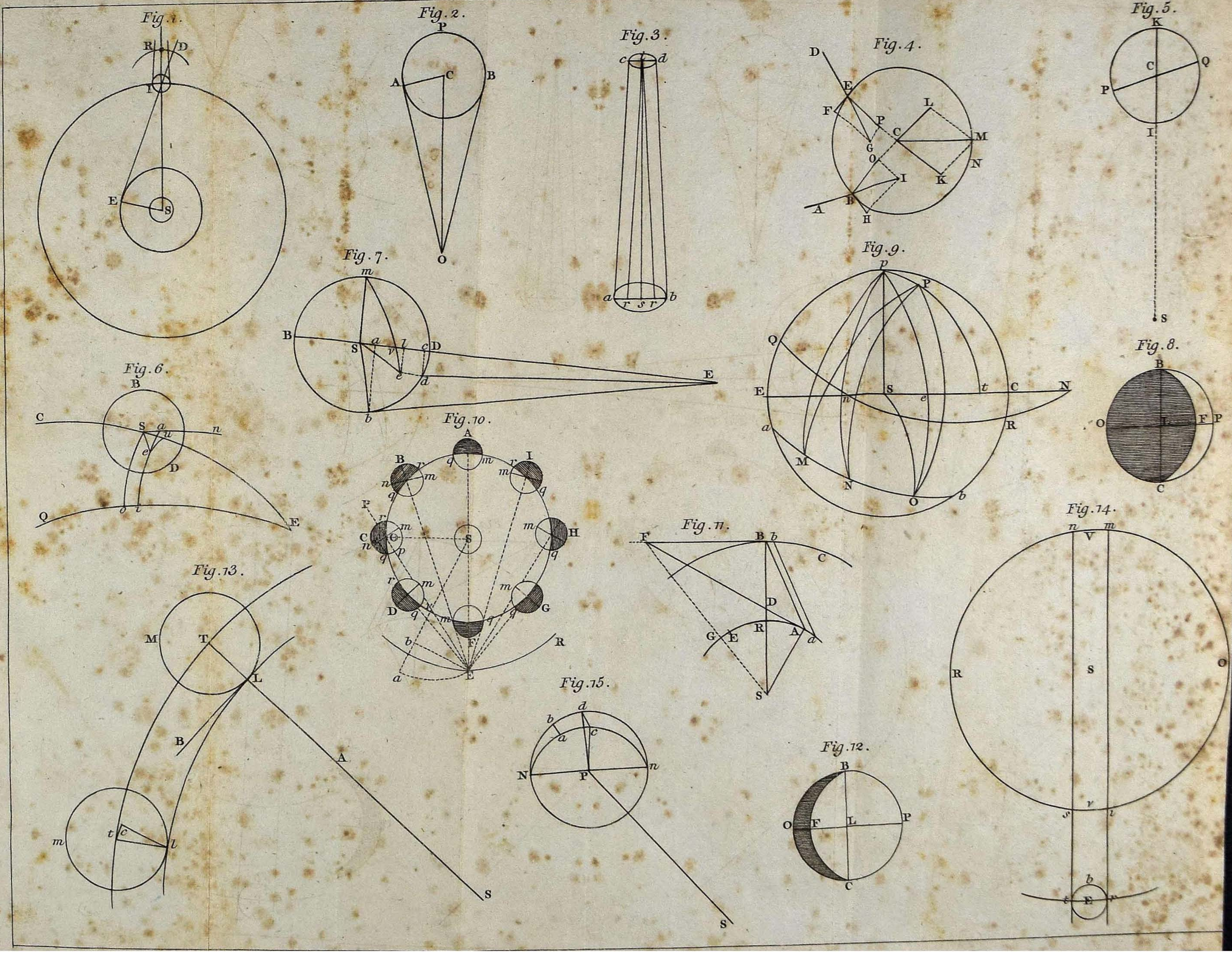
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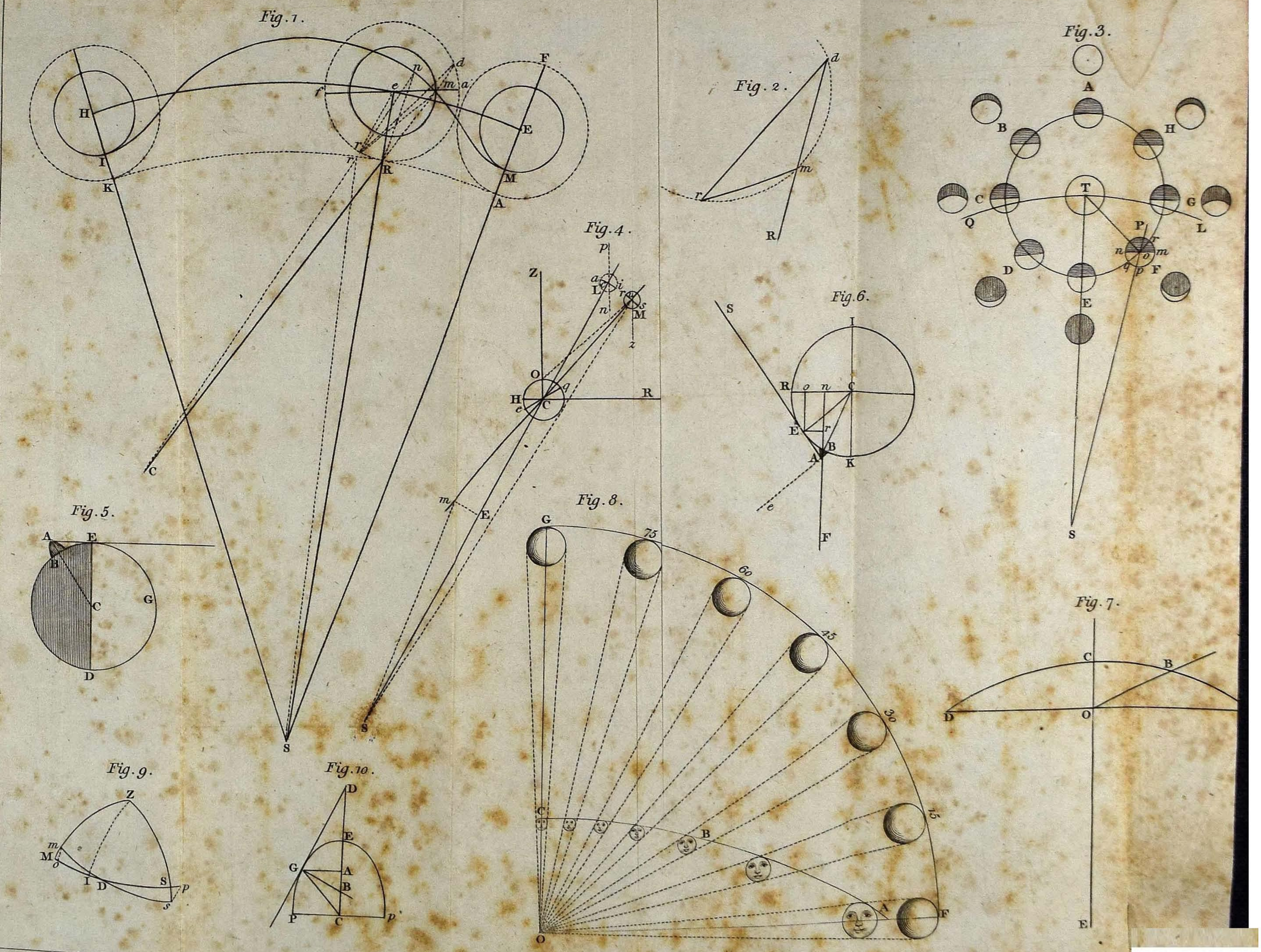
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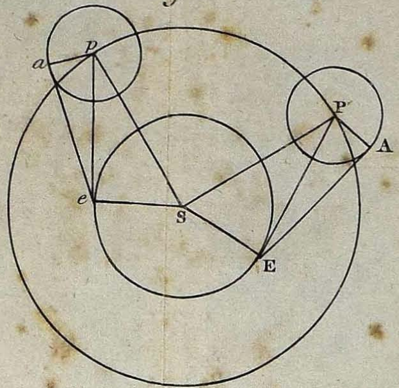

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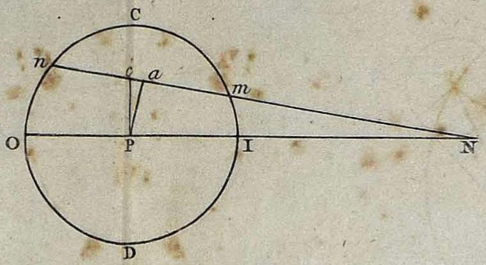

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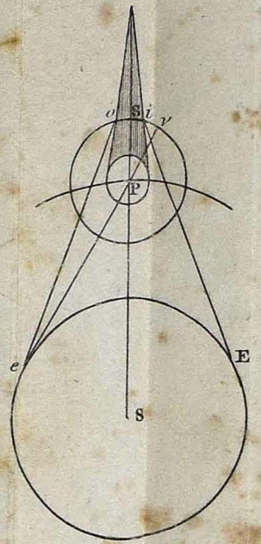

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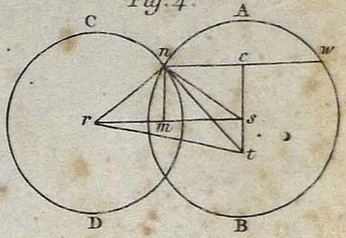

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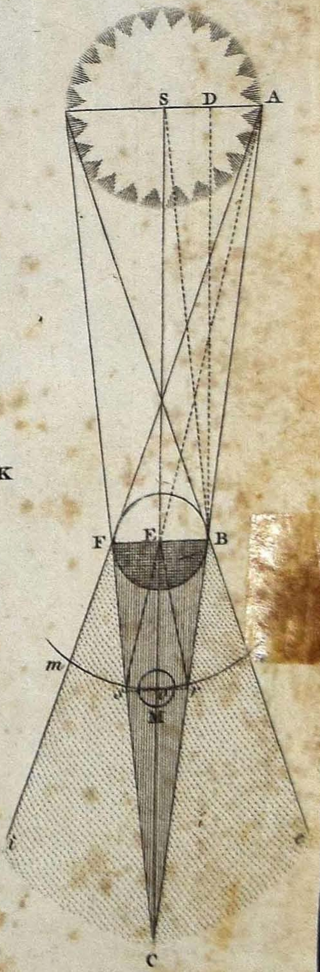

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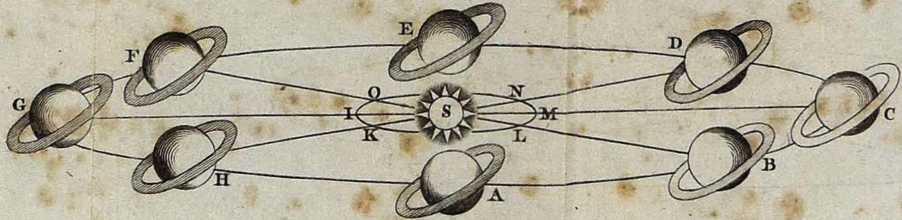

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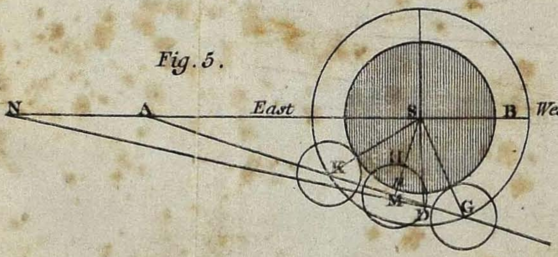

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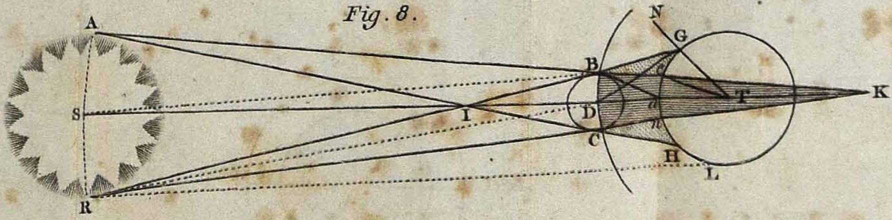


Fig. 9.

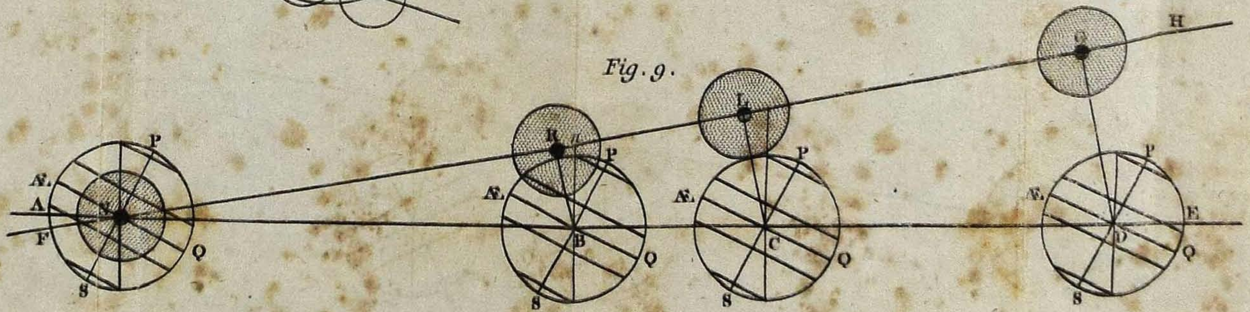


Fig. 1.

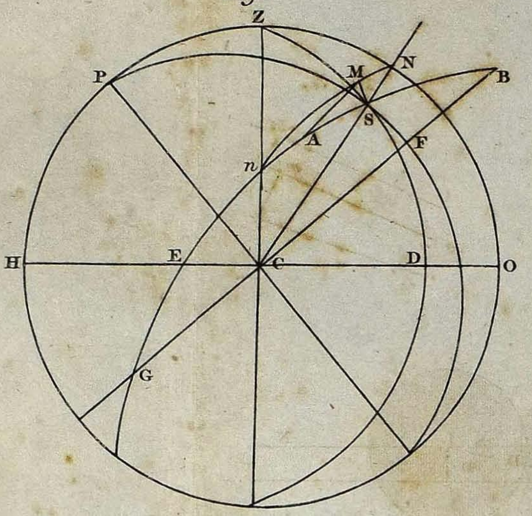
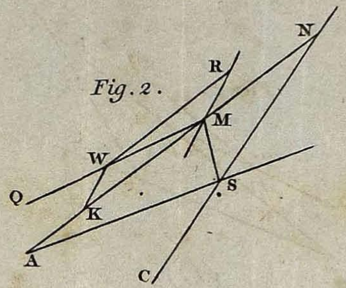


Fig. 2.



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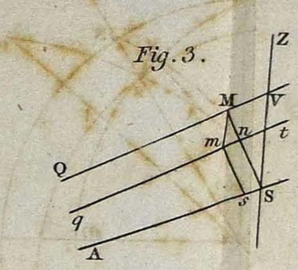

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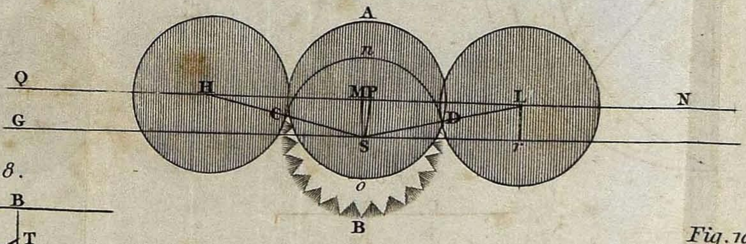

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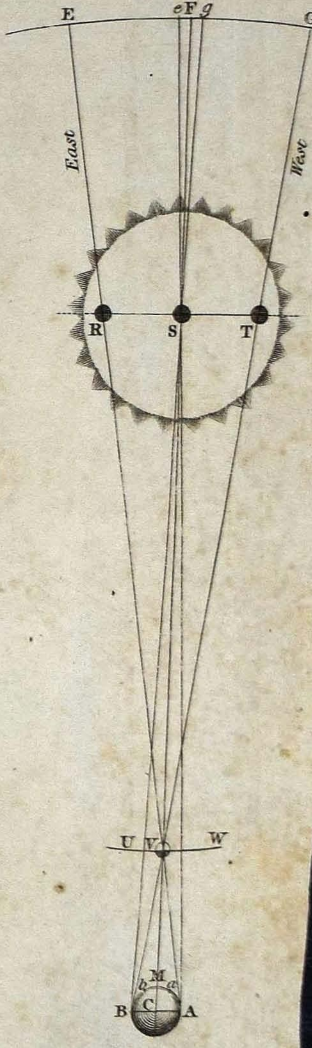

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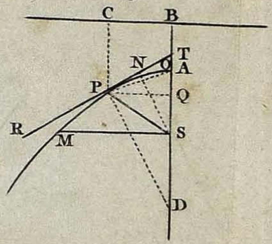

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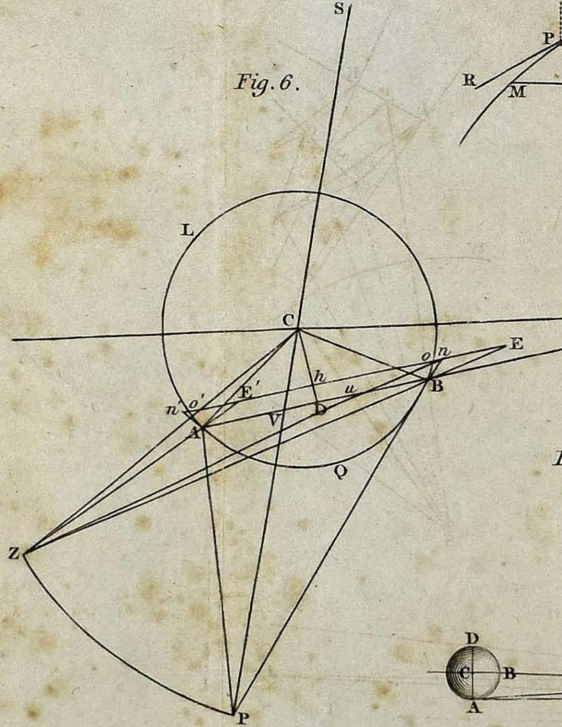

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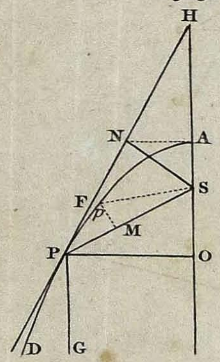

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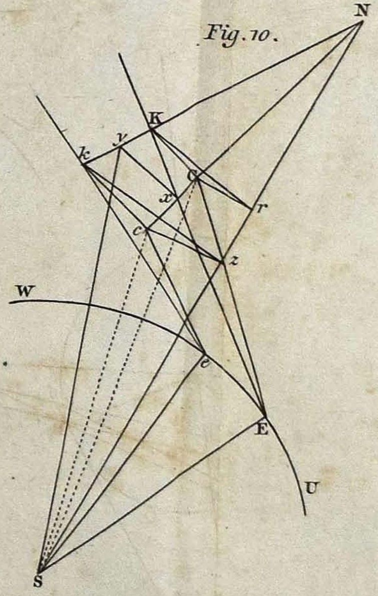

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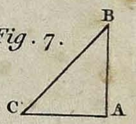

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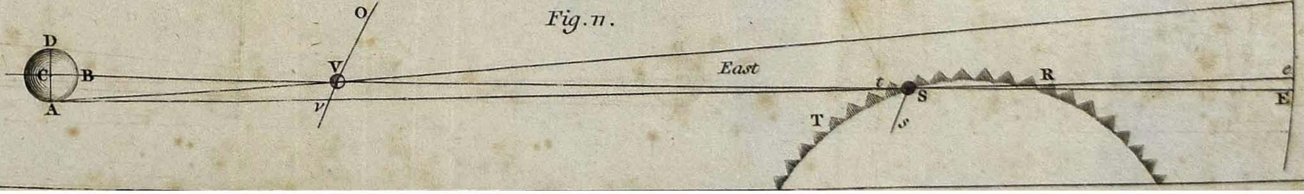
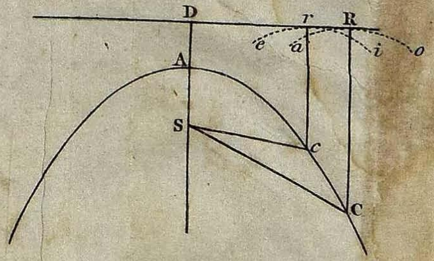


Fig. 1.



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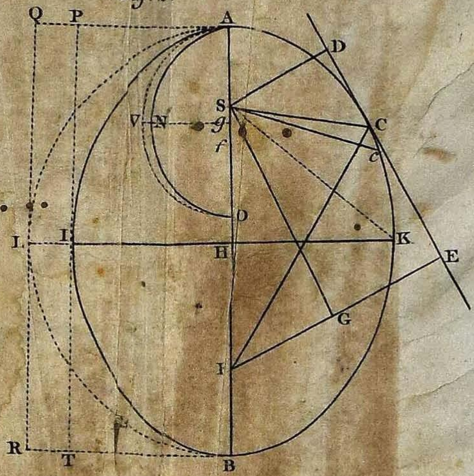

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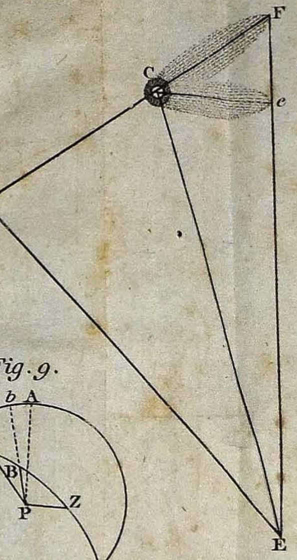

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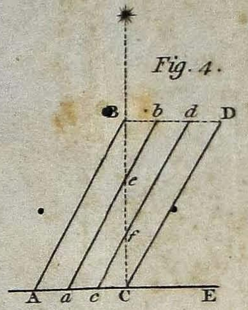

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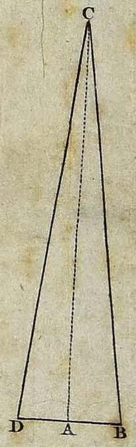

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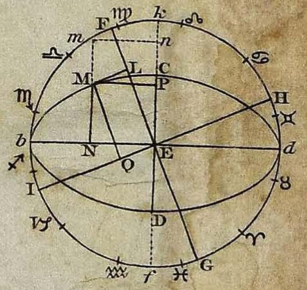

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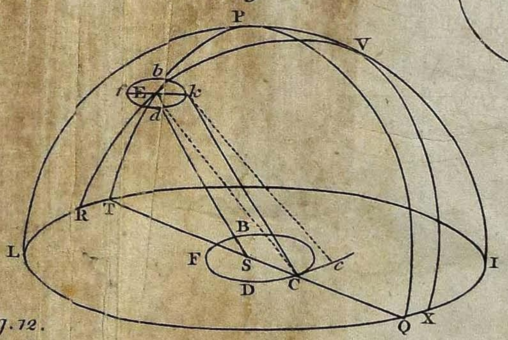

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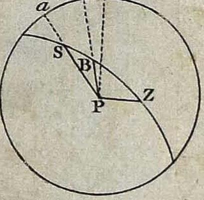

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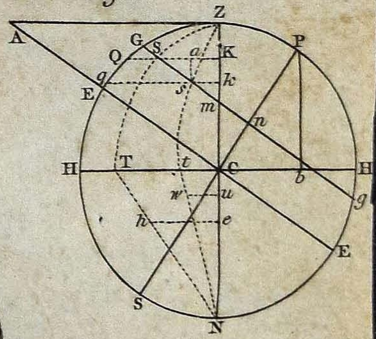

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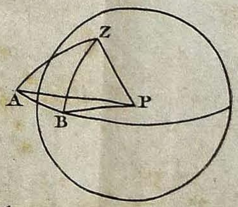

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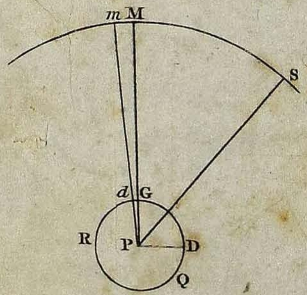

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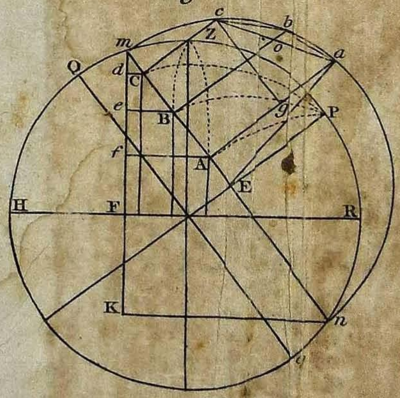

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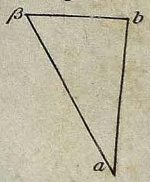

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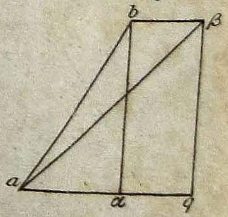

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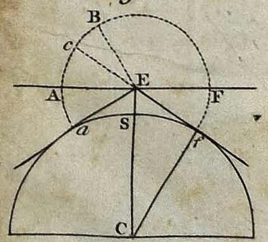
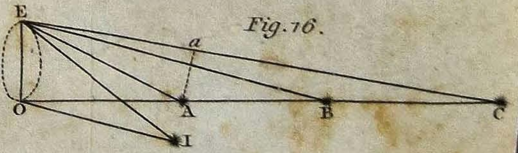
Fig. 16.